A Study of the Most Efficient Earth-to-Titan Orbit Trajectory Using the N-Body Problem

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ABSTRACT

A Study of the Most Efficient Earth-to-Titan Orbit Trajectory Using the N-Body Problem

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This paper explores the theory of sending a satellite to orbit Saturn's moon, Titan, using the N-body problem. The study examines what is the most efficient flight path to entire the Titan atmosphere, and what makes that specific flight path efficient. The equations of relative motion and related orbital mechanics equations are used to accomplish these objectives. The N-body simulation is run in MATLAB and simulates the Solar System as the satellite makes its path.

Nomenclature

a	= semi-major axis
ω	= angular velocity
e_h	= eccentricity
G	= Universal Gravitational Constant
h	= angular momentum
M	= Mass
Q	= Spacecraft
r	= radius
SOI	= Sphere of Influence
v	= velocity
v_a	= arrival velocity
v_d	= departure velocity
v_∞	= excess velocity
δ	= turn angle
π_1	= mass ratio
ϕ_a	= angle of arrival velocity
ϕ_d	= angle of departure velocity
θ	= true anomaly
$\theta_{initial}$	= Initial Phase Angle
θ_{final}	= Final Phase Angle
n_{planet}	= mean motion
τ	= period
t_{12}	= Synodic Period

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1.0 Introduction

1.1 Motivation

The current problem is developing the most efficient flight path from Earth to orbit Titan using N-body simulations. Titan is the largest of Saturn's 82 moons. Titan is believed to have the potential to sustain life, being a celestial body that possesses lakes and seas filled with visible fluid. The distance between the Sun and Titan is approximately 1.4 billion km, meaning the sunlight is not as present as on Earth [1]. Multiple missions prior, such as the Cassini spacecraft, have provided a significant amount of information that allows for a reason for further study and exploration on Titan.

There have been plenty of studies about the two-body problem, the three-body problem, and the four body problem for transfer design. However, the question has not been asked if it is possible to create an n-body simulation for an efficient flight path from Earth to Titan. This paper will uncover the answer to this problem.

1.2 Literature Review

1.2.1 Stability of P-type orbits around stellar binaries: An extension to counter-rotating orbits

Chaelin Hong and Maurice H. P. M. van Putten explored the idea of counter-rotating planetary orbits around stellar binaries. The motivation behind this concept is that an understanding of circumbinary systems can be reached. In order to reach this understanding, Hong and van Putten first discuss the initial conditions of P-type orbits. These conditions are the positions of three bodies, which all have a coplanar configuration. Studying the dynamical stability of this three-body problem, initial distance and angle can be found in order to answer if the orbit is stable or unstable. The equations of motion are explored next in the two-body and three-body problems. The importance of these referenced problems is to establish which equations are to be used in order to accurately determine the stability of the orbits. Using an N-body simulation in MATLAB, the results of the study show an increase in the upper critical orbit (UCO) and the lower critical orbit (LCO), which are the orbital radius of the third object. Hong and van Putten concluded that counter rotating orbits are more stable than corotation orbits. This assessment is essential in uncovering the habitability based on the P-type orbit stability changes [2].

1.2.2 Numerical multistep methods for the efficient solution of quantum mechanics and related problems

Zacharias A. Anastassi and Theodore E. Simos present an in-depth study of linear multistep methods and hybrid methods. The difference between linear multistep methods and hybrid methods is that the main objective of the hybrid method is to retain the amount of steps in the method while optimizing it, whereas linear multistep methods use additional approximations to increase algebraic order. Both methods were tested on the Schrödinger equation, the N-body problem, the inhomogeneous equation, the nonlinear equation, and the two-body problem that uses various low eccentricities. These five systems of ordinary differential equations (ODE) were tested to see if they have oscillatory solutions. Each ODE system had a chart that displayed the efficiency for each method. The mathematical steps to each system were presented with the theory behind each of the five systems of ODEs. Anastassi and Simos concluded that high order of fitting provides efficient data for the Schrödinger equation integration. They also concluded that the method for an oscillatory problem, such as the N-body problem, needs to have the key aspect of a long interval of periodicity for integration. The third point the authors established was that the high order fitted explicit symmetric method was the most efficient for the inhomogeneous equation, nonlinear equation and the low eccentricity two-body problem [3].

1.2.3 Dynamics of the terrestrial planets from a large number of N-body simulations

Rebecca A. Fischer and Fred J. Ciesla examine the dynamic properties of the Solar System. The incentive behind this research was to understand historical accretion ranges of the planets under various dynamic circumstances. The second objective in this research was to evaluate how applicable N-body simulations are in understanding the formation of Earth. In order to accomplish this task, Fischer and Ciesla performed 100 N-body simulations of the planet accretion. This is accomplished by calculating the gravitational interactions of celestial bodies that evolve into a planetary formation. The orbital configurations that were tested were Jupiter and Saturn with two different cases, The first is the Eccentric Jupiter and Saturn (EJS) case, which are planets current orbits. The second is the Circular Jupiter and Saturn Case (CJS), where both planets are given non-eccentric orbits. The N-body simulations resulted in accurate values for the EJS configuration. Fischer and Ciesla concluded that the formation of Earth is correlated with the late veneer mass and the timescale of Earth. This study helps promote a deeper understanding and method development for N-body simulations [4].

1.2.4 Plasma environment at Titan's orbit with Titan present and absent

Wei, Russell, Wellbrock, Doughert, and Coates performed a study on Titan's plasma environment when Titan is present and absent. The objective behind this study was to understand the influence of the plasma environment on Titan. This was done by examining the magnetic field measurements that were obtained from the Cassini magnetometer and the Cassini plasma spectrometer. The authors concluded that Titan being present displays an effective impact on its plasma environment [5].

1.2.5 An empirical model for the plasma environment along Titan's orbit based on Cassini plasma observations

Todd Smith and Abigail M. Rymer modeled the plasma environment along Titan's orbit. Titan was identified as having a dense atmosphere of nitrogen that Cassini observed while in Saturn's atmosphere. Smith and Rymer provided evidence that Saturn's magnetopause was impacted by the location of Titan. The technique to detect ionization particles leaving Titan depends on the surrounding plasma environment. There were four plasma categories formed after the analysis of 54 Titan interactions. The categories are the plasma sheet, lobe-like, magnetosheath, and bimodal plasma. Smith and Rymer created an empirical model based upon Saturn local time that applies the probability of each category [6].

1.2.6 Titan aerogravity-assist maneuvers for Saturn/Enceladus missions

Ye Lu and Sarag J. Saikia developed a design method for a Saturn and Enceladus mission using Titan aerogravity assist (AGA) maneuvers. AGA refers maneuvers where the spacecraft uses a hyperbolic trajectory to enter and exit the atmosphere. This study can be broken down into three sections: arrival, atmospheric flight, and post-orbit. When designing the trajectory, research needs to be done on Titan. Titan's orbit about Saturn is almost circular with near zero inclination and axial tilt to Saturn. This allows for a spacecraft to interact with Titan at different points. The AGA maneuver will have an impact on the excess velocity that is dependent on the direction of the orbital velocity. The two missions have differences in the approach, however, Lu and Saikia concluded that AGA is possible for both a Saturn mission and an Enceladus mission. The promising aspect of this study is the foundation of mission design and trade analysis as another method for interplanetary travel, instead of other methods such as gravity assists or moon tours [7].

1.2.7 Reduction of Saturn Orbit Insertion Impulse using Deep-Space Low Thrust

Elena Fantino, Roberto Flores, Jesus Pelaez, and Virginia Raposo-Pulido establish a system to reduce the hyperbolic excess velocity to Saturn using electric propulsion. The motive behind this study is that understanding planets such as Jupiter and Saturn can potentially lead to understanding how the Solar System was formed. The missions dedicated to these planets require a substantial amount of propellant, thus, finding the most efficient flight path while searching for alternative propulsion systems would reduce costs greatly. The trajectory that was designed is an Earth to Saturn path that involves a gravity assist at Jupiter. The strategy to reduce excess speed from Jupiter to Saturn is a low-thrust (LT) transfer. The mission requirements for this study involved the transfer time from Earth to Jupiter to be within a three year time frame and the post gravity assist thrust should not last more than four years. The authors obtained results that would have the lowest propellant budget and had a total mission time of 13 years. The assumption of the spacecraft had a mass of 1000 kg. This design presents possible trajectories for interplanetary travel and introduces an inexpensive strategy to go to Saturn [8].

1.2.8 The spatial Hill four-body problem I—An exploration of basic invariant sets

In Jaime Burgos-Garcia, Abimael Bengochea, and Luis Franco-Perez's study of the spatial Hill four-body problem, the authors examine the basic invariant sets. Since the spatial Hill three-body problem is insufficient in modeling dynamics of specific celestial bodies, the objective behind this research is to build upon the three-body problem. This paper introduces the restricted four-body problem (R4BP), where three points with gravitational forces come into contact with a massless particle. The equations of motions involved have the assumption that the three points move in circular orbits and have a constant angular velocity. The invariant sets are examined and were computed numerically, reaching a possibility that polar orbits bifurcate. The authors developed the study of the four-body problem that has an emphasis on equilibrium points and symmetric periodic orbits. The symmetric periodic orbits were numerically solved with boundary value problems. The Jacobi constant was found to have multiple bifurcations at various values for mass. It was determined that this study needs further development to reach a more comprehensive understanding [9].

1.2.9 Comparisons between the circular restricted three-body and bi-circular four body problems for transfers between the two smaller primaries

Allan Kardec de Almeida Junior and Antonio Fernando Bertachini de Almeida Prado discuss the distinctions between the circular restricted three body problem (CR3BP) and the bi-circular four body problem (4BP) and develop a method that measures those differences. One major factor of this study is the inclusion of the Sun in the equations of motion. The four systems that were looked at were the Sun-Earth-Moon system, the Sun-Mars-Phobos system, the Sun-Saturn-Titan system, and the Sun-Ida-Dactyl system. The orbit transfer for all four systems involves a bi-impulsive maneuver, where two impulses are initiated, the first is at the main celestial mass, and the second is at a certain point in the orbit of the corresponding moon. The authors concluded that the shorter the distance of the spacecraft from the Earth-moon barycenter, the magnitude of the perturbation is lower. The Sun-Ida-Dactyl system had the fewest distinctions between the CR3BP and the 4BP, while the Sun-Mars-Phobos system had the most differences. The analysis drawn from these results is that the higher the cumulative mass of the celestial bodies, the more differences are found [10].

1.2.10 Orbital dynamics in the planar Saturn-Titan system

In Euaggelos E. Zotos' paper "Orbital dynamics in the planar Saturn-Titan system", Zotos explores the orbital dynamics of varying bodies in orbit about the Saturn and Titan system. The research involves understanding how escaping orbits is not a concept that has been thoroughly studied. The model used for this project was the planar circular restricted three-body problem (PCRTBP). This involves two primaries with circular orbits and a particle that interacts with the primaries on the same plane. The variable that is changing in this study is the Jacobi constant. The first conclusion Zotos drew is that Jacobi constants with high values correlate with collisional orbits. Jacobi constants with low values correlate to orbits escaping. The second conclusion drawn is that an increasing Jacobi constant means an increasing collisional time, while a decreasing Jacobi constant lowers the average escape time. The results and conclusions presented further advances the current information about the Saturn-Titan system and its corresponding orbital dynamics [11].

1.2.11 Generalizing the restricted three-body problem. The Bianular and Tricircular coherent problems

In Gabern and Jorba's paper "Generalizing the restricted three-body problem. The Bianular and Tricircular coherent problems", the authors create two models with the Sun, Jupiter, Saturn and Uranus and have a particle interact with the gravitational forces. The reason behind this study is that the restricted three-body problem is not as accurate as it does not include specific conditions. The first dynamic model is the Sun-Jupiter-Saturn system. The second model is the Sun-Jupiter-Saturn-Uranus system. The N-planetary problem is examined and is defined as a planar problem N bodies revolve around the main body in orbit. Simulations were run for the N-body problem for the motion of an asteroid in the models. Quasi-periodic solutions have been found for both the Sun-Jupiter-Saturn system and the Sun-Jupiter-Saturn-Uranus system. Gabern and Jorba state that the models created still resemble the restricted three-body problem too closely, and cannot be considered as an alternative. The models are to be viewed as an optimization and simplification of certain aspects of the three-body problem [12].

1.2.12 Reflections on the Hohmann transfer

In Meile, Ciarcia, and Mathwig's paper "Reflections on the Hohmann Transfer", the authors complete an in-depth study of the Hohmann transfer maneuver. There are important properties that are required to understand the transfer in itself. The first assumption made is that the orbits are circular and coplanar. The second assumption made is that there is a singular gravitational force. The third assumption is that the departure and arrival of the spacecraft are assumed to have circular motions. The final assumption is that the velocity impulses are tangential and exclusively for the terminal points. The velocity impulses accelerate for an ascending Hohmann transfer, whereas the descending transfer the impulses are braking. The inclination is affected for an ascending transfer as well, where it is positive everywhere except the endpoints, where it does not exist. An optimization of the Hohmann transfer was also proven, where the assumption is based upon having non-tangential velocity impulses at departure and arrival. The authors concluded that there is a maximum point on the inclination path. It was also determined that for the Hohmann transfer, the spacecraft velocity and the local circular velocity are the same at departure, mid-radius, and arrival [13].

1.2.13 Summary of References

The references discussed involve topics such as plasma on Titan, four-body problems, N-body problems, and trajectory planning. Each topic was specifically chosen to broaden the understanding and provide the mission objective direction. An understanding of the two-body, three-body and four-body problems are essential to understanding the N-body problem. The research provided mentions how accurate the N-body problem works and how accurate it can be.

1.3 Project Proposal

The objective of this study is to develop an efficient orbital transfer from Earth to orbit Titan using N-body simulations. The report will go in-depth on two main topics. The first is to design and calculate the ideal flight path from Earth to Titan's sphere of influence. The second is to apply this design using N-body simulations. These simulations will include major elements of the solar system in order to accurately predict a flight path. To execute the N-body problem, the simulations will be run in MATLAB.

1.4 Methodology

The process of launching a spacecraft from Earth is a careful and meticulous process that requires an agenda. This project will be organized in three main segments. The first segment will be the research phase, where the historical data is collected and utilized to have a further understanding of this project. The second segment will consist of trajectory designs and calculations of the celestial bodies involved. The design phase will determine the most efficient method to send a spacecraft to orbit Titan. The third segment, which is predicted to be the most time-consuming, will handle the n-body simulations of the intended celestial bodies. These N-body simulations will be run in MATLAB and incorporate specific characteristics to accurately portray the flight path.

2.0 Trajectory Planning

2.1 Measures of Merit

To achieve the objective of the mission of finding the most efficient Earth to Titan trajectory, the exact method requires calculations and planning. The efficiency of the mission is dependent on how many maneuvers the flight would have and how much time the spacecraft takes to get to Titan's orbit.

2.2 Methods

There will be nine paths that will be closely examined for this study, as shown in Fig. 3.1. These nine paths were chosen based on inspiration from the Cassini-Huygens mission.

