Physics 51 "Study Guide" for Final ("Laundry List" of important concepts) Todd Sauke

Concept (important **concepts** in **bold; vectors** also shown in **bold**)	**Symbol or Equation**
Prerequisites:	
Physics quantities are typ. either scalars or **vectors** (magnitude & direction)	*components* of vectors add
From **mechanics**, total external **force** on a body = mass x acceleration	$\Sigma \mathbf{F}_{ext} = m\,\mathbf{a}$ (SI newton, "N")
Mass (SI kilogram, "kg") resists change in motion (via "**momentum**", **p**)	$\mathbf{p} = m\mathbf{v}$, $\mathbf{F}_{ext} = d\mathbf{p}/dt$
A mass moving in a circle undergoes **centripetal acceleration**	$a_{centr} = v^2 / r$
Conservation of linear momentum: Isolated system ($\Sigma\mathbf{F}_{ext} = 0$) → $\Delta\mathbf{p}=0$; $\mathbf{p}_f=\mathbf{p}_i$	$m_1\mathbf{v}_{f1} + m_2\mathbf{v}_{f2} = m_1\mathbf{v}_{i1} + m_2\mathbf{v}_{i2}$
A moving mass has energy of motion, "**Kinetic Energy**" (SI joule, "J")	$KE = \frac{1}{2} m v^2$ (a scalar)
A spring being compressed pushes back proportional to compression	$\mathbf{F} = -k\,\mathbf{x}$
A compressed spring has energy of compression, elastic "**Potential Energy**"	$U = \frac{1}{2} k x^2$
For conservative forces, mechanical energy is conserved	$E=KE+PE=\text{constant}$ ($W_{nc}=0$)
Electromagnetics:	
Electric Charge is the fundamental quantity in Electrostatics	Q (SI coulomb, "C")
Charge is conserved, quantized, and comes in "positive" and "negative"	$e = 1.602 \times 10^{-19}$ C
Like charges repel (**radially**); opposite charges attract; **Coulomb's Law**	$F = 1/(4\pi\varepsilon_0)\, q_1 q_2 / r^2$
The constant ε_0 is numerically related (by definition) to the speed of light, c	$\varepsilon_0 = 10^7/(4\pi c^2) = 8.854 \times 10^{-12}$
All "normal" matter is made up of **protons, neutrons** and **electrons**	Coulomb's $k=1/(4\pi\varepsilon_0)=8.99\times10^9$
Protons have +e charge; electrons have –e. Their mutual attraction holds	$m_p = 1.67 \times 10^{-27}$ kg
everything together. In a conductor, electrons are free to move around.	$m_e = 9.11 \times 10^{-31}$ kg
Total force (**vector**) is the vector sum of individual forces (**superposition**)	$\mathbf{F} = \Sigma\,\mathbf{F}_i$
The Electric field vector is the force per unit charge on a "test charge", q_0	$\mathbf{E} = \mathbf{F}_0 / q_0$ $\quad \mathbf{F} = q\,\mathbf{E}$
For distributions of charge (eg. λ, σ), vector integrate over the distribution	$\mathbf{E} = \int d\mathbf{E} = \int dq/(4\pi\varepsilon_0 r^2)\,\hat{\mathbf{r}}$
Field lines provide a graphical **representation** of **E** (and **B**) fields	E strong where lines are dense
An **Electric Dipole** is a separation of equal magnitude, opposite charges	$\mathbf{p} = q\mathbf{d}$ (**d**=separation - → +)
An Electric Dipole, **p**, in an Electric field, **E**, experiences a torque	$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ $\quad \tau = p\,E \sin(\phi)$
An Electric Dipole oriented in an Electric field has potential energy, U	$U = -\mathbf{p} \cdot \mathbf{E} = -p\,E\cos(\phi)$
Electric Flux; "flow" of **E** through a surface. (d**A** is a **vector** ⊥ to surface)	$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ (through surface)
