Objective:

To use a ballistic pendulum to determine the velocity of a projectile. To verify this velocity by measuring the range of the projectile.

Experiment:

1. We use measurements obtained from a ballistic pendulum to review energy conservation, momentum conservation, and projectile motion. From the measured recoil speed of the pendulum we determine the speed of a ball fired by a spring-loaded cannon. We check this speed by measuring the range of a ball fired horizontally from the cannon from the bench top to the floor.

2. The spring-loaded cannon fires a steel ball with sufficient speed to shatter eyeglasses. Use appropriate caution. Don’t look down the barrel; you can see from the side whether the spring is cocked and if the ball is in place. Don’t leave a cocked and loaded cannon on the bench.

3. Cock the spring with the ball in the barrel. Use the ramrod to push the ball against the plunger until the trigger mechanism latches in one of its three range positions. To fire, pull the trigger away from the barrel. Hold the base firmly against the bench top to prevent recoil of the cannon and pendulum apparatus.

4. To determine the initial velocity of the steel ball, we apply two conservation laws, the conservation of momentum (during the collision) and the conservation of energy (after the collision).

   Conservation of momentum relates the velocity of the ball before the collision with the velocity of ball plus pendulum after the collision, according to

   \[ m_b v_0 = (m_b + m_p) v_f \]

   Conservation of energy relates the initial (kinetic) energy of the pendulum to its final (potential) energy, according to

   \[ \frac{1}{2} (m_b + m_p) v_f^2 = (m_b + m_p) gh \]

   where \( h \) is the maximum height reached by the center of mass of the pendulum.

   Using these two relations, we can determine the velocity \( v_0 \) of the ball when it is shot from the cannon.
Conservation of linear momentum
(Before and immediately after the collision)

\[ m_b v_0 = (m_p + m_b) v_r \]

Initial velocity \( v_0 = \frac{(m_p + m_b)}{m_b} v_r \)

Conservation of energy
(During the swing of the pendulum arm)

\[ \frac{1}{2} (m_p + m_b) v_r^2 = (m_p + m_b) gh_{cm} \]

Recoil velocity \( v_r = \sqrt{2gh_{cm}} \)

Procedure:

1. Remove the pendulum from the stand by unscrewing the pivot axle. Weigh and record the masses of the pendulum \( m_p \) and the steel ball \( m_b \).

2. With the ball latched in the catcher, balance the pendulum on a ruler or pencil to locate the center of mass. Measure and record as \( r_{cm} \), the distance from the pivot axis to the center of mass.

3. Reattach the pendulum to the stand. Check that it swings freely. With the pendulum hanging vertically, move the angle pointer to just contact the pendulum arm. Record this angle as \( \theta_0 \).

4. With the pendulum latched at 90°, insert the steel ball and use the ramrod to cock the spring to its middle or maximum range position. Let the pendulum hang vertically and move the angle pointer to \( \theta_0 \). While holding the base firmly against the table fire the cannon by pulling up on the trigger. Write down the maximum angle recorded by the angle indicator.

5. Repeat this process five times to check the reproducibility of the apparatus. Be sure to cock the cannon to the same setting each time. To minimize drag on the pendulum set the angle indicator initially to about 5° below the maximum recoil angle. Record the maximum recoil angle, \( \theta_{max} \), for each shot.

Experiment 8 Ballistic Pendulum
**Analysis:**

1. Compute the average value of the recoil angle \( \theta = \theta_{\text{max}} - \theta_0 \).

2. Find the change in height of the center of mass \( h_{cm} = r_{cm}(1 - \cos \theta) \).

3. Calculate the recoil velocity \( v_R = \sqrt{2gh_{cm}} \) being sure to use three significant figures for \( g \); \( g=981 \text{ cm/s}^2 \).

4. Calculate the initial velocity of the steel ball \( v_0 = \left( \frac{m_p + m_b}{m_b} \right) v_R \). Express this quantity as a value plus or minus its uncertainty, thus: \( v_0 \pm \Delta v_0 \).

5. Now remove the pendulum and fire the ball horizontally. (Find an appropriate place to shoot the ball, making sure it does not hit anyone.) Observe where the ball hits the floor. By measuring the horizontal distance (range) of the projectile, you can determine its initial velocity. This gives you an independent check on the velocity \( V_0 \) with which the ball is fired by the cannon.

6. The range is given by \( x = v_0 t \)

where \( V_0 \) is the initial velocity of the ball and \( t \) is the time of flight. The time of flight can be determined from the fact that the distance the ball falls vertically is given by \( y = \frac{1}{2}gt^2 \).
7. Once you have roughly determined the range x where the ball hits the floor lay a sheet of paper on the floor. Use masking tape to keep it from moving. Lay a sheet of carbon paper on top of the page. Fire the cannon five or six times to make a reasonable pattern of dots on the page. Use the center of the cluster of dots to determine $x$, the range.

8. Calculate $v_0$, the initial velocity from the range and express it as a value with uncertainty, thus: $v_0 \pm \Delta v_0$.

9. Do the two values of the velocity of the projectile agree to within measurement uncertainty? (That is, is the percent difference between the two values less than the sum of the percent uncertainty in each value?)

10. Finally, compute the percentage of the ball’s initial kinetic energy that was lost when the ball collided with the pendulum arm.

$$\text{Percentage of KE lost is} \quad 1 - \frac{\frac{1}{2} (m_b + m_p) v_r^2}{\frac{1}{2} m_b v_0^2} \times 100\%$$

**Report:**

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1) A neat and organized presentation of all measured and calculated values.
2) A calculation of the percent uncertainty in the two values of initial velocity and the percent difference between these values. List the sources of error that contributed to this percent difference.
3) A final comment on whether or not mechanical energy was conserved. If not, is that what you would expect in this type of a collision?