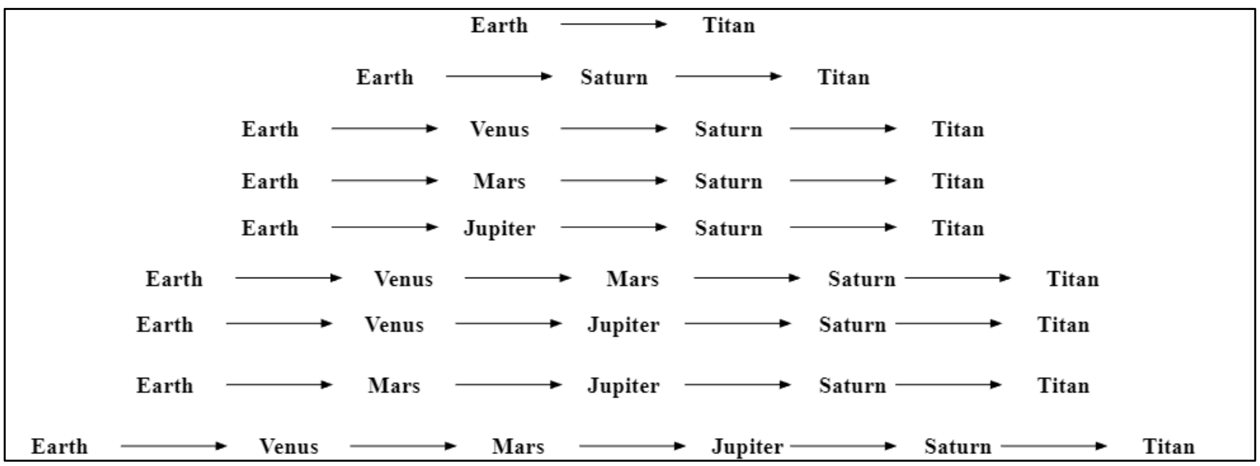

Figure 2.1 Trajectory paths

2.3 Time Frame

The ideal frame for the study is for Saturn and Earth to be at a true anomaly of 180 degrees, which would allow for simpler calculations. Another factor for this frame would be for Saturn and Earth to be within a closer distance of each other for a more fuel-efficient orbit.

2.4 Assumptions

In order to design a trajectory that can be understood, there will be assumptions made. The first is to assume all the planetary orbits are circular. The second assumption is that inclination will be the same, meaning the orbital bodies all lie on the same Z-plane. The third assumption will be assuming that the flight path for each transfer is 180 degrees.

3.0 Astrodynamics

3.1 Orbital Mechanics

3.1.1 Basics of Orbital Mechanics

The study of orbital mechanics focuses on problems concerning spacecraft motions. The solutions resolving these issues deal with specific equations of motion that involve Newton's laws of motion and gravitation. Newton's law of gravitation describes the force of gravity as two bodies with a distance between the two, written as

$$F_g = G\frac{m_1 m_2}{r^2} \qquad (3.1)$$

3.1.2 Orbital Parameters

The mass of the sun is represented as

$$M_{sun} = 1.989 \ x \ 10^{30} \ kg$$

The Universal Gravitational Constant is

$$G = 6.67 \ x \ 10^{-20} \ \frac{km^3}{kgs^2}$$

Table 3.1 Mass of each planet

Planet	M (kg)
Venus	$4.87 \ x \ 10^{24}$
Earth	$5.97 \ x \ 10^{24}$
Mars	$6.42 \ x \ 10^{23}$
Jupiter	$1.898 \ x \ 10^{27}$
Saturn	$5.683 \ x \ 10^{26}$

Table 3.2 Radius of planets

Planet	r (km)
Venus	$108.2 \ x \ 10^{6}$
Earth	$149.6 \ x \ 10^{6}$
Mars	$228.0 \ x \ 10^{6}$
Jupiter	$778.5 \ x \ 10^{6}$
Saturn	$143.2 \ x \ 10^{7}$

3.2 Relative Motion Equations

There are required parameters to calculate prior to making the calculations for the trajectory to Titan. These are determined by the equations of relative motion.

$$ {}^{sun}v^{planet} = \sqrt{\frac{GM_{sun}}{r_{planet}}} \tag{3.2} $$

Table 3.3 Circular orbital velocity

Planet Velocity w/ respect to the Sun	v (km/s)
${}^{sun}v^{Venus}$	35.016
${}^{sun}v^{Earth}$	29.7793
${}^{sun}v^{Mars}$	24.122
${}^{sun}v^{Jupiter}$	13.0542
${}^{sun}v^{Saturn}$	9.6252

$$ {}^{PA}a^{PB} = \frac{1}{2}(r_{PA} + r_{PB}) \tag{3.3} $$

Equation (3.2) represents the circular orbital velocity of the given planet that is relative to the sun. Equation (3.3) is a method to calculate the semi-major axis, which is the mean distance between one celestial body and another body.

3.3 Sphere of Influence

$$ r_{SOI} = r_{planet} \left(\frac{M_{planet}}{M_{sun}}\right)^{2/5} \tag{3.4} $$

Table 3.4 Sphere of influence

Planet	SOI (km)
Venus	616289.732
Earth	924415.913
Mars	577424.152
Jupiter	48208452.07
Saturn	54743849.22

The Sphere of Influence (SOI) is critical to understand the distance required for the spacecraft to leave the planet's gravitational influence, which is calculated with Eq. (3.4).

4.0 Interplanetary Transfer

4.1 Introduction to Hohmann Transfers

The Hohmann transfer was developed in 1925 by Wolfgang Hohmann. In order to be fuel efficient, this method was developed for transferring one spacecraft from one circular orbit to another circular orbit. The Hohmann transfer is critical to understand in order to grasp the concept of an interplanetary flyby. The beginning of a transfer is the same process for the interplanetary flyby. The objective of the beginning steps is to obtain the arrival velocity of the spacecraft with respect to the sun, ${}^{sun}v_a^Q$.

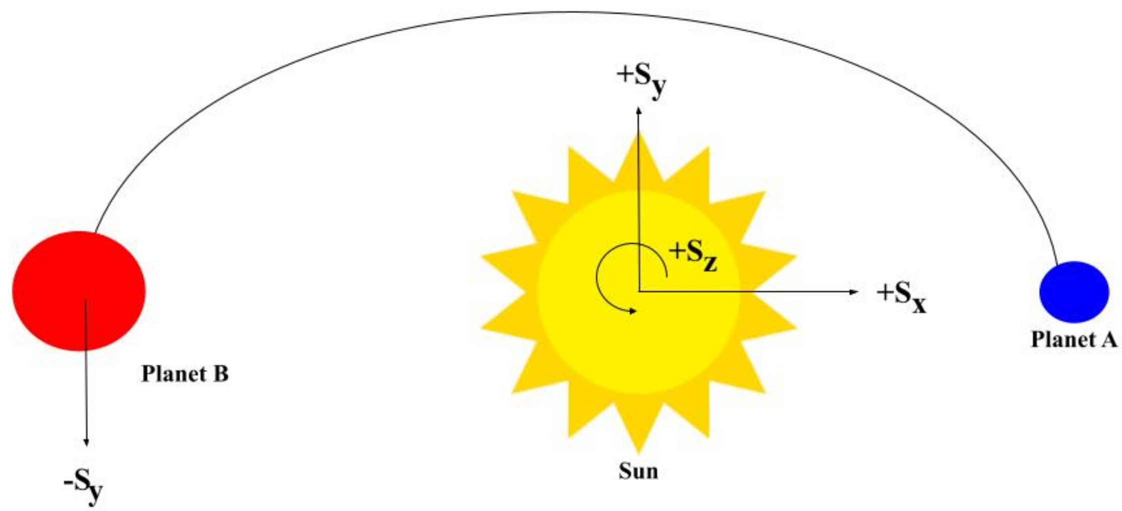

Figure 4.1 Sun reference frame

Fig. (4.1) displays the Sun reference frame, where Planet A is where the spacecraft departs and Planet B is where the spacecraft approaches. Sun reference frame is defined as $\mathbf{S_x}$, $\mathbf{S_y}$, and $\mathbf{S_z}$.

4.2 Interplanetary Transfer Approach

In order to fully understand the mission, the process of interplanetary flybys can be broken into three phases: departure, transit, and arrival. Fig. (4.2) shows the reference frame for an Earth to Saturn transfer.

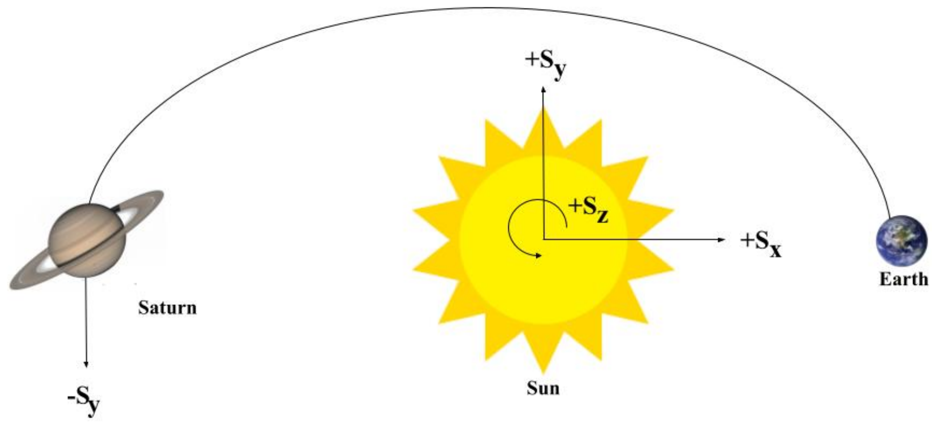

Figure 4.2 Earth to Saturn diagram

4.2.1 Transit Phase

The transit phase begins when the spacecraft exits the SOI of the planet it departs from. In this specific example, the transit phase will begin once the spacecraft leaves Earth's SOI. The objective of the transit phase is to find the hyperbolic excess speed, v_∞.

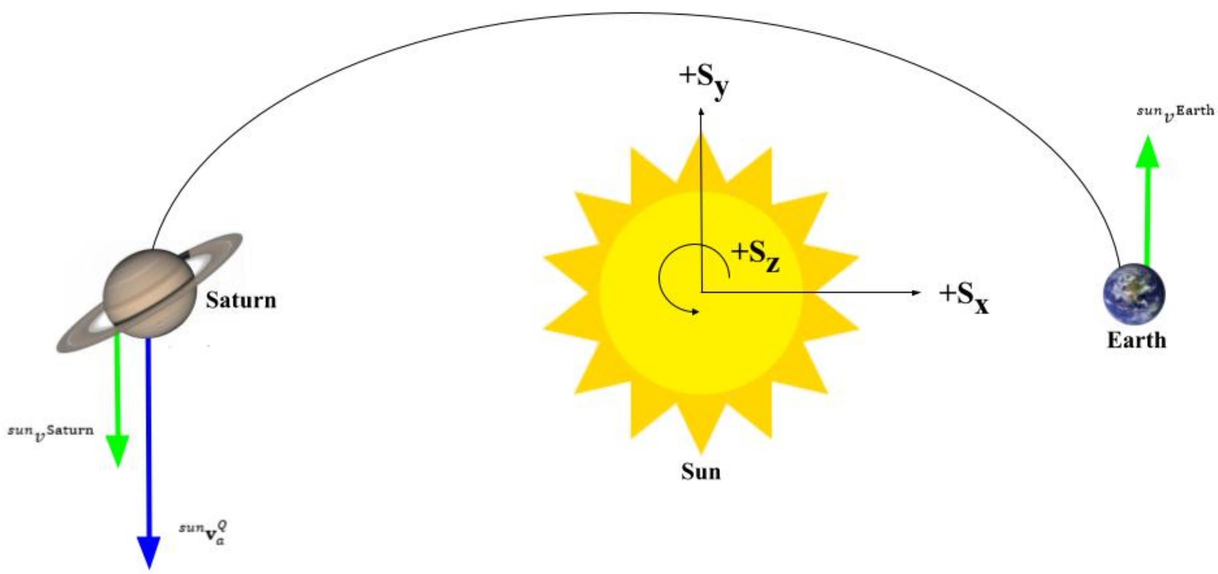

Figure 4.3 Earth to Saturn velocity visual

The first step in this stage is to find the circular orbit speeds of both planets using Eq. (3.2), in this case, Earth and Saturn. According to Table 3.3, the circular orbit velocities are:

$$^{sun}v^{Earth} \;=\; 29.779\ \text{km/s}\ \mathbf{Sy}$$

$$^{sun}v^{Saturn} \;=\; -9.885\ \text{km/s}\ \mathbf{Sy}$$

The next step of the transit phase is to calculate the velocity of the spacecraft departing Earth, $^{sun}\mathbf{v}_d^Q$.

$$^{sun}\mathbf{v}_d^Q \quad = \left(\sqrt{GM_{Sun}}\right)\left(\sqrt{\frac{2}{r_{PA}} - \frac{1}{_{PA}a^{PB}}}\right)\mathbf{S}_\mathrm{y} \tag{4.1}$$

Using Eq. (3.3) to solve for $^{Earth}a^{Saturn}$,

$$^{Earth}a^{Saturn} = \frac{1}{2}(r_{Earth} + r_{Saturn})$$
$$^{Earth}a^{Saturn} = \frac{1}{2}(149.6\ x\ 10^6 + 143.2\ x\ 10^7)$$
$$^{Earth}a^{Saturn} = \ 7.908\ x\ 10^8 \text{ km}$$

Substituting the values into Eq. (3.2),

$$^{sun}\mathbf{v}_d^Q \quad = 364233.8535\ x\left(\sqrt{\frac{2}{149.6\ x\ 10^6} - \frac{1}{7.908\ x\ 10^8}}\right)\mathbf{S}_\mathrm{y}$$
$$^{sun}\mathbf{v}_d^Q \quad = \ +40.073\ km/s\ \mathbf{Sy}$$

$\mathbf{v}_{\infty_d}$ is the velocity of the spacecraft departing Earth with respect to the Sun.

$$^{sun}\mathbf{v}_d^Q \quad = \ ^{sun}\mathbf{v}^{planet} \ + \ ^{planet}\mathbf{v}_d^Q \tag{4.2}$$

$$^{planet}\mathbf{v}_d^Q \ = \ ^{sun}\mathbf{v}_d^Q \ - \ ^{sun}\mathbf{v}^{planet} \tag{4.3}$$

$$^{planet}\mathbf{v}_d^Q \ = \ \mathbf{v}_{\infty_d} \tag{4.4}$$

Using Eq. (4.2) – (4.4), the $\mathbf{v}_{\infty_a}$can be solved with the values of the circular orbital velocity of Earth and the spacecraft velocity departing Earth.

$$^{sun}\mathbf{v}_d^Q \quad = \ ^{sun}\mathbf{v}^{Earth} \ + \ ^{Earth}\mathbf{v}_d^Q$$

$$^{Earth}\mathbf{v}_d^Q \ = \ ^{sun}\mathbf{v}_d^Q \ - \ ^{sun}\mathbf{v}^{Earth}$$

$$^{Earth}\mathbf{v}_d^Q \ = \ 40.073\ km/s\ \mathbf{Sy} - \ 29.779\ km/s\ \mathbf{Sy}$$

$$^{Earth}\mathbf{v}_d^Q \ = \ 10.294\ km/s\ \mathbf{Sy}$$

$$^{Earth}\mathbf{v}_d^Q \ = \ \mathbf{v}_{\infty_d}$$

$$\mathbf{v}_{\infty_d} \ = \ 10.294\ km/s\ \mathbf{Sy}$$

The next step of the transit phase is to calculate the velocity of the spacecraft approaching Saturn, ${}^{sun}\mathbf{v}_a^Q$.

$$ {}^{sun}\mathbf{v}_a^Q = \left(\sqrt{GM_{Sun}}\right)\left(\sqrt{\frac{2}{r_{PB}} - \frac{1}{PA_aPB}}\right)\mathbf{S_y} \tag{4.5}$$

$$ {}^{sun}\mathbf{v}_a^Q = \left(\sqrt{GM_{Sun}}\right)\left(\sqrt{\frac{2}{r_{Saturn}} - \frac{1}{Earth_aSaturn}}\right)\mathbf{S_y} $$

Substituting the values into Eq. (3.2),

$$ {}^{sun}\mathbf{v}_a^Q = 364233.8535\ x\left(\sqrt{\frac{2}{143.2\ x\ 10^7} - \frac{1}{7.908\ x\ 10^8}}\right)\mathbf{S_y} $$

$$ {}^{sun}\mathbf{v}_a^Q = -\,4.186\ km/s\ \mathbf{Sy} $$

$\mathbf{v}_{\infty_a}$ is the velocity of the spacecraft approaching Saturn with respect to the Sun.