Gauss's Law expresses the fact that the source of (static) flux is charge	$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = Q_{encl} / \varepsilon_0$
Charge on a conductor at rest resides on its *surface*. Also for conductor→	$\mathbf{E}_{inside} = 0$ (for **static** case)
Use Gauss's Law to determine **E** field for **symmetric** charge distributions	eg. $E = \sigma / 2\varepsilon_0$ (for sheet)
Gauss's Law easily shows E from a line of charge (instead of nasty integral)	$E = \lambda / (2\pi\varepsilon_0 r)$
A symmetric distribution will be easier to solve for E using Gauss's Law	eg. $E=\sigma/\varepsilon_0$ (between two cond. plates)
Electric force from a **static** charge distribution is a **conservative** force	
Work done on "test charge" is path-independent change in **potential energy**	$U=qV$ $\quad U=q_0/(4\pi\varepsilon_0)\Sigma q_i/r_i$
Electric Potential is potential energy per unit charge (SI volt, "V")	$V=U/q$ $\quad V=1/(4\pi\varepsilon_0)\Sigma q_i/r_i$
We always speak of "**potential difference**" (the zero is chosen for convenience)	$V_a - V_b = \int \mathbf{E} \cdot d\mathbf{l}$
The reverse of this is that **E** field is the (minus) gradient of the potential	$\mathbf{E} = -\,\mathbf{Grad}\,(V)$
Equipotential surfaces are everywhere perpendicular to the E field lines	
A **capacitor** (any pair of separated conductors) holds charge per volt	$C = Q / V_{ab}$ (SI farad, "F")
Capacitance depends ONLY on geometry (& what's between conductors)	$C = \varepsilon_0 A / d$ (parallel plates)
When capacitors are connected in **parallel**, the **equivalent capacitance** is:	$C_{eq} = C_1 + C_2 + C_3 + \ldots$
When in **series**, capacitors have an **equivalent capacitance** given by:	$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$
It takes work (energy) to charge a capacitor. W = potential energy, U	$U = \frac{1}{2}Q^2/C = \frac{1}{2}CV^2 = \frac{1}{2}QV$
Energy (U) stored in a capacitor **"resides"** ***in*** **the electric field**	$u = \frac{1}{2}\varepsilon_0 E^2$ (u = U-density)
If the insulation separating capacitor conductors is dielectric, not vacuum:	just replace ε_0 with $\varepsilon = k\,\varepsilon_0$!

Current is the amount of charge flowing through an area per unit time	$I = dQ/dt$ (SI ampere, "A")
In a conductor (non-static case now!), free charges (typ. electrons) can move (with drift velocity, v_d) in response to an Electric field	$I = n\, q\, v_d\, A$ (A = area) (above: n = charge density)
The current per unit area is called **current density** (a vector!)	$\mathbf{J} = n\, q\, \mathbf{v}_d$
Even conductors have resistance to the flow of charge (material dependent)	$\rho = E / J$ (ρ = "**resistivity**")
The resistivity of a material is temperature dependent, typ. increasing w/ T	$\rho(T) = \rho_0 [1 + \alpha (T - T_0)]$
A source of **electromotive force** (emf or $\mathcal{E}$) makes current flow in a circuit	$\mathcal{E}$ is provider of voltage, V
For materials obeying **Ohm's Law**, current is proportional to voltage	$V = I R$
The ratio V/I is called **resistance** and is related to a material's resistivity	$R = \rho L / A$ (SI ohm, "Ω")
Current flowing through a resistor is accompanied by a **voltage drop**	label R w/ '+' & '-' in diagrams!