$$ {}^{sun}\mathbf{v}_a^Q = {}^{sun}\mathbf{v}^{planet} + {}^{planet}\mathbf{v}_a^Q \tag{4.6}$$

$$ {}^{planet}\mathbf{v}_a^Q = {}^{sun}\mathbf{v}_a^Q - {}^{sun}\mathbf{v}^{planet} \tag{4.7}$$

$$ {}^{planet}\mathbf{v}_a^Q = \mathbf{v}_{\infty_a} \tag{4.8}$$

Using Eq. (4.6) – (4.8), the $\mathbf{v}_{\infty_a}$can be solved with the values of the circular orbital velocity and the spacecraft velocity approaching Saturn.

$$ {}^{sun}\mathbf{v}_a^Q = {}^{sun}\mathbf{v}^{Saturn} + {}^{Saturn}\mathbf{v}_a^Q $$

$$ {}^{Saturn}\mathbf{v}_a^Q = {}^{sun}\mathbf{v}_a^Q - {}^{sun}\mathbf{v}^{Saturn} $$

$$ {}^{Saturn}\mathbf{v}_a^Q = -\,4.186\ km/s\ \mathbf{Sy} - (-9.6252\ km/s)\ \mathbf{Sy} $$

$$ {}^{Saturn}\mathbf{v}_a^Q = 5.4388\ km/s\ \mathbf{Sy} $$

$$ {}^{Saturn}\mathbf{v}_a^Q = \mathbf{v}_{\infty_a} $$

$$ \mathbf{v}_{\infty_a} = 5.4388\ km/s\ \mathbf{Sy} $$

4.2.2 Rendezvous Phase

Once the spacecraft enters the orbit of the approaching planet, the spacecraft will either rendezvous or perform a flyby. In order to do this, the flyby hyperbola needs to be defined with a given altitude, which would allow eccentricity, e_h, to be solved for.

$$e_h = 1 + \frac{r_p v_\infty^2}{GM_{planet}} \quad (4.9)$$

Eq. (4.9) is used to find the eccentricity of the flyby hyperbola. The radius will have a range of values to understand the relationship between the radius and the eccentricity. The eccentricity is critical in solving for the turn angle, δ, which affects the departure angle, ϕ_d. Eq. (4.10) represents the angle between the departure velocity and the arrival velocity with respect to the Sun reference frame. Eq. (4.11) is the angle between the departure velocity and the arrival velocity of the target planet.

$$\delta = 2sin^{-1}\left(\frac{1}{e}\right) \quad (4.10)$$

$$\phi_d = \phi_a + \delta \quad (4.11)$$

After arriving at the planet, the spacecraft will make a flyby at a calculated departure angle. Similar to Eq. (4.6) and (4.7), finding the departure velocity of the flyby can be solved with Eq. (4.12) - (4.15).

$$^{sun}\mathbf{v}_{df}^{Q} = {}^{sun}\mathbf{v}^{planet} + {}^{planet}\mathbf{v}_{df}^{Q} \quad (4.12)$$

$$^{planet}\mathbf{v}_{df}^{Q} = {}^{planet}\mathbf{v}_{df}^{Q} - {}^{sun}\mathbf{v}^{planet} \quad (4.13)$$

$$^{planet}\mathbf{v}_{df}^{Q} = \mathbf{v}_{\infty_{df}} \quad (4.14)$$

$$\mathbf{v}_{\infty_{df}} = v_{\infty_a} cos_\delta \mathbf{S}_\mathrm{y} - v_{\infty_a} cos_\delta \, \mathbf{S}_\mathrm{x} \quad (4.15)$$

Eq. (4.16) is the change in velocity between the spacecraft departure velocity and the spacecraft arrival velocity with respect to the sun.

$$\Delta\mathbf{v} = {}^{sun}\mathbf{v}_{df}^{Q} - {}^{sun}\mathbf{v}_{a}^{Q} \quad (4.16)$$

The final steps are to calculate specific values of the flyby departure trajectory. The first is angular momentum, h_2, where v is the velocity tangent to the ellipse. The velocity that is tangent to the ellipse in the scenario would be in the $\mathbf{S}_\mathrm{y}$ direction.

$$h_2 = r_{planet} v_\theta \quad (4.17)$$

The second value to calculate would be the magnitude of the velocity, v_2, which is crucial for the energy equation, Eq. (4.19).

$$v_2 = \left| {}^{sun}\mathbf{v}_{df}^{Q} \right| \tag{4.18}$$

$$v_2^2 = GM_{sun}\left(\frac{2}{r_{planet}} - \frac{1}{a_2}\right) \tag{4.19}$$

Eq. (4.20) represents the eccentricity of the flyby hyperbola upon departure. Eq. (4.21) is the orbit equation which can be manipulated into Eq. (4.22) to find the angle between the transfer ellipse and the perihelion.

$$e_2 = \sqrt{1 - \frac{h_2^2}{a_2 GM_{sun}}} \tag{4.20}$$

$$r_{planet} = \frac{h_2^2}{GM_{sun}(1+e_2 \cos\theta_2)} \tag{4.21}$$

$$\cos\theta_2 = \frac{h_2^2 - r_{planet}GM_{sun}}{r_{planet}GM_{sun}e_2} \tag{4.22}$$

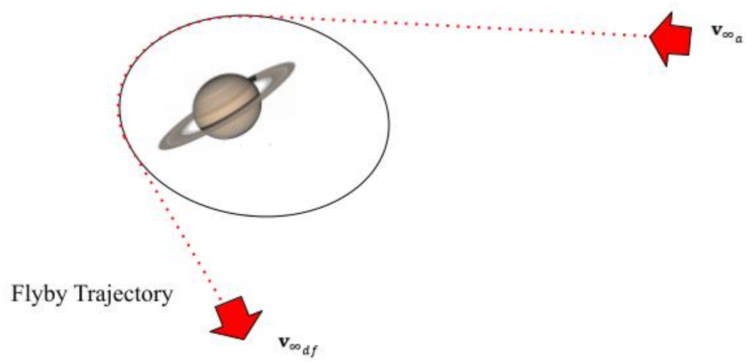

Figure 4.4 Saturn flyby trajectory

Fig. (4.4) shows the purpose of the calculations, to manipulate the flyby trajectory by having an ideal approaching velocity, which is critical in designing the next flyby departure velocity.

4.3 Rendezvous Opportunities

For an interplanetary transfer to succeed, the timing of the launch needs to be correct. This process involves determining how long it will take for the spacecraft to arrive and when it should leave based on the position of the planets.

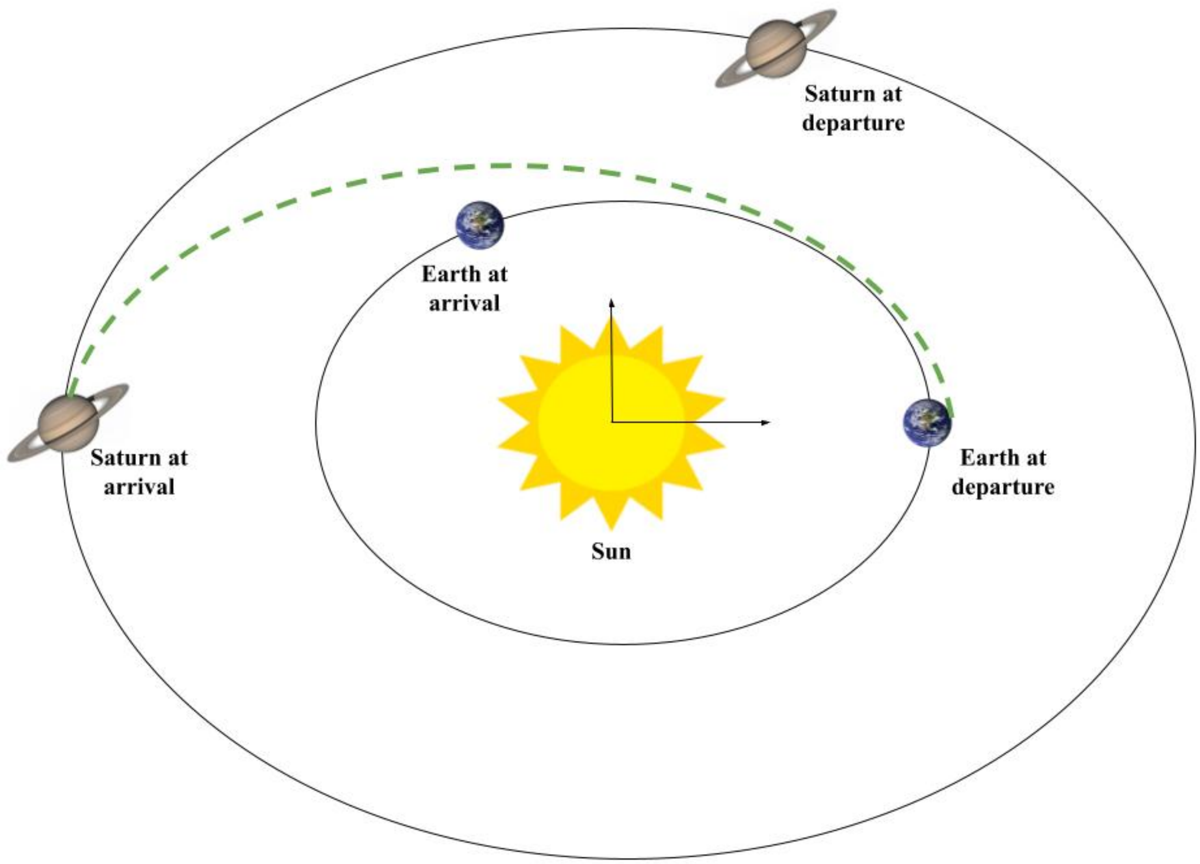

Figure 4.5 Earth to Saturn rendezvous

4.3.1 Initial Phase

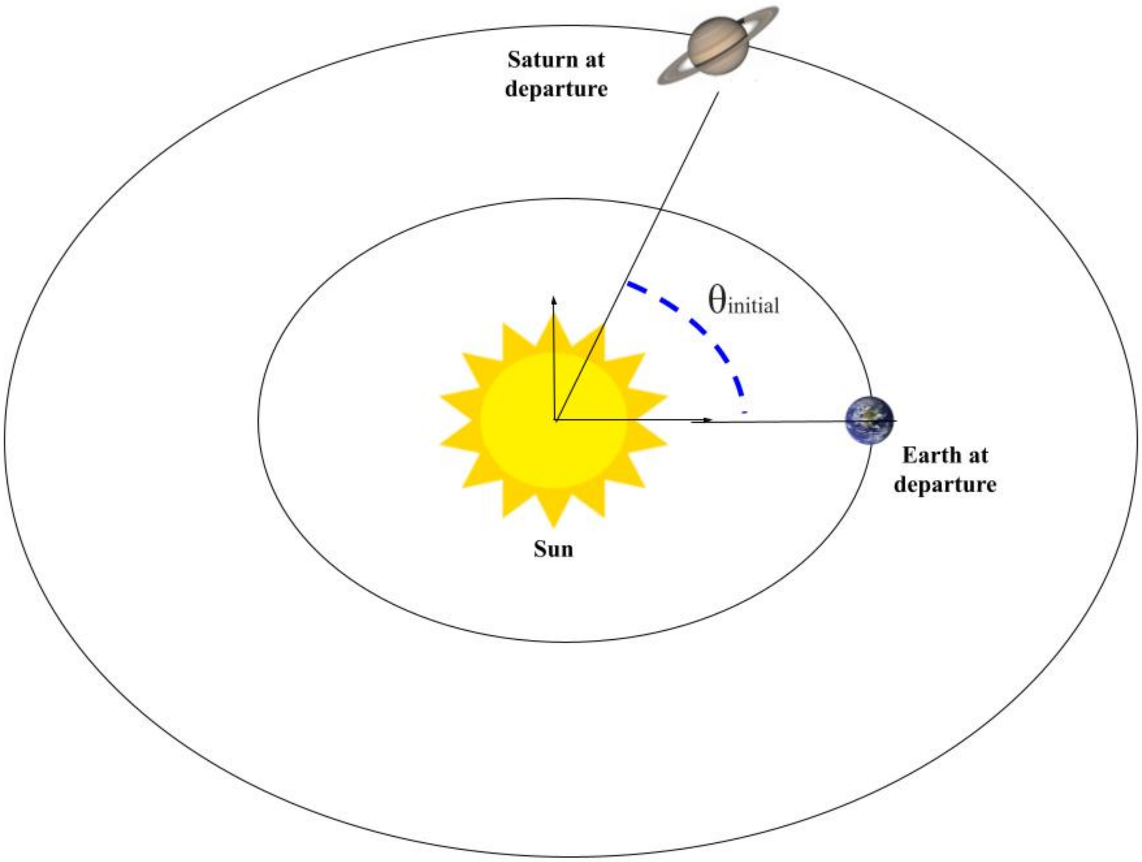

Figure 4.6 Earth to Saturn departure

$\theta_{initial}$ is the required angle for the planets to have between each other to have an arrival at the line of apses.

$$\theta_{initial} = \pi - n_2 t_{12} \tag{4.23}$$

n_1 and n_2 are the mean motions of the planets, in this case, n_1 and n_2 are Earth and Saturn, respectively.

Table 4.1 Orbital period of planets

Planet	Orbital Period (T_{Planet})
Venus	225.0
Earth	365.256
Mars	687.0
Jupiter	4333.0
Saturn	10759.0

$$n_{Planet} = \frac{2\pi}{T_{Planet}} \quad (4.24)$$

$$n_{Earth} = \frac{2\pi}{T_{Earth}}$$

$$n_{Earth} = \frac{2\pi}{365.256}$$

$$n_{Earth} = .0172 \text{ rad/day}$$

Using Eq. (4.24) and Table 4.1, the mean motions of Earth can be found to be .0172 rad/day and Saturn can be found to be 5.8399 x 10^{-4} rad/day. The next step to solve Eq. (4.23) would be to solve for the synodic period, which is given by:

$$t_{12} = \frac{\pi}{\sqrt{\mu_{sun}}}\left(\frac{R_1+R_2}{2}\right)^{3/2} \quad (4.25)$$

R_1 and R_2 represent the radius of the planets. Plugging in the values for an Earth and Saturn synodic period, t_{12} is 2.2200 x 10^3 days. Substituting n_2 and t_{12} into Eq. (4.23),

$$\theta_{initial} = \pi - (5.8399 \text{ x } 10^{-4} \text{ rad/day})(2.2200 \text{ x } 10^3 \text{ day})$$

$$\theta_{initial} = \pi - 1.2965\, rad$$

$$\theta_{initial} = 1.8541\, rad \text{ or } 105.7175°$$

This means that the angle between Earth and Saturn would need to be 1.8541 rad or 105.7175° upon departure.

4.3.2 Final Phase

When the spacecraft arrives at the desired planet, there will be a new phase angle between the departure planet and the arrival planet, θ_{final}.

$$\theta_{final} = \pi - n_1 t_{12} \quad (4.26)$$

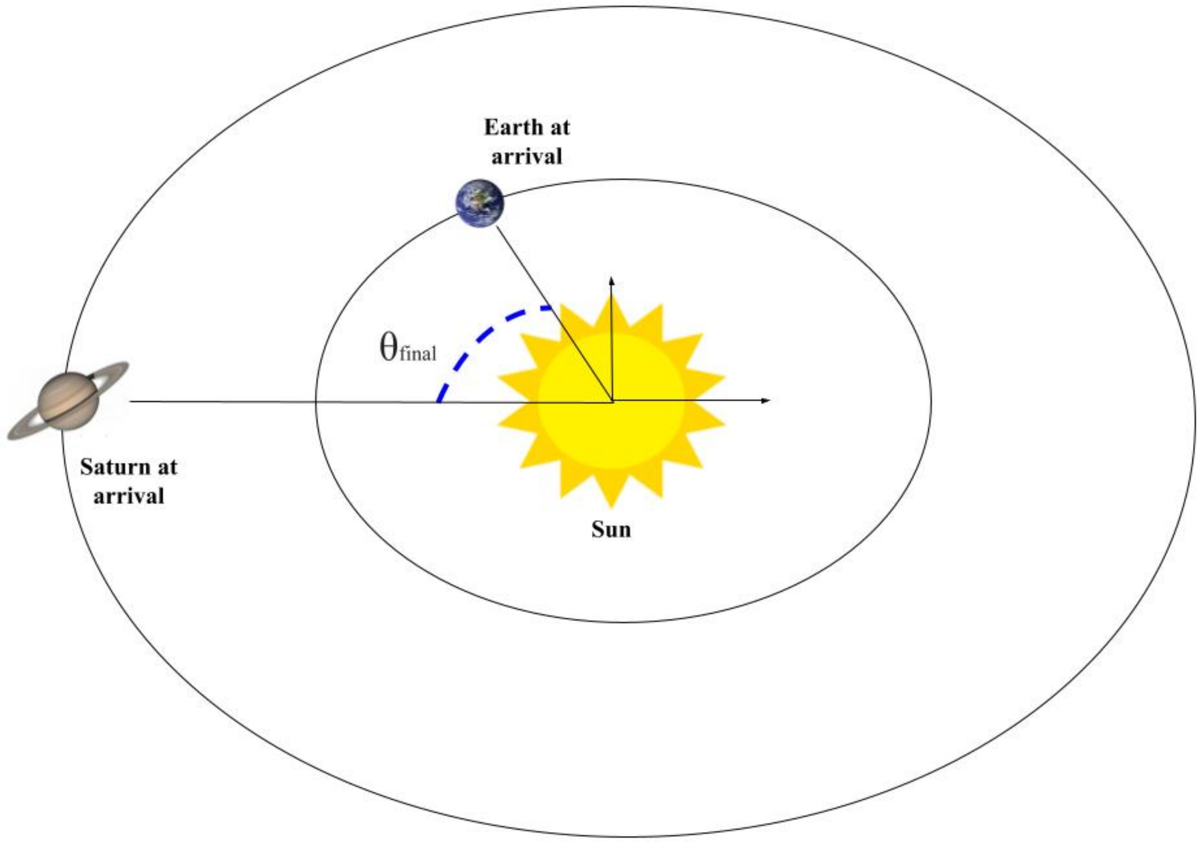

Figure 4.7 Earth to Saturn arrival

Using Eq. (4.26) and Eq. (4.25), θ_{final} can be found after the spacecraft departs Earth.

$\theta_{final} = \pi - (0.0172)(2.2200 \text{ x } 10^3 \text{ day})$

$\theta_{final} = \pi - 38.184$

$\theta_{final} = -35.042\, rad\ or\ -2007.8°$

This is the angle when between Earth and Saturn when the spacecraft arrives at Saturn.

5.0 Saturn to Titan

5.1 Characteristics of Titan

Table 5.1 Titan properties

Mass	$1.345\ x\ 10^{23}$ kg

Perigee	1186680 km
Apogee	1257060 km
Radius	1221870 km
radius	2575.4 km
SOI	43306.04056 km

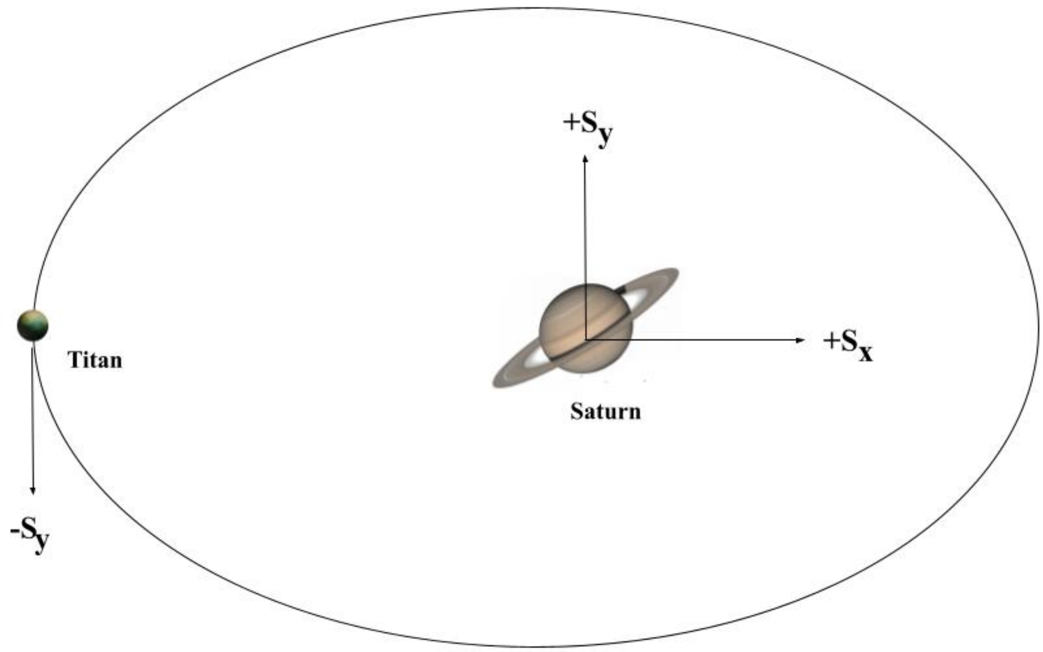

Figure 5.1 Saturn and Titan reference frame

Modifying the Eq. (3.2),

$$^{Saturn}v^{Titan} = \sqrt{\frac{GM_{Saturn}}{r_{Titan}}} \tag{5.1}$$

$$^{Saturn}v^{Titan} = \sqrt{\frac{(6.67\ x\ 10^{-20})(5.683\ x\ 10^{26})}{1221870}}$$

$$^{Saturn}v^{Titan} = 5.569795\ \text{km/s}$$

$$v_e^{Titan} = \sqrt{\frac{2GM_{Titan}}{radius}} \tag{5.2}$$

$$v_e^{Titan} = \sqrt{\frac{2(6.67 \ x \ 10^{-20})(1.345 \ x \ 10^{23})}{2575.4}}$$

$$v_e^{Titan} = 2.6394 \ km/s$$

The circular orbital velocity of Titan is 5.5698 km/s. The escape velocity of Titan is 2.6394 km/s.

Using Eq. (3.3), the semi-major axis of the transfer orbit can be found. For this example, arbitrary values of 10000 km for $a_{initial}$ and 1257000 for a_{final} .

$$^{f}a^{i} = \frac{1}{2}\left(a_{final} + a_{initial}\right)$$
$$^{f}a^{i} = \frac{1}{2}(1257000 + 10000)$$
$$^{f}a^{i} = 633500 \ km$$

$$^{Saturn}\mathbf{v}_{\infty_d}^{Q} \ = \ \left(\sqrt{GM_{Saturn}}\right)\left(\sqrt{\frac{2}{a_{initial}} - \frac{1}{^{f}a^{i}}}\right)$$

$$^{Saturn}\mathbf{v}_{\infty_d}^{Q} \ = \ \left(\sqrt{37905610}\right)\left(\sqrt{\frac{2}{10000} - \frac{1}{633500}}\right)$$
$$^{Saturn}\mathbf{v}_{\infty_d}^{Q} \ = \ 86.725 \ km/s$$

$$\Delta\mathbf{v} \ = \ ^{Saturn}v_{\infty_d}^{Q} \ - \ ^{Saturn}v^{Titan}$$

$$\Delta\mathbf{v} \ = \ 86.725 \ - \ 5.5698$$

$$\Delta\mathbf{v} \ = \ 81.1555 \ km/s$$

$$v \ = \ \left(\sqrt{GM_{Saturn}}\right)\left(\sqrt{\frac{2}{a_{final}} - \frac{1}{^{f}a^{i}}}\right)$$

$$^{Saturn}v_{\infty_a}^{Q} \ = \ \left(\sqrt{37905610}\right)\left(\sqrt{\frac{2}{1257000} - \frac{1}{633500}}\right)$$
$$^{Saturn}v_{\infty_a}^{Q} \ = \ .6899 \ km/s$$

$${}^{Saturn}v^{Q}_{\infty_{tr}} = \left(\sqrt{GM_{Saturn}}\right)\left(\sqrt{\frac{2}{r}-\frac{1}{a}}\right)$$

$${}^{Saturn}v^{Q}_{\infty_{tr}} = \left(\sqrt{37905610}\right)\left(\sqrt{\frac{2}{1257000}-\frac{1}{1257000}}\right)$$

${}^{Saturn}v^{Q}_{\infty_{tr}} = 5.491\ km/s$ velocity of transfer orbit

$$\Delta\mathbf{v} = {}^{Saturn}v^{Q}_{\infty_{tr}} - {}^{Saturn}v^{Q}_{\infty_{a}}$$

$$\Delta\mathbf{v} = 5.491 - .6899$$

$$\Delta\mathbf{v} = 4.8015\ km/s$$

Now that this orbit is inserted into Titan's SOI, the velocity must not exceed Titan's escape velocity, which is 2.6394 km/s. Seeing how the insertion orbit exceeds the escape velocity, the spacecraft needs a burn to slow down the spacecraft. In this example, the spacecraft will orbit Titan at an arbitrary altitude of 20000.

$${}^{Titan}v^{Q}_{\infty_{a}} = \left(\sqrt{GM_{Titan}}\right)\left(\sqrt{\frac{2}{r}-\frac{1}{a}}\right)$$

$${}^{Titan}v^{Q}_{\infty_{a}} = \left(\sqrt{8971.15}\right)\left(\sqrt{\frac{2}{20000}-\frac{1}{20000}}\right)$$

$${}^{Titan}v^{Q}_{\infty_{a}} = .6697\ km/s$$

$${}^{Titan}\mathbf{v}^{Q}_{a} = {}^{Titan}\mathbf{v}^{Saturn} + {}^{Saturn}\mathbf{v}^{Q} \quad (5.3)$$

$${}^{Titan}\mathbf{v}^{Q}_{a} = -{}^{Saturn}\mathbf{v}^{Titan} + {}^{Saturn}\mathbf{v}^{Q}_{a} \quad (5.4)$$

The circular orbit velocity of Titan with respect to Saturn was solved using Eq. (5.1), which is 5.5698 km/s . The arrival velocity of the spacecraft with respect to Saturn departing from Earth was solved using Eq. (4.8) and calculated to be 5.699 km/s.

$${}^{Titan}\mathbf{v}^{Q}_{a} = -5.5698 + 5.699$$

$${}^{Titan}\mathbf{v}^{Q}_{a} = 0.1292$$

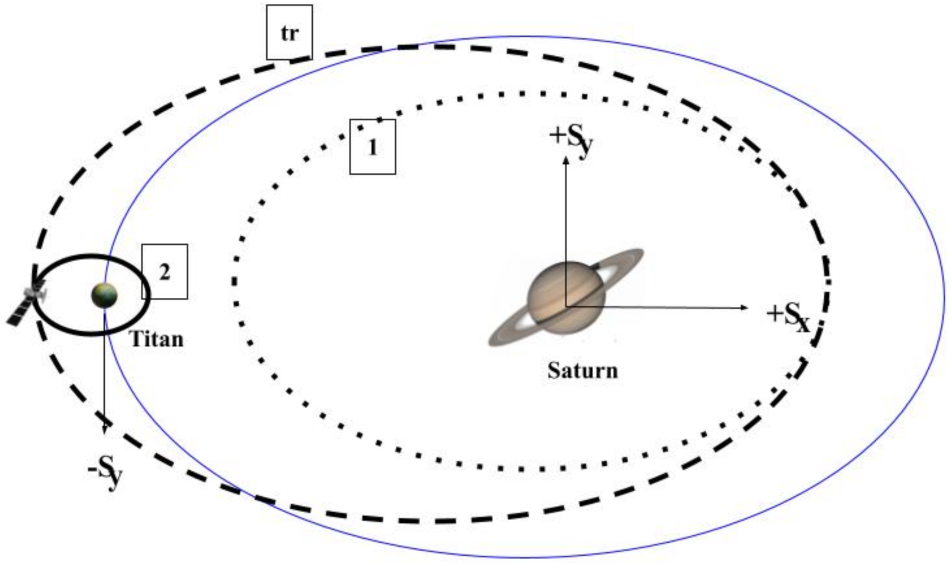

Figure 5.2 Saturn and Titan transfer

5.2 Three-Body Problem

The three-body problem is a situation where three bodies of similar mass are within a relative distance between one another. Each body is affected by the gravitational influence of the other two bodies. If one of the masses is significantly smaller than the other two masses that it could be considered negligible, then the problem would turn into a restricted three body problem. The spacecraft arriving to Titan from Saturn would be a perfect example of this problem.

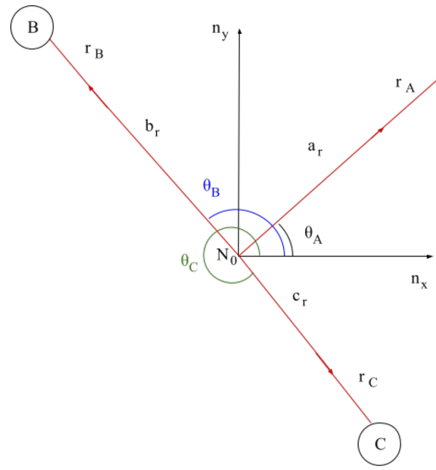

Figure 5.3 Three-body problem

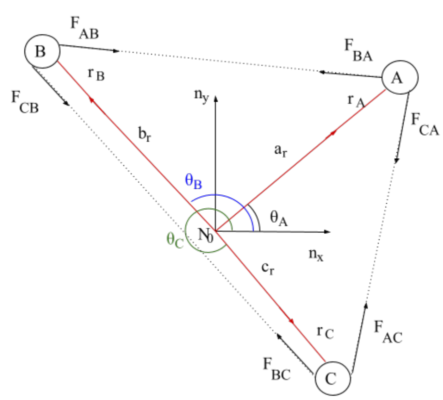

Figure 5.4 Three-body problem forces

As shown in Fig. 3.2, the only forces are the ones acting on the planets are the forces that have mutual gravitation. The forces F_{AB} and F_{Ba} are equal and opposite.