An "ideal" source of emf supplies a perfectly constant voltage to the circuit	
A real source of emf (eg. a battery) has **internal resistance**, r (voltage drop)	$V_{ab} = \mathcal{E} - I r$
A circuit element with potential difference, V_{ab}, across it and current, I, flowing through it is a source or sink of **power** depending on sign of I	$P=V_{ab}I=I^2R=V^2/R$ (SI watt, "W") (a resistor **always** takes energy out)
When resistors are connected in **series**, the **equivalent resistance** is:	$R_{eq} = R_1 + R_2 + R_3 + \ldots$
When in **parallel**, resistors have an **equivalent resistance** given by:	$^1/R_{eq}= {}^1/R_1+{}^1/R_2+{}^1/R_3+ \ldots$
Series resistors all have the same current; when in parallel, the same voltage	
Kirchhoff's junction rule (based on conservation of charge)	$\Sigma\ I = 0$
Kirchhoff's loop rule (based on conservation of energy)	$\Sigma\ V = 0$
Use Kirchoff's rules to generate equations ("n equations in n unknowns")	
An "ideal" ammeter has zero resistance and measures the current through it	$\Delta V_{ammeter} = 0$
An "ideal" voltmeter has ∞ r_{in} & measures voltage across probed points	$I_{in\_voltmeter} = 0$ (admits **no** current)
When a circuit **("RC circuit")** has a capacitor being charged or discharged by a series resistor, the current and charge are not constant. Kirchhoff's loop-rule equation for the circuit results in a differential equation, the solution of which involves decaying exponentials:	$q =C\mathcal{E}(1-e^{-t/(RC)})= Q_f(1-e^{-t/(RC)})$ $i = I_0\, e^{-t/(RC)}$ (for charging) $q = Q_0\, e^{-t/(RC)}$ $i = I_0\, e^{-t/(RC)}$ (for discharging)
The product of R & C has units of time and is called the "**time constant**" τ	$\tau = RC$, (in $e^{-t/\tau}$)
Magnetic interactions are interactions between **moving** charged particles	
Magnetic interactions are described by the **vector** field, **B** (SI tesla, "T")	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ (**right-hand-rule**)
The magnetic force is **always perpendicular** to **v** (and **B**): no work done	
Just like for **E**, field lines provide a graphical representation of **B** fields	**B** field lines form **closed** loops
Magnetic Flux; "flow" of **B** through a surface (again, just like with **E)**	$\Phi_{mag} = \int \mathbf{B} \cdot d\mathbf{A}$
Gauss's law for magnetism: there are **no "magnetic monopoles"**	$\Phi_{mag}=\oint \mathbf{B}\cdot d\mathbf{A}=0$ (closed surface)
In a uniform **B** field: charged particle goes in circle (or spiral) of radius, R	$R = mv / (qB)$; $\omega = qB / m$
Crossed **E** and **B** fields: velocity selector (when $F_{net}(v) = 0$, no deflection)	$v_{selected} = E / B$
For a current carrying wire in a magnetic field, there is a force on the wire	$\mathbf{F} = \int d\mathbf{F} = \int I\, d\mathbf{l} \times \mathbf{B}$
A current loop of area A & current I in a uniform magnetic field **B** has $\mathbf{F}_{net} = 0$	$\boldsymbol{\mu} = n\ I\ \mathbf{A}$ (for n loops)
But it does experience a torque, $\boldsymbol{\tau}$, in terms of magnetic moment, $\boldsymbol{\mu}$.	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ $\quad \tau = \mu B \sin(\phi)$
The work done by the torque can be described as a potential energy	$U = -\boldsymbol{\mu} \cdot \mathbf{B} = - \mu B \cos(\phi)$
A moving charge q with velocity **v** creates a magnetic field **B** that depends on distance as $1/r^2$, & is perpendicular to both **v** and $\hat{\mathbf{r}}$ the **unit** vector **from** the "source point" (at q) **to** the "field point" (at **B**).	$\mathbf{B} = (\mu_0 / 4\pi)\ q\mathbf{v} \times \hat{\mathbf{r}} / r^2$ $\mu_0 = 4 \pi \times 10^{-7}$