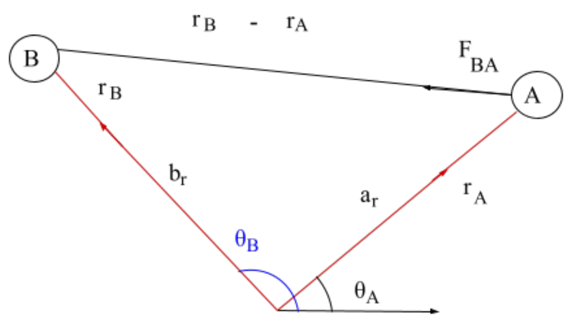

Figure 5.5 Three-body problem force of masses

The equations of motion of particle A are:

$$F_A = {}^N\frac{d}{dt}{}^N\mathbf{r}^A \tag{5.5}$$

$$F_A = F_{BA} + F_{CA} = Gm_A\left[\frac{m_B(r_B - r_A)}{|r_B - r_A|^3} - \frac{m_C(r_C - r_A)}{|r_C - r_A|^3}\right] \tag{5.6}$$

$$\boldsymbol{r_A} = {}^N\mathbf{r}^A = \boldsymbol{r_A a_r} = \boldsymbol{r_A}\cos\boldsymbol{\theta_A n_x} + \boldsymbol{r_A}\sin\boldsymbol{\theta_A n_y} \tag{5.7}$$

$$\begin{aligned} a_A = {}^N a^A = &(r''_A\cos\boldsymbol{\theta_A} - 2r'_A\boldsymbol{\theta_A}\sin\boldsymbol{\theta_A} - r_A\boldsymbol{\theta''_A}\sin\boldsymbol{\theta_A} - r_A\boldsymbol{\theta'^2_A}\cos\boldsymbol{\theta_A})\boldsymbol{n_x} \\ &+ (r''_A\sin\boldsymbol{\theta_A} - 2r'_A\boldsymbol{\theta_A}\sin\boldsymbol{\theta_A} - r_A\boldsymbol{\theta''_A}\cos\boldsymbol{\theta_A} - r_A\boldsymbol{\theta'^2_A}\sin\boldsymbol{\theta_A})\boldsymbol{n_y} \end{aligned} \tag{5.8}$$

$$r''_A = \frac{Gm_B}{r_{AB}^3}\left[r_B\cos(\boldsymbol{\theta_B} - \boldsymbol{\theta_A}) - \boldsymbol{r_A}\right] + \frac{Gm_C}{r_{AC}^3}\left[r_C\cos(\boldsymbol{\theta_C} - \boldsymbol{\theta_A}) - \boldsymbol{r_A}\right] + r_A\boldsymbol{\theta'^2_A} \tag{5.9}$$

$$\boldsymbol{\theta''_A} = \frac{Gm_B r_B}{r_{AB}^3 r_A}\sin(\boldsymbol{\theta_B} - \boldsymbol{\theta_A}) + \frac{Gm_C r_C}{r_{AC}^3 r_A}\sin(\boldsymbol{\theta_C} - \boldsymbol{\theta_A}) - \frac{2r'_A\theta'^2_A}{r_A} \tag{5.10}$$

$$r_{AB} = |r_A - r_B| = |r_B - r_A| \tag{5.11}$$

Assume the three particles in the three-body problem were Saturn, Titan and a spacecraft, where the spacecraft is in Titan's SOI. Since Titan is in Saturn's SOI, the spacecraft would be in both Titan and Saturn's SOI. Assuming the spacecraft's mass is small in comparison to Titan and Saturn, the spacecraft's mass can be negligible. This would turn the three-body problem into the restricted three body problem.

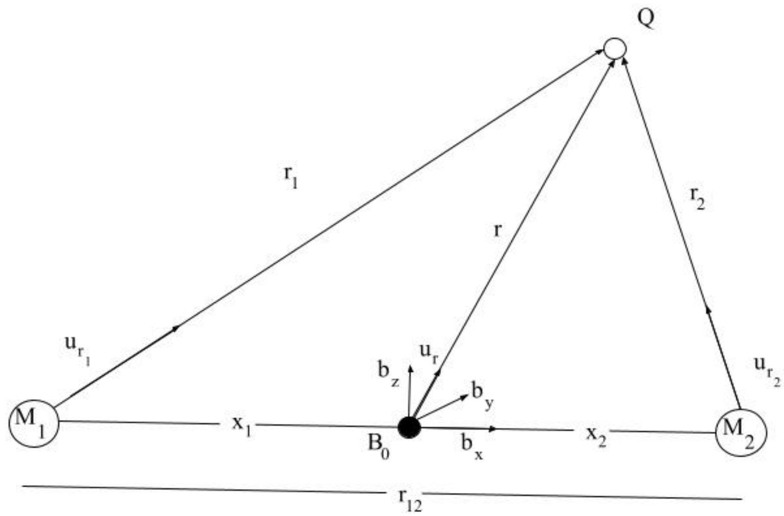

Figure 5.6 Restricted three-body problem

$^{B}r^{M1} = -x_1 b_x$ (5.12)

$^{B}r^{M2} = x_2 b_x$ (5.13)

$^{B}r^{Q} = x b_x + y b_y + z b_z$, where $\sqrt{x^2 + y^2 + z^2} = r$ (5.14)

$^{M1}r^{Q} = (x + x_1) b_x + y b_y + z b_z$ (5.15)

$^{M2}r^{Q} = (x - x_2) b_x + y b_y + z b_z$ (5.16)

Angular Velocity

$$\omega = \frac{2\pi}{\tau} \quad (5.17)$$

$$\tau = \frac{2\pi}{\sqrt{G(M_1+M_2)}} r_{12}^{3/2} \quad (5.18)$$

$$\omega = \frac{\sqrt{G(M_1+M_2)}}{r_{12}^{3/2}} \quad (5.19)$$

$$-M_1 x_1 + M_2 x_2 = 0 \rightarrow x_1 = \frac{M_2}{M_1} x_2 \quad (5.20)$$

$$r_{12} = x_2 + x_1 \quad (5.21)$$

$$r_{12} = x_2 + (\frac{M_2}{M_1} x_2) = (\frac{M_1+M_2}{M_1}) x_2 \quad (5.22)$$

$$r_{12} = (\frac{M_1+M_2}{M_1})x_1 \quad (5.23)$$

Defining $\pi_1 = \frac{M_1}{M_1+M_2}$ and $\pi_2 = \frac{M_2}{M_1+M_2}$, then:

$$x_1 = \pi_2 r_{12} \quad (5.24)$$

$$x_2 = \pi_1 r_{12} \quad (5.25)$$

Applying these steps to the Saturn and Titan system,

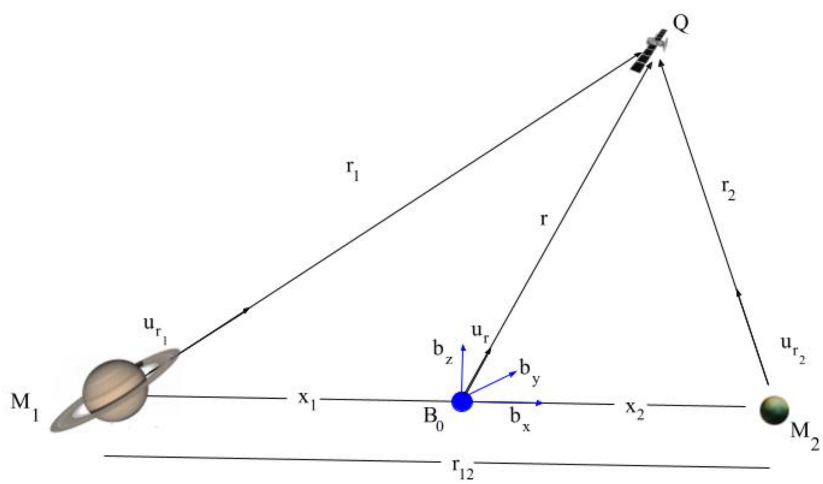

Figure 5.7 Restricted Saturn-Titan problem

$${}^{B}r^{Saturn} = -x_1 b_x \quad (5.26)$$

$${}^{B}r^{Titan} = x_2 b_x \quad (5.27)$$

$${}^{B}r^{Q} = xb_x + yb_y + zb_z \text{, where } \sqrt{x^2+y^2+z^2} = r \quad (5.28)$$

$${}^{Saturn}r^{Q} = (x + x_1)b_x + yb_y + zb_z \quad (5.29)$$

$${}^{Titan}r^{Q} = (x - x_2)b_x + yb_y + zb_z \quad (5.30)$$

Angular Velocity

$$\omega = \frac{2\pi}{\tau} \quad (5.31)$$

$$\tau = \frac{2\pi}{\sqrt{G(M_{Saturn}+M_{Titan})}} r_{12}^{3/2} \quad (5.32)$$

$$\tau = \frac{2\pi}{\sqrt{6.67 \; x \; 10^{-20} \, \frac{km^3}{kgs^2}(5.683 \; x \; 10^{26} \; kg + 1.345 \; x \; 10^{23} \; kg)}}(1221870 \; km)^{3/2}$$

$$\tau = 1378206.637 \; s$$

$$\omega = \frac{\sqrt{G(M_1 + M_2)}}{r_{12}^{3/2}} \tag{5.33}$$

$$\omega = \frac{\sqrt{6.67 \; x \; 10^{-20} \, \frac{km^3}{kgs^2}(5.683 \; x \; 10^{26} \; kg + 1.345 \; x \; 10^{23} \; kg)}}{(1221870 \; km)^{3/2}} \tag{5.34}$$

$\omega = 4.5589583 \; x \; 10^{-6}$ rad/s

$$-M_{Saturn}x_1 + M_{Titan}x_2 = 0 \rightarrow x_1 = \frac{M_{Titan}}{M_{Saturn}}x_2 \tag{5.35}$$

$$r_{12} = x_2 + x_1 \tag{5.36}$$

According to Eq. (3.24) and (3.25), $x_1 = \pi_2 r_{12}$ and $x_2 = \pi_1 r_{12}$.

Defining $\pi_1 = \frac{M_{Saturn}}{M_{Satur,n} + M_{Titan}}$ and $\pi_2 = \frac{M_{Titan}}{M_{Saturn} + M_{Titan}}$, then:

$$\pi_1 = \frac{5.683 \; x \; 10^{26} \; kg}{5.683 \; x \; 10^{26} \; kg + 1.345 \; x \; 10^{23} \; kg}$$

$$\pi_1 = 0.999763 \; kg$$

$$\pi_2 = \frac{1.345 \; x \; 10^{23} \; kg}{5.683 \; x \; 10^{26} \; kg + 1.345 \; x \; 10^{23} \; kg}$$

$$\pi_2 = 2.3661 \; x \; 10^{-4} \; kg$$

$$x_1 = \pi_2 r_{12} \tag{5.37}$$

$$x_1 = (2.3661 \; x \; 10^{-4} \; kg)(1221870 \; km)$$

$$x_1 = 289.1127$$

$$x_2 = \pi_1 r_{12} \tag{5.38}$$

$$x_2 = (0.999763 \; kg)(1221870 \; km)$$

$$x_2 = 1221580.887$$

Deriving the EOM for the Saturn and Titan, the position vectors can be rewritten as:

$^{M1}r^{Q} = (x + \pi_2 r_{12})b_x + yb_y + zb_z$

$^{M2}r^{Q} = (x - \pi_1 r_{12})b_x + yb_y + zb_z$

The particle EOM can be seen in Eq. (5.39).

$$\sum F = m^S a^Q \tag{5.39}$$

The forces considered are the gravitational forces of Saturn and Titan.

$$-\frac{GM_1 m}{r_1^2} u_{r_1} - \frac{GM_2 m}{r_2^2} u_{r_2} = m^S a^Q \tag{5.40}$$

$$-\frac{GM_1 m}{r_1^2} u_{r_1} - \frac{GM_2 m}{r_2^2} u_{r_2}$$
$$= m\{(x'' - 2\omega y' - \omega^2 x)b_x + (y'' + 2\omega x' - \omega^2 y)b_y + (z'')b_z\} \tag{5.41}$$

$$u_{r_1} = \frac{^{M1}r^Q}{|^{M1}r^Q|} = \frac{(x+\pi_2 r_{12})b_x + yb_y + zb_z}{r_1} \tag{5.42}$$

$$u_{r_2} = \frac{^{M2}r^Q}{|^{M2}r^Q|} = \frac{(x - \pi_1 r_{12})b_x + yb_y + zb_z}{r_2} \tag{5.43}$$

$$-\frac{GM_1}{r_1^3}\{(x + \pi_2 r_{12})b_x + yb_y + zb_z\} - \frac{GM_2}{r_2^3}\{(x - \pi_1 r_{12})b_x + yb_y + zb_z\}$$

$$= \{(x'' - 2\omega y' - \omega^2 x)b_x + (y'' + 2\omega x' - \omega^2 y)b_y + (z'')b_z\} \tag{5.44}$$

After simplifying, this leaves the scalar equations for the Saturn-Titan system.

$$\text{(bx) } x'' - 2\omega y'' - \omega^2 x = -\frac{GM_1}{r_1^3}\{(x + \pi_2 r_{12}) - \frac{GM_2}{r_2^3}(x - \pi_1 r_{12})\} \tag{5.45}$$

$$\text{(by) } (y'' + 2\omega x' - \omega^2 y) = (-\frac{GM_1}{r_1^3} - \frac{GM_2}{r_2^3})y \tag{5.46}$$

$$\text{(bz) } z'' = (-\frac{GM_1}{r_1^3} - \frac{GM_2}{r_2^3})z \tag{5.47}$$

6.0 Results

6.1 Calculations

The calculations from Tables (6.1) to (6.8) were calculated with MATLAB simulations.

Table 6.1 Velocity of spacecraft on arrival to planet w/ respect to the Sun calculations

Planet A	Planet B	Result (km/s)
Earth	Venus	-37.7230
Earth	Mars	-21.4723
Earth	Jupiter	-7.4120
Earth	Saturn	-4.1864
Venus	Mars	-19.3527
Venus	Jupiter	-6.4490
Venus	Saturn	-3.6079
Mars	Jupiter	-8.7867
Mars	Saturn	-5.0447
Jupiter	Saturn	-8.0781

Table 6.2 Velocity of spacecraft on arrival to planet w/ respect to planet "B" calculations

Planet A	Planet B	Result (km/s)
Earth	Venus	-2.7070
Earth	Mars	2.6497
Earth	Jupiter	5.6422
Earth	Saturn	5.4388
Venus	Mars	4.7692
Venus	Jupiter	6.6052
Venus	Saturn	6.0173
Mars	Jupiter	4.2675
Mars	Saturn	4.5805
Jupiter	Saturn	1.5471

Table 6.3 Time of flight from planet "A" to planet "B"

Planet A	Planet B	Result (s)
Earth	Venus	1.2623×10^7
Earth	Mars	2.2375×10^7
Earth	Jupiter	8.6222×10^7
Earth	Saturn	1.9181×10^8
Venus	Mars	1.8798×10^7
Venus	Jupiter	8.0517×10^7
Venus	Saturn	1.8433×10^8
Mars	Jupiter	9.7374×10^7
Mars	Saturn	2.0625×10^8
Jupiter	Saturn	3.1693×10^8

Table 6.4 Time of flight of designated path

Path	Result (s)	Result (days)
1	1.9181×10^8	2.200×10^3
2	1.9695×10^8	2.2795×10^3
3	2.3767×10^8	2.7508×10^3
4	4.1007×10^8	4.7462×10^3
5	4.4572×10^8	5.1588×10^3
6	2.2862×10^8	2.6461×10^3
7	4.3668×10^8	5.0541×10^3
8	4.0315×10^8	4.6661×10^3

Table 6.5 Synodic period

Planets	Time (days)
Earth relative to Venus	585.9471
Mars relative to Venus	334.5779
Jupiter relative to Venus	237.3211
Saturn relative to Venus	229.8059
Mars relative to Earth	779.9085
Jupiter relative to Earth	398.8734
Saturn relative to Earth	378.0918
Jupiter relative to Mars	816.4200
Saturn relative to Mars	733.8595
Saturn relative to Jupiter	7.2569×10^3

Table 6.6 Mean motions

Planet	Velocity (rad/day)
Venus	0.0279
Earth	0.0172
Mars	0.0091
Jupiter	0.0014
Saturn	5.8399×10^{-4}

Table 6.7 Phase angles

Planets	Initial Phase Angle (degrees)	Final Phase Angle (degrees)
Venus to Earth	36.0073	-53.7519
Venus to Mars	65.9873	-168.1187
Venus to Jupiter	102.5878	-1.3111×10^3
Venus to Saturn	108.6150	-3.2335×10^3
Earth to Mars	44.2923	-75.2489
Earth to Jupiter	97.1034	-803.5765
Earth to Saturn	105.7175	-2.0081×10^3
Mars to Jupiter	86.3808	-410.5780
Mars to Saturn	100.1263	-1.0709×10^3
Jupiter to Saturn	57.2625	-124.7055

Table 6.8 Wait time

Planets	Time (days)
Venus to Earth	-1.0025×10^4
Venus to Mars	-1.7905×10^4
Venus to Jupiter	-9.9040×10^4
Venus to Saturn	-2.3653×10^5
Earth to Mars	-1.8681×10^4
Earth to Jupiter	-1.0203×10^5
Earth to Saturn	-2.4167×10^5
Mars to Jupiter	-1.0670×10^5
Mars to Saturn	-2.5015×10^5
Jupiter to Saturn	-2.8806×10^5

6.2 Flyby Hyperbola Calculations

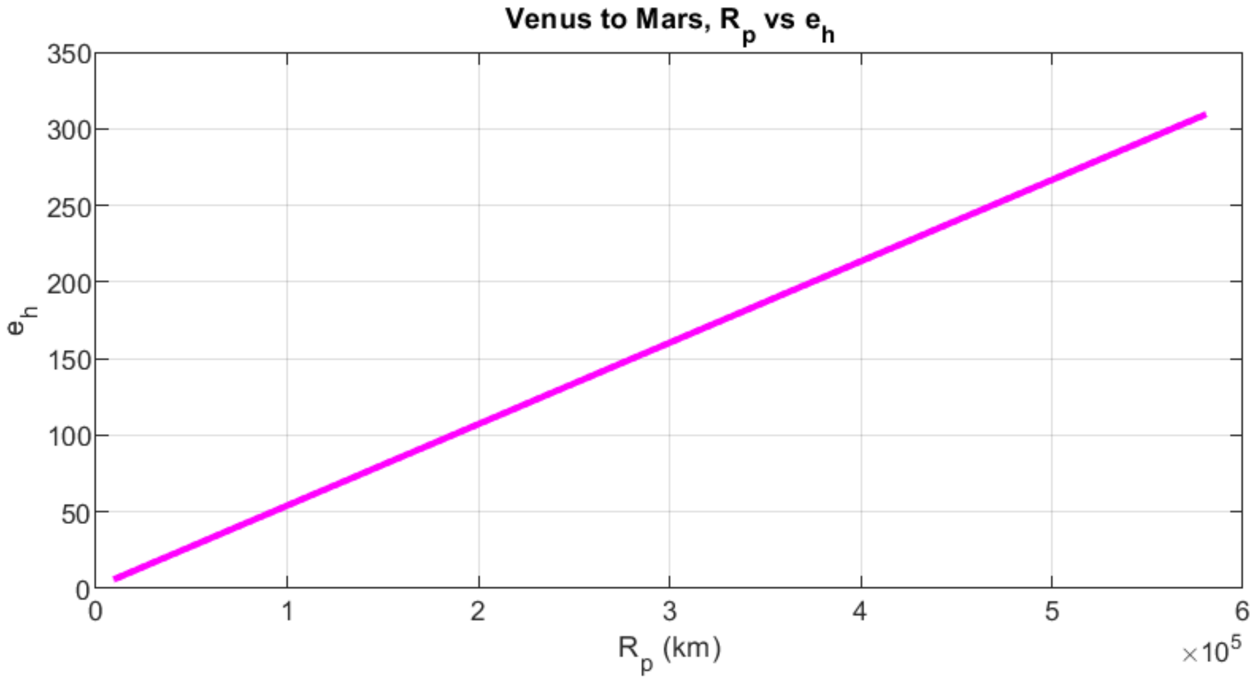

Figure 6.1 Venus to Mars, R_p vs. e_h

Fig. (6.1) displays the relationship between the radius and the eccentricity where the spacecraft departs from Venus and approaches Mars. The values for R_p are a percentage of the Mars Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This indicates that as the Mars R_p increases, the eccentricity also increases.

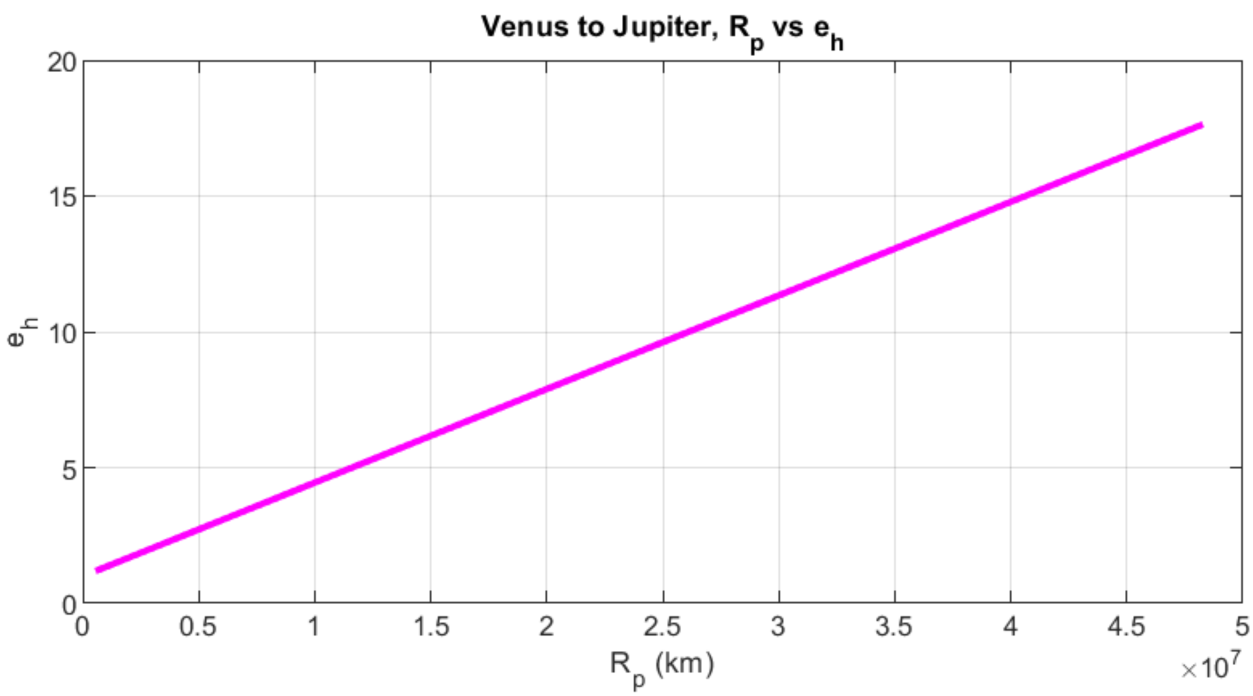

Figure 6.2 Venus to Jupiter, R_p vs. e_h

Fig. (6.2) shows the relationship between the radius and the eccentricity where the spacecraft departs from Venus and approaches Jupiter. The values for R_p are a percentage of the Jupiter Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This shows that as the Jupiter R_p increases, the eccentricity also increases.

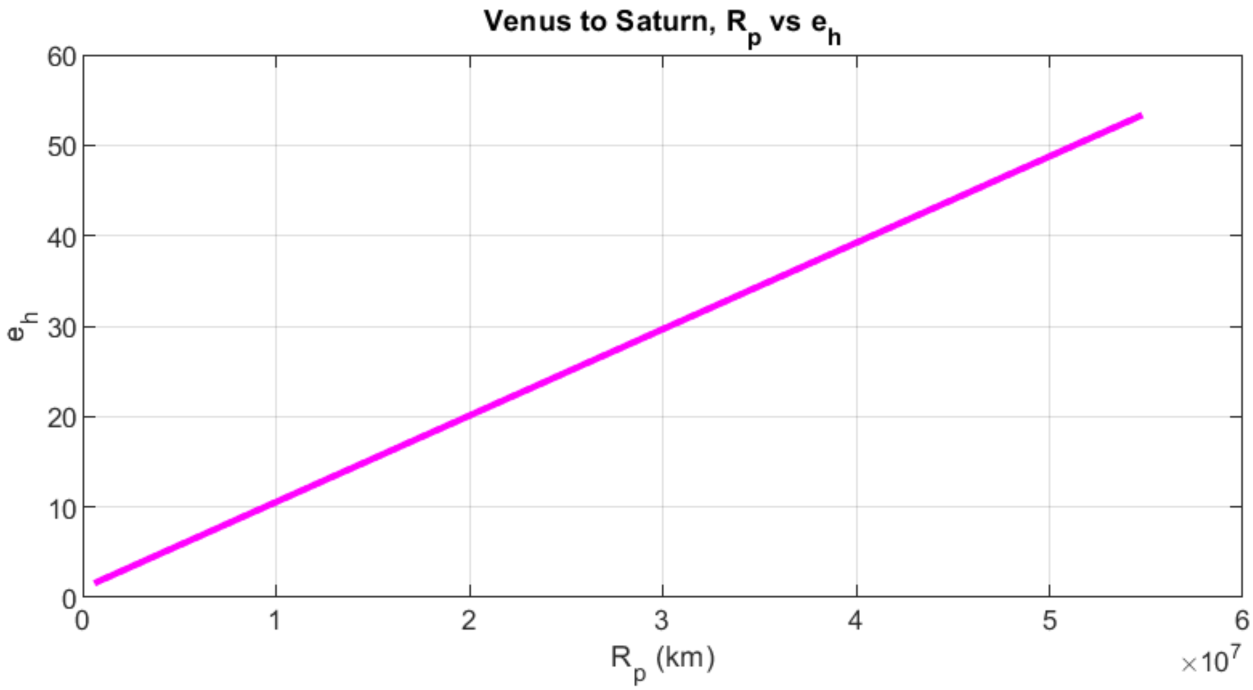

Figure 6.3 Venus to Saturn, R_p vs. e_h

Fig. (6.3) presents the relationship between the radius and the eccentricity where the spacecraft departs from Venus and approaches Saturn. The values for R_p are a percentage of the Saturn Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This represents that as the Saturn R_p increases, the eccentricity also increases.

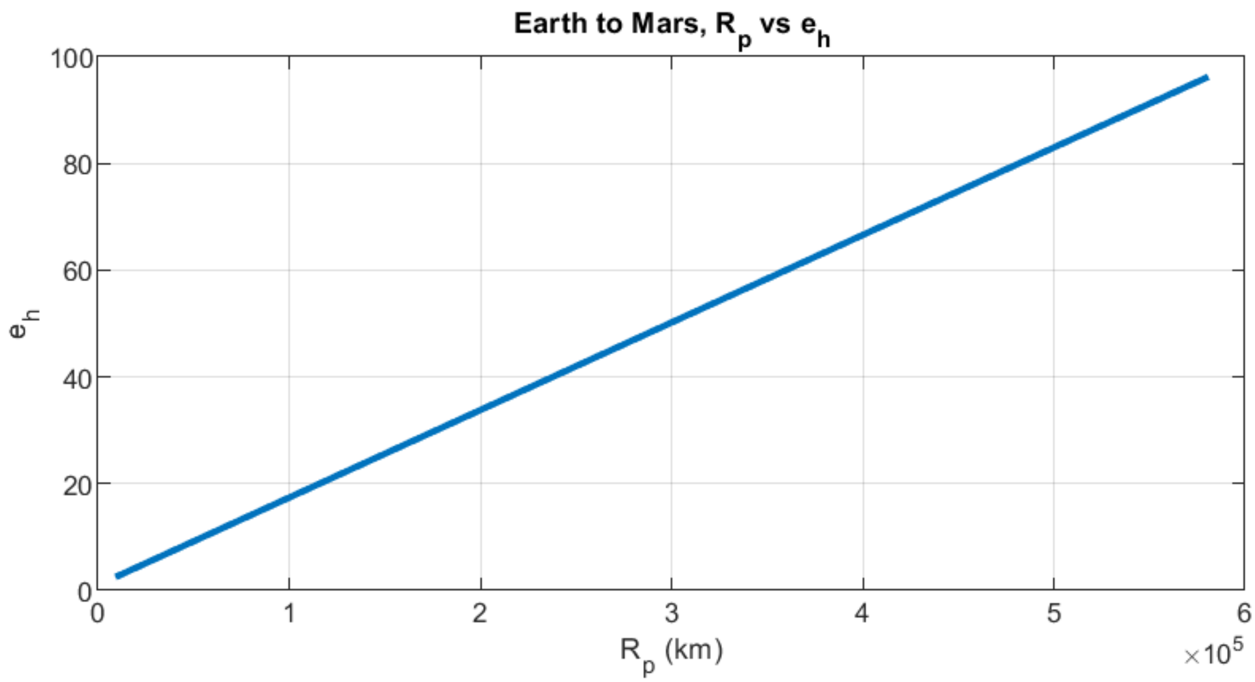

Figure 4.8 Earth to Mars, R_p vs. e_h

Fig. (6.4) shows the relationship between the radius and the eccentricity where the spacecraft departs from Earth and approaches Mars. The values for R_p are a percentage of the Mars Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This indicates that as the Mars R_p increases, the eccentricity also increases.

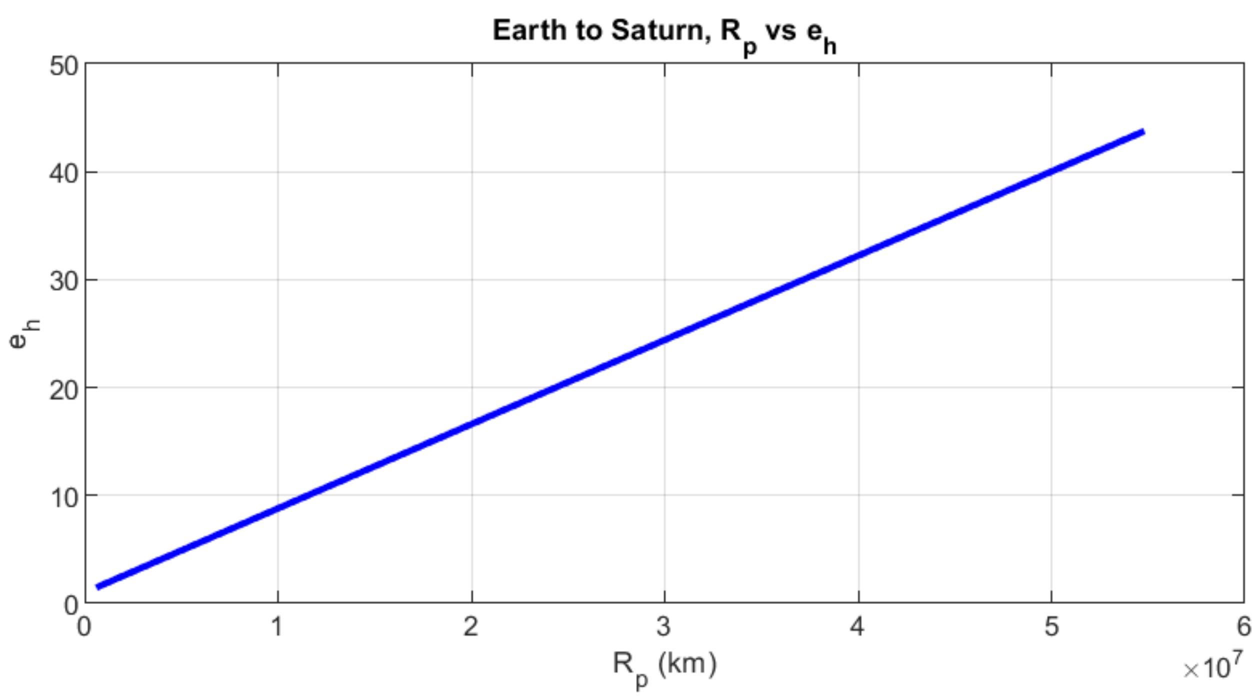

Figure 6.5 Earth to Saturn, R_p vs. e_h

Fig. (6.5) displays the relationship between the radius and the eccentricity where the spacecraft departs from Earth and approaches Saturn. The values for R_p are a percentage of the Saturn Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This shows that as the Saturn R_p increases, the eccentricity also increases.

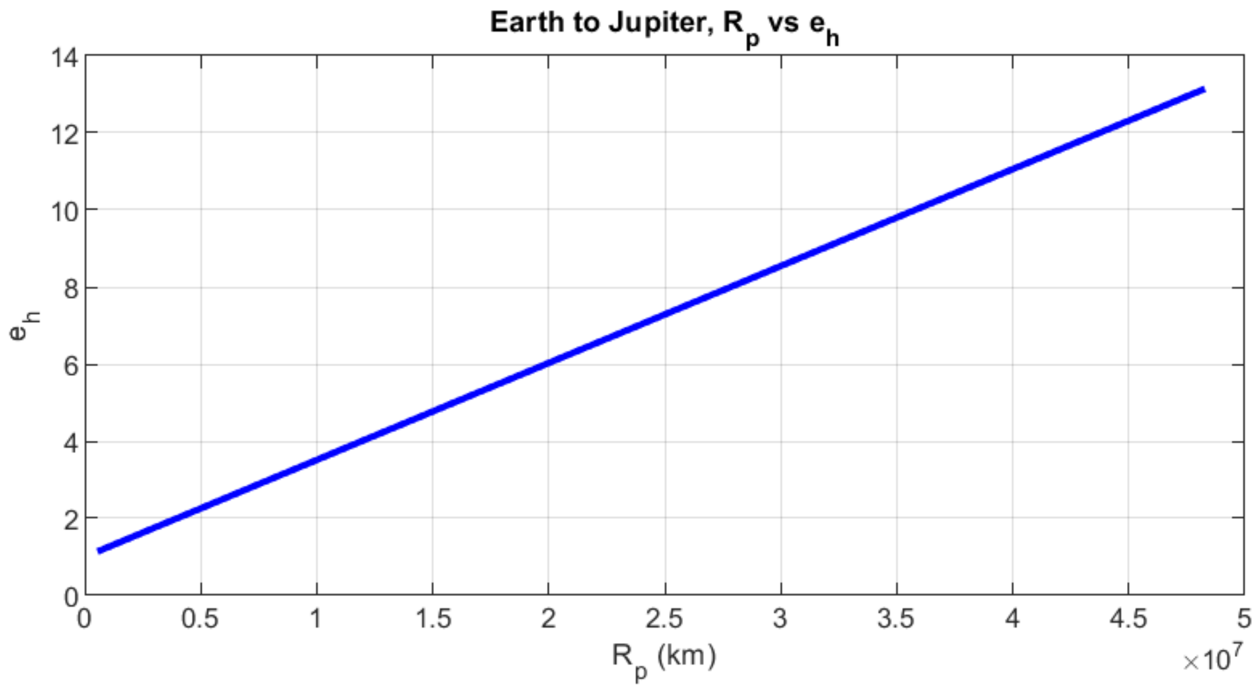

Figure 6.6 Earth to Jupiter, R_p vs. e_h

Fig. (6.6) displays the relationship between the radius and the eccentricity where the spacecraft departs from Earth and approaches Jupiter. The values for R_p are a percentage of the Jupiter Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This indicates that as the Jupiter R_p increases, the eccentricity also increases.

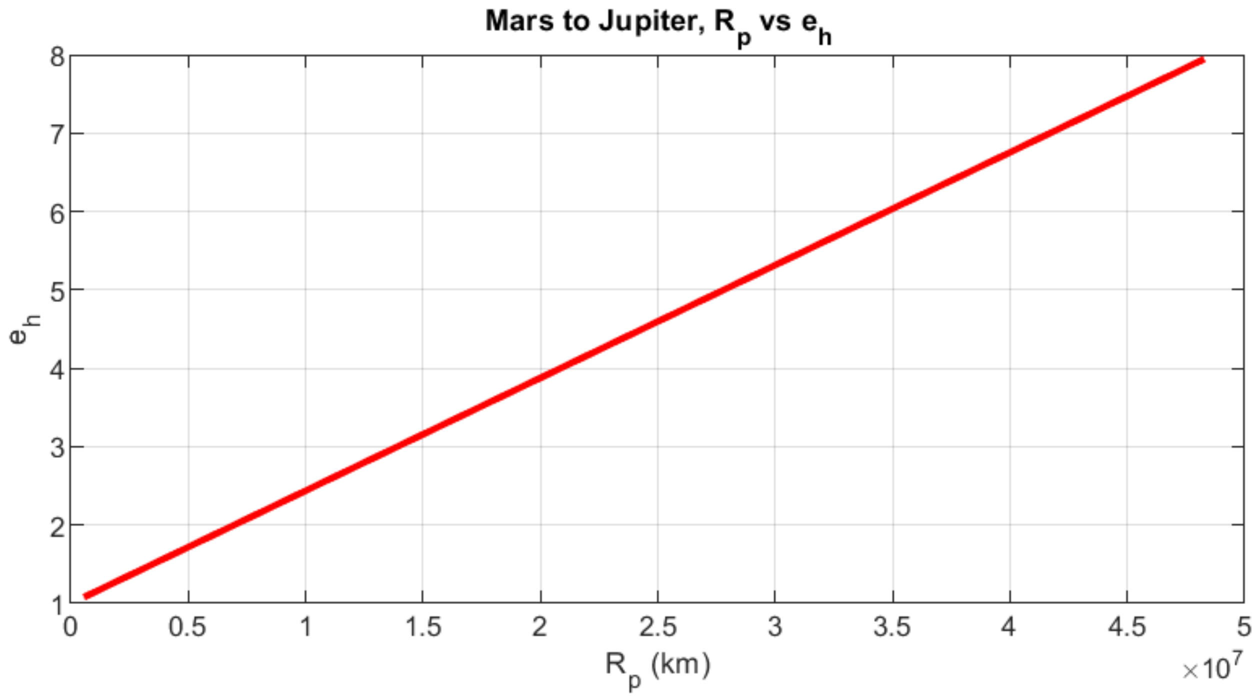

Figure 6.7 Mars to Jupiter, R_p vs. e_h

Fig. (6.7) shows the relationship between the radius and the eccentricity where the spacecraft departs from Mars and approaches Jupiter. The values for R_p are a percentage of the Jupiter Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This shows that as the Jupiter R_p increases, the eccentricity also increases.

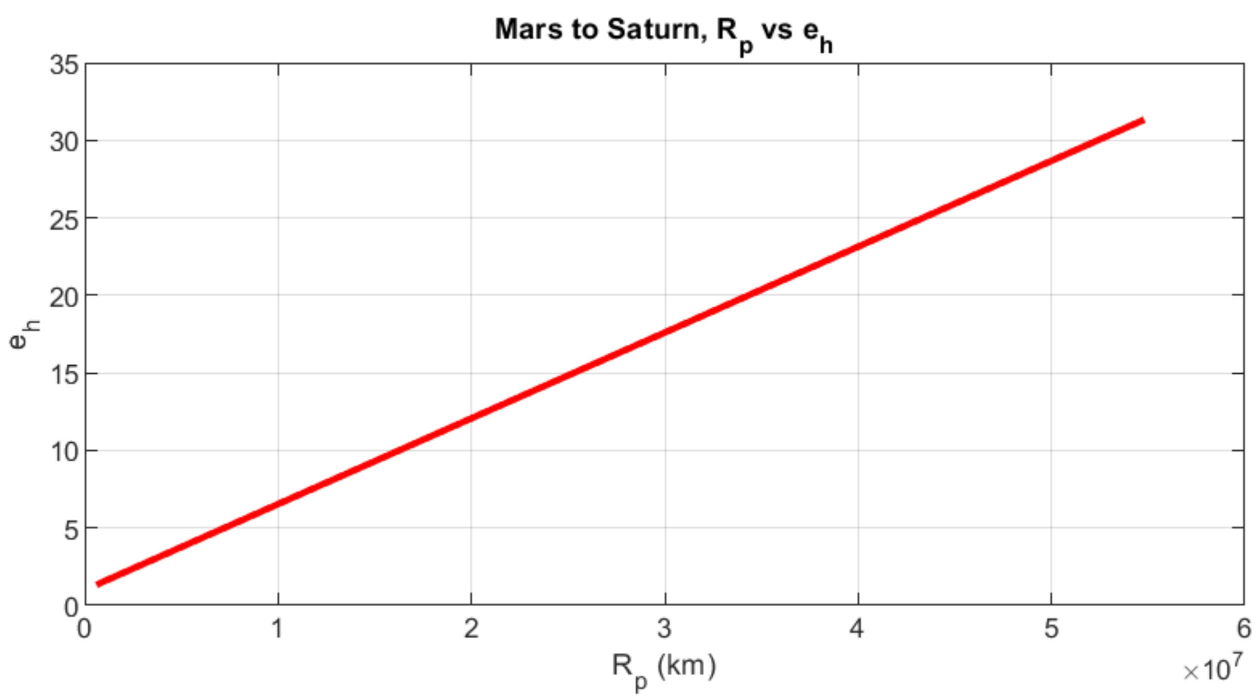

Figure 6.8 Mars to Saturn, R_p vs. e_h

Fig. (6.8) displays the relationship between the radius and the eccentricity where the spacecraft departs from Mars and approaches Saturn. The values for R_p are a percentage of the Saturn Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This indicates that as the Saturn R_p increases, the eccentricity also increases.

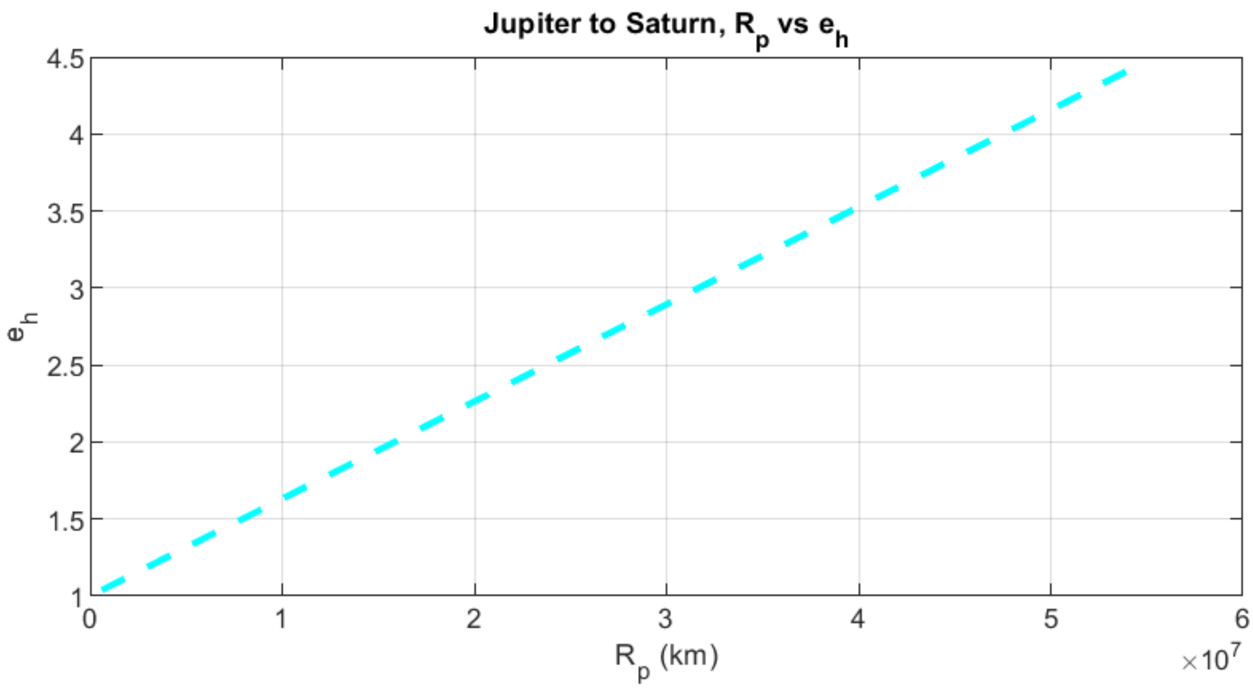

Figure 6.9 Jupiter to Saturn, R_p vs. e_h

Fig. (6.9) displays the relationship between the radius and the eccentricity where the spacecraft departs from Jupiter and approaches Saturn. The values for R_p are a percentage of the Saturn Sphere of Influence (SOI) that ranges from 1% to 100% of the SOI. This indicates that as the Saturn R_p increases, the eccentricity also increases.

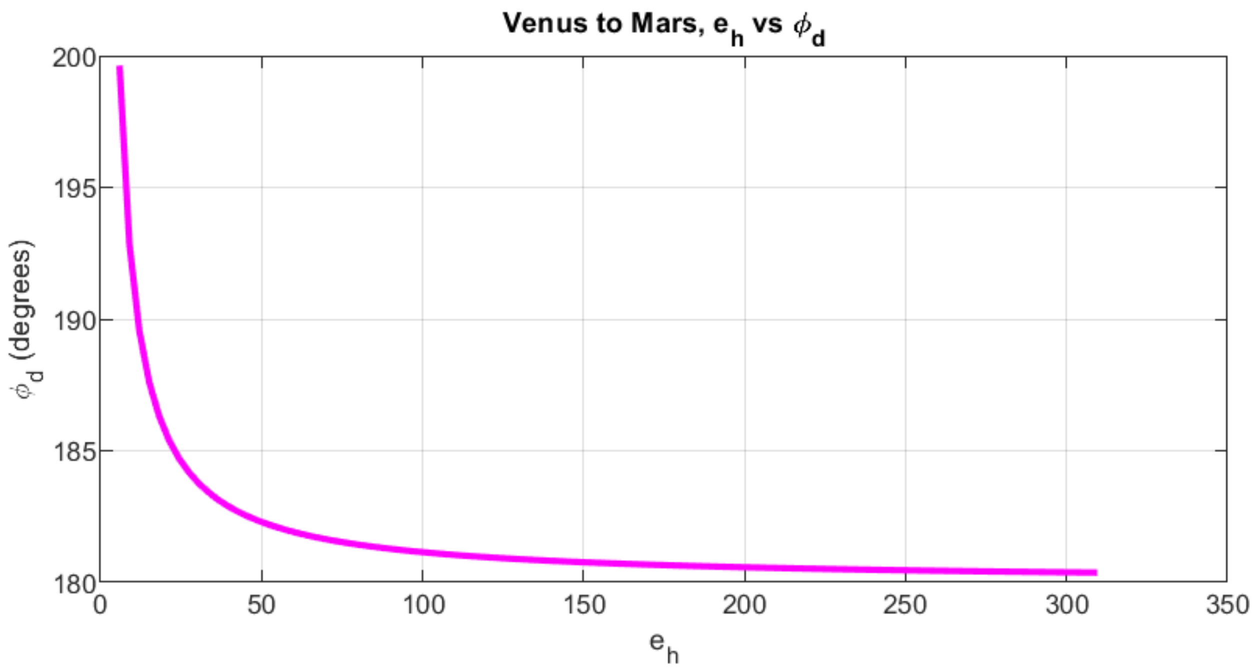

Figure 6.10 Venus to Mars, e_h vs. ϕ_d

Fig. (6.10) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Venus and approaches Mars. The values for e_h are the values calculated from the R_p from Fig. (6.1). This indicates that as the Mars R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