The constant μ_0 is defined so that, together with ε_0, they relate to c	$c^2 = {}^1 / (\varepsilon_0\, \mu_0)$
The total **B** from several moving charges is the vector sum of the fields produced by the individual charges (**superposition**)	$\mathbf{B}=\int d\mathbf{B}=\int(\mu_0/4\pi)\ dq\mathbf{v} \times \hat{\mathbf{r}} / r^2$
The **Biot-Savart law** gives the previous relations in terms of current in wire	$\mathbf{B}=\int d\mathbf{B}=\int(\mu_0/4\pi)\ I\, d\mathbf{l} \times \hat{\mathbf{r}} / r^2$

From this law, the **B** field from a long, straight current carrying wire is: and the **right-hand-rule** gives the direction that **B** curls around the wire	$B = \mu_0 I \,/\, (2\pi r)$ (at distance r)
The force per length between two long, parallel current carrying wires is attractive if currents are in the same direction, repulsive if opposite	$F / L = \mu_0 I I' / (2\pi r)$
B at distance x along axis of conducting loop (N turns, radius a, current I)	$\mathbf{B}_x = \mu_0 N I a^2 / 2 (x^2 + a^2)^{3/2}$
Ampere's Law relates the line integral of **B** around any closed path to the net current through any area bounded (encircled) by the path	$\int \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{enc}$ (closed loop) I_{enc} includes displacement current
We apply Ampere's law to a highly symmetric situation where we can choose the integration loop (through field point P) to have constant B aligned with the path (or ⊥)	Choose integration *path* through P to make the integral easy
The **B** field inside of a long solenoid with n turns per unit length is:	$B = \mu_0 n I$ (near the center)
B inside a toroidal solenoid (N turns) at distance r from symmetry axis:	$B = \mu_0 N I / (2 \pi r)$
B field outside the space enclosed by a tightly wound solenoid is near 0	$B \approx 0$ (outside solenoid)
B field **inside** a long cylindrical conductor of radius R is easy using Ampere's Law	$B = \mu_0 I r / (2 \pi R^2)$ inside cylinder
When magnetic materials are present, there is an effect on the B field	just replace μ_0 with $\mu = K_m \mu_0$!
Changing **B** flux (Φ_B) through a closed loop induces emf (**Faraday's Law**)	$\mathcal{E} = - d\Phi_B / dt$
The minus sign is **Lenz's Law**; induced current or $\mathcal{E}$ **opposes** the change	(lest the universe explode)
A "**search coil**" with N loops of area A, and resistance R, can be used to measure B fields	$Q_{transferred} = B A N / R$
Motional emf is caused by a conductor moving in a magnetic field	$\mathcal{E} = \int (\mathbf{v} \times \mathbf{B}) \bullet d\mathbf{l}$
Associated with the induced emf is an induced **E** field (non-conservative)	$\mathcal{E} = \int \mathbf{E} \bullet d\mathbf{l} = - d\Phi_B / dt$
A time-varying electric field generates a "**displacement current**", i_d	$i_d = \varepsilon \, d\Phi_E / dt$

The fundamental relations between electric and magnetic fields have	$\int \mathbf{E} \bullet d\mathbf{A} = Q_{encl} / \varepsilon_0$
been presented, but are collected together again here to show the unity	$\int \mathbf{B} \bullet d\mathbf{A} = 0$
and symmetry of the relations. They are called **Maxwell's Equations**	$\int \mathbf{B} \bullet d\mathbf{l} = \mu_0 (I_c + \varepsilon_0 \, d\Phi_E/dt)_{enc}$
and together form a **complete basis** for all of **electromagnetism**.	$\int \mathbf{E} \bullet d\mathbf{l} = - d\Phi_B / dt$
For reference, full force equation is repeated here (completes all of E&M)	$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Changing currents in circuits with shared magnetic flux induce emf in one another. **Mutual inductance** M (SI henry, "H") is geometry dependent.	$\mathcal{E}_2 = -M di_1/dt$; $\mathcal{E}_1 = -M di_2/dt$ eg. $M = N_2 \Phi_{B2} / i_1 = N_1 \Phi_{B1} / i_2$
A changing current, i, in any circuit causes a self-induced emf $\mathcal{E}$	$\mathcal{E} = -L \, di/dt$
The self **inductance**, L (SI henry, "H") is enhanced by number N of loops	$L = N \Phi_B / i$
An **inductor** is a circuit device intended to have substantial inductance	
The "–" sign (again Lenz's Law) means that an inductor opposes current change	again lest the universe explode
An inductor (L) carrying current I has **energy associated with the B field**	$U = \frac{1}{2} L I^2$
If what's inside an inductor is not vacuum, replace μ_0 with μ (permeability)	$u = \frac{1}{2} B^2 / \mu_0$ (u = U-density)
When a circuit **("RL circuit")** has an inductor connected to an emf and a series resistor, the current is not constant. Kirchoff's loop-rule equation for the circuit gives a diff. eq., with decaying exponential solutions.	$i = \mathcal{E}/R(1-e^{-tR/L}) = I_f(1-e^{-tR/L})$ $i = I_0 e^{-tR/L}$ (with no emf) $\tau = L/R$, (in $e^{-t/\tau}$)
When a circuit **("LC circuit")** has an inductor (L) and capacitor (C), it has electrical oscillations **analogous** to a mechanical harmonic oscillator	$i = I \cos(\omega t + \phi)$ $\omega = \sqrt{(^1/_{LC})}$ (**ω in radians / s**)
L is analogous to mass m; $^1/_C$ is like the spring constant k; q is like x; i like v	$U = \frac{1}{2} L I^2$ is like $KE = \frac{1}{2} m v^2$
Many other analogies hold; L i (= L $^{dq}/_{dt}$) is like m $^{dx}/_{dt}$ = mv (momentum) and it requires an **electromotive force** (voltage) to change it (like **force** changing **p**)	F=ma (=m $^{dv}/_{dt}$) is like V=L $^{di}/_{dt}$
For the LC circuit, the energy exchanges back and forth (in the E, B fields)	label L and C with '+' & '-'
When you add resistance R **("LRC circuit")** the oscillations (of ω') decay in time	$i = I e^{-tR/(2L)} \cos(\omega' t + \phi)$
For small enough R, circuit is "underdamped"; for $R^2 = {}^{4L}/_C$ → "critical damping", ω'=0	$\omega' = \sqrt{(^1/_{LC} - R^2/(4L^2))}$

An ac source produces an emf that varies sinusoidally with time. We can	$I_{rms} = I_{max} / \sqrt{2}$
represent an sinusoidal voltage or current with a **phasor vector** rotating	$V_{rms} = V_{max} / \sqrt{2}$
counterclockwise with ang. freq. ω. Instantaneous quantity is x projection	$i = I \cos(\omega_{applied}\, t)$
In general, the (sinusoidal) voltage can be "out of phase" with the current	$v = V \cos(\omega_{applied}\, t + \phi)$
The voltage v across a resistor is "**in phase**" with the i (power is dissipated)	$V_R = I R$
The v across an inductor "**leads**" the current i by 90˚ (no power dissipated)	$V_L = I X_L$
The v across a capacitor "**lags**" the current i by 90˚ (no power dissipated)	$V_C = I X_C$
X_L and X_C are the inductive and capacitive "**reactance**", respectively	$X_L = \omega L$; $X_C = {}^{1}/_{\omega C}$
In a general ac circuit, the v and i amplitudes are related by "**impedance**"	$V = IZ \quad Z = \sqrt{(R^2 + [X_L - X_C]^2)}$
and the **phase angle ϕ** is seen from the phasor diagram to be given by:	$\tan(\phi) = (\omega L - 1 / _{(\omega C)}) / R$
Impedance is frequency dependent, making high-pass, low-pass circuits	(Z and ϕ above for *series* circuit)
The average power delivered to an ac circuit depends on the amplitudes	$P_{ave} = \frac{1}{2} V I \cos(\phi)$
of the v and i and on the phase angle ϕ (like a dot product of **phasors**)	or $P_{ave} = V_{rms} I_{rms} \cos(\phi)$
In the RLC circuit, the i is maximum (Z minimum) at "**resonance**"	$\omega_o = \sqrt{({}^{1}/_{(LC)})}$
At the resonant frequency, i and v are "in phase", total reactance = 0 and:	$Z = R$ (at resonance)
A **transformer** transforms the voltage and current levels in an ac circuit.	
The ratio of V_{in} (V_1) to V_{out} (V_2) is = to the turns-ratio (ideal transformer)	$V_2 / V_1 = N_2 / N_1$
The current ratio is opposite, so that [power in] = [power out]	$V_1 I_1 = V_2 I_2$
Maxwell's equations predict the existence of **electromagnetic waves** that	
propagate in vacuum at the **speed of light c**. That's what **light** *is*.	$c = \sqrt{{}^{1}/(\varepsilon_0\, \mu_0)} = 2.998 \times 10^8$ m/s
E and B are uniform over a plane perpendicular to propagation (plane wave)	$E = c B$
Electromagnetic waves are **transverse** w/ "in phase" sinusoidal field	$E(x,t) = E_{max} \cos(kx - \omega t)$
oscillations. **E**, **B** are ⊥ to each other and the propagation direction.	$B(x,t) = B_{max} \cos(kx - \omega t)$
"k" is the **wave number** of the propagating electromagnetic wave	$k = 2\pi/\lambda \qquad E_{max} = c B_{max}$
There are two possible **polarizations** of light for x-propagation (E in y or E in z)	
For an electromagnetic wave traveling in a dielectric (eg. glass) v < c	$v^2 = {}^{1} / (\varepsilon\, \mu) = c^2 / K K_m$
The period T, wavelength λ, and frequency *f* are related to wave speed v:	$f = {}^{1}/_{T} \qquad v = {}^{\lambda}/_{T} = f\lambda$
The **energy density** of a traveling electromagnetic wave is ½ in E, ½ in B	$u_{tot} = u_E + u_B = 2\, u_E = \varepsilon_0 E^2$
Poynting vector S is energy flow rate (power per unit area) for EM wave	$\mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0$
The magnitude of the time-averaged value of S is called **intensity** I of wave	$I = S_{av} = E_{max} B_{max} / (2\mu_0) = E_{max}^2 / (2\mu_0 c)$
Electromagnetic waves also carry momentum, exerting **radiation pressure**	$p_{rad} = I / c$ (⊥ absorbed)
The radiation pressure p_{rad} of full intensity sunlight is small but measurable	$p_{rad} = 2 I / c$ (⊥ reflected)
The **flow rate of electromagnetic momentum** (**p**) is related to S and c	${}^{1}/_{A}\, dp/dt = S / c = EB / (\mu_0 c)$
If a perfectly reflecting surface is placed at x=0, the incident and reflected	E-nodes at $kx = n\pi$
waves add up to form a "**standing wave"** with **nodal planes** of **E** & **B**	B-nodes at $kx = (n + \frac{1}{2})\pi$
Standing E-M waves have stationary sinusoidal oscillations	$E_y(x,t) = -2E_{max} \sin(kx) \sin(\omega t)$
There is an E-node at the mirror: There can be no tangential E on a conducting surface	
The E & B variations for a standing E-M wave have **E** & **B** "out of phase"	$B_z(x,t) = -2B_{max} \cos(kx) \cos(\omega t)$
Because the E and B are "out of phase", there is no net flow of energy	$I = S_{av} = 0$ (standing wave)
If there is **another** mirror parallel to the first one, there **must** be an E-node **there** too. This **optical cavity** is the basis of the **laser**, and can only support the **special** wavelengths of light that have nodes at both ends.	
The different **colors** of light correspond to different wavelengths λ	$\lambda = c / f = 2\pi c / \omega$
Visible light is but a small part of the **spectrum** of electromagnetic waves	$\lambda_\gamma < \lambda_{xray} < \lambda_{uv} < \lambda_{\mathbf{vis}} < \lambda_{ir} < \lambda_{\mu wav} < \lambda_{tv/radio}$