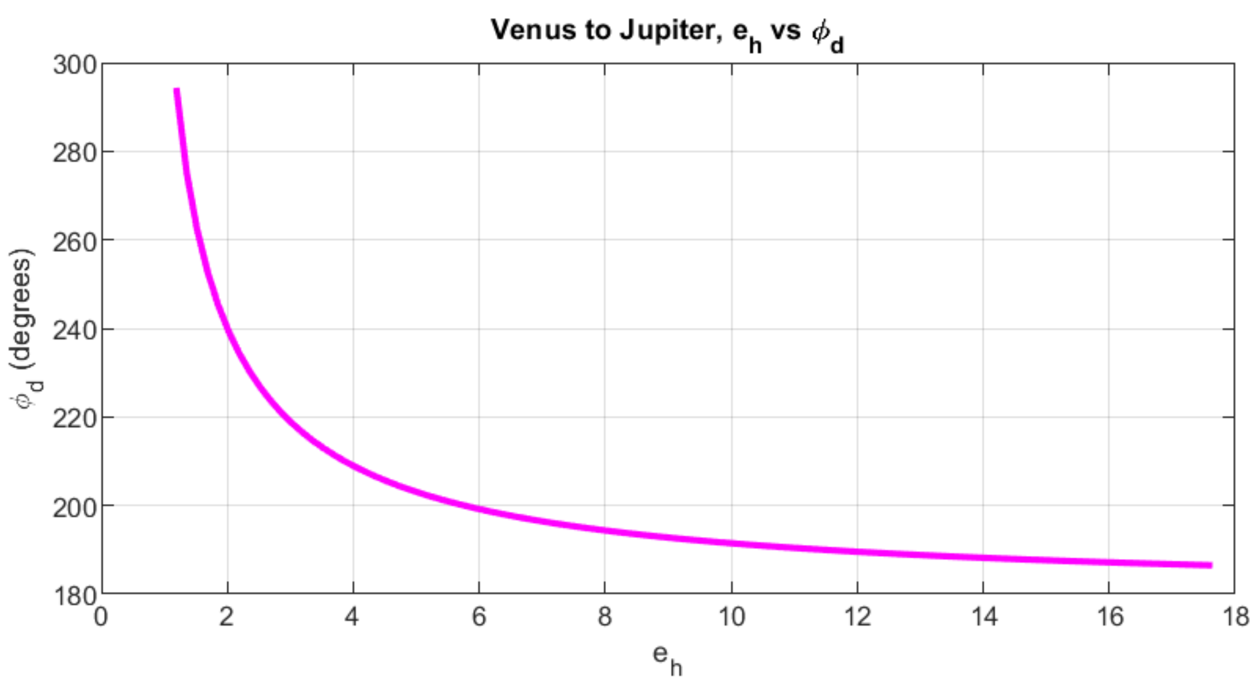

Figure 6.11 Venus to Jupiter, e_h vs. ϕ_d

Fig. (6.11) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Venus and approaches Jupiter. The values for e_h are the values calculated from the R_p from Fig. (6.2). This shows that as the Jupiter R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

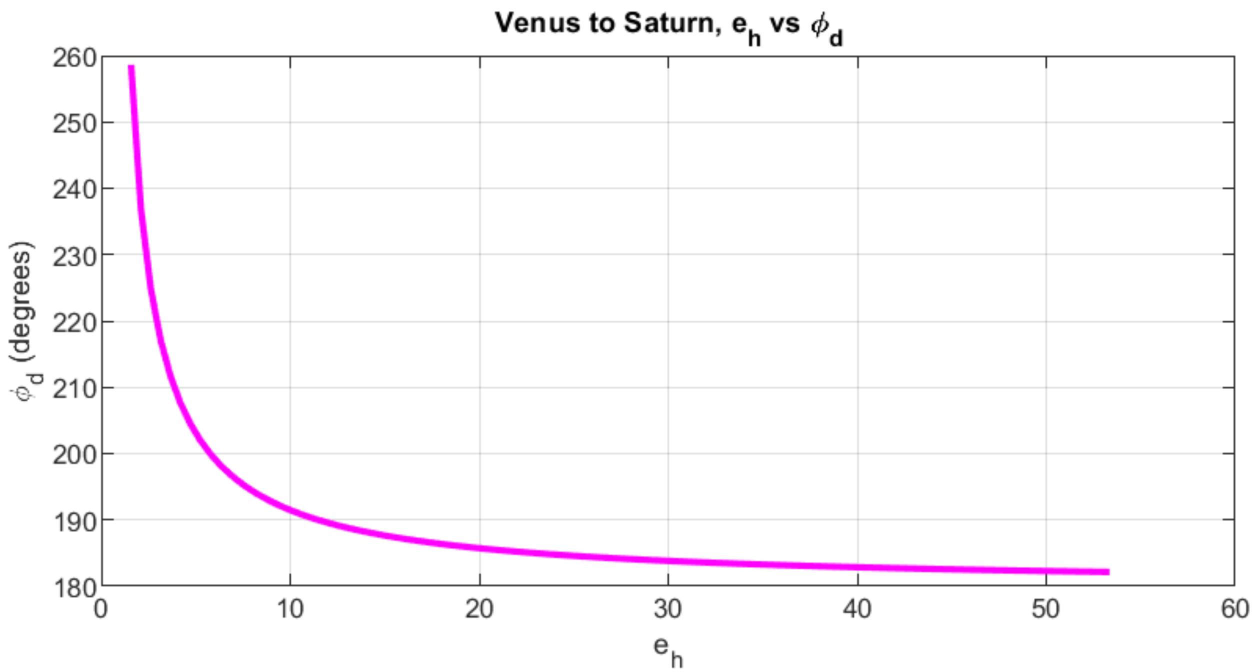

Figure 6.12 Venus to Saturn, e_h vs. ϕ_d

Fig. (6.12) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Venus and approaches Saturn. The values for e_h are the values calculated from the R_p from Fig. (6.3). This shows that as the Saturn R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

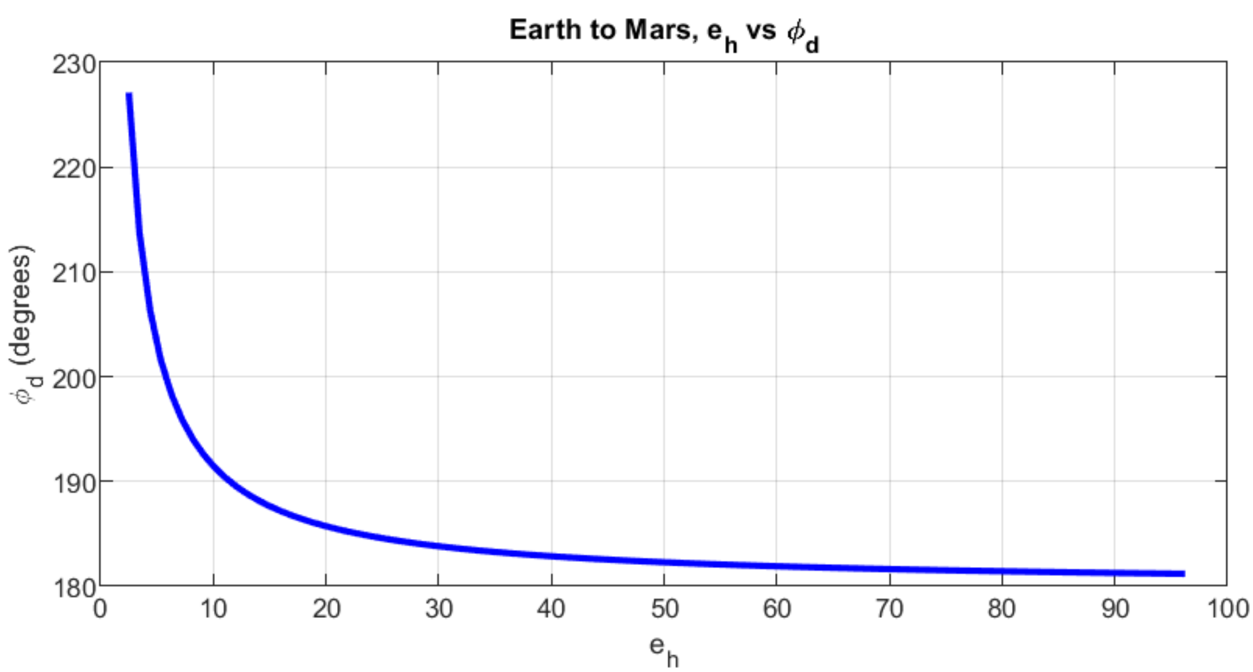

Figure 6.13 Earth to Mars, e_h vs. ϕ_d

Fig. (6.13) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Earth and approaches Mars. The values for e_h are the values calculated from the R_p from Fig. (6.4). This represents that as the Mars R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

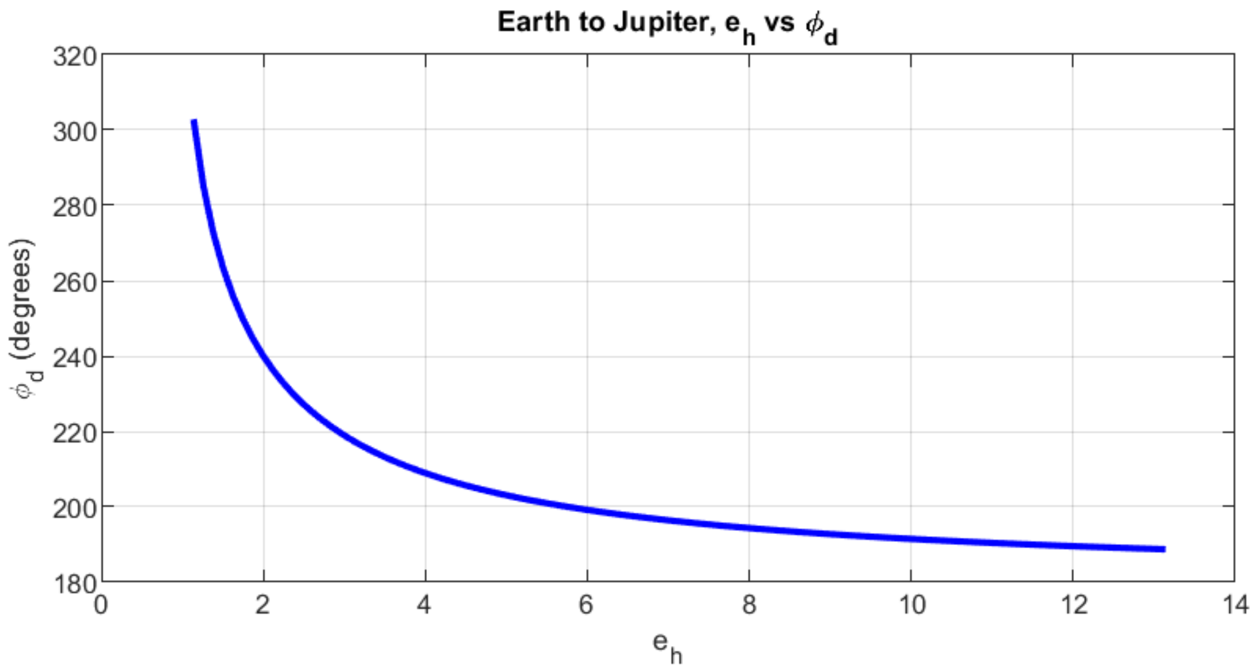

Figure 6.14 Earth to Jupiter, e_h vs. ϕ_d

Fig. (6.14) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Earth and approaches Jupiter. The values for e_h are the values calculated from the R_p from Fig. (6.5). This shows that as the Jupiter R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

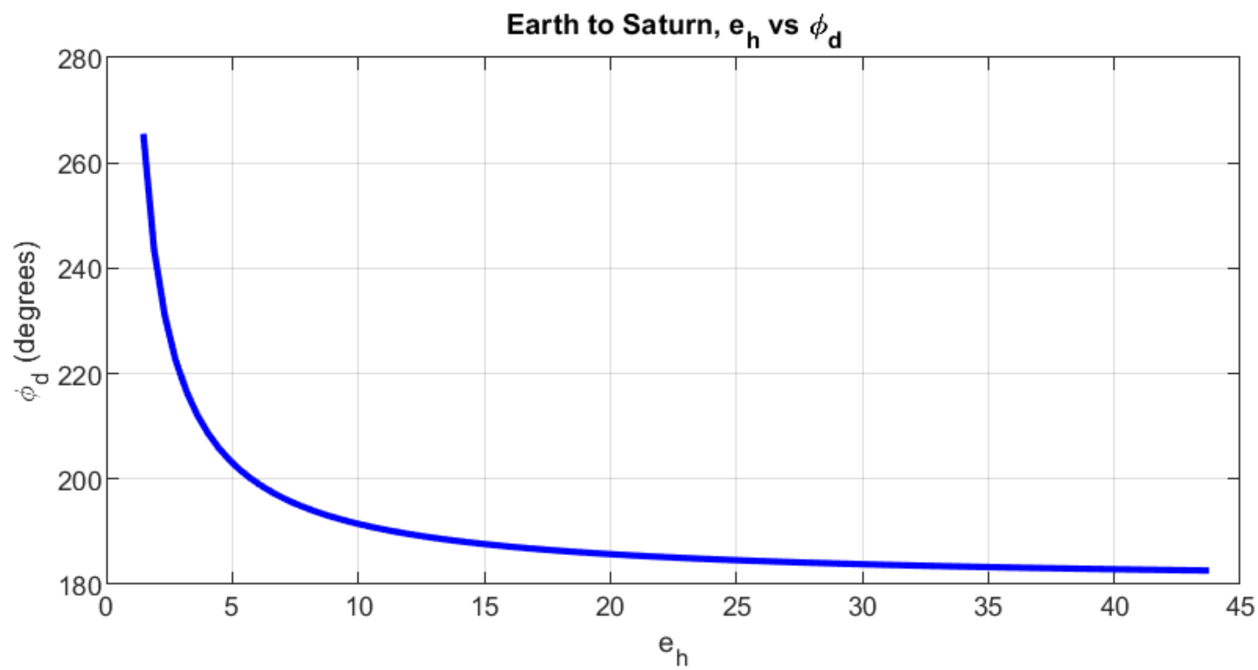

Figure 6.15 Earth to Saturn, e_h vs. ϕ_d

Fig. (6.15) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Earth and approaches Saturn. The values for e_h are the values calculated from the R_p from Fig. (6.6). This indicates that as the Saturn R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

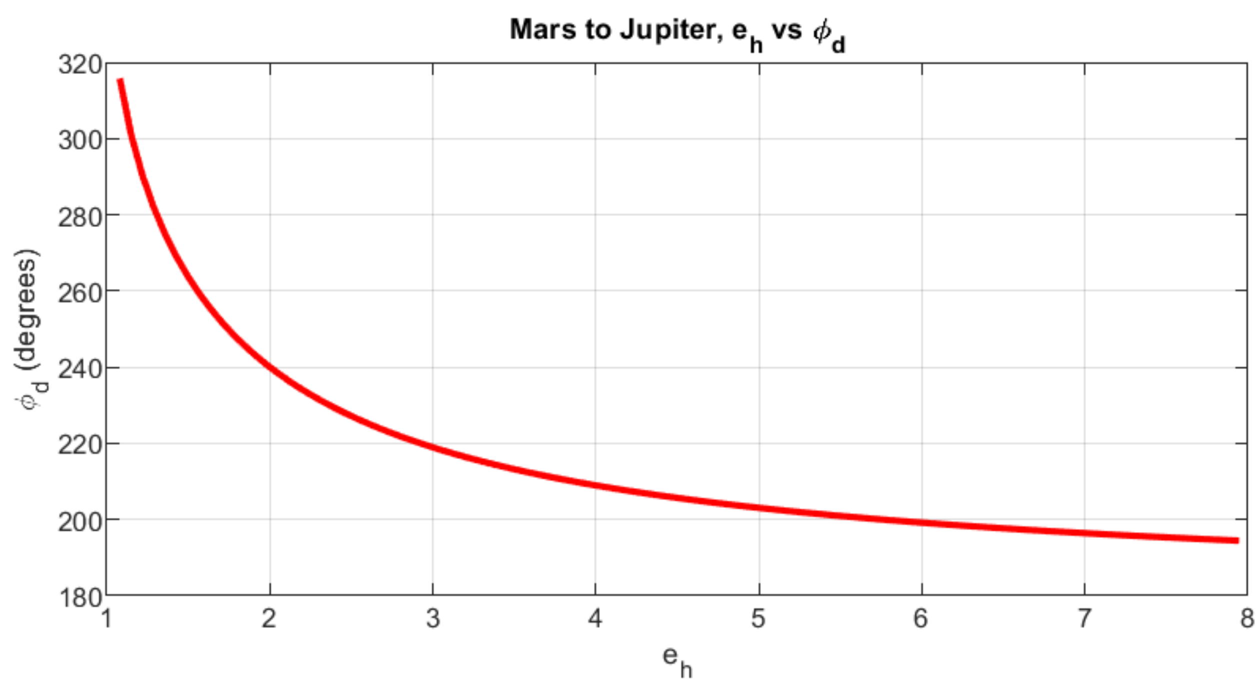

Figure 6.16 Mars to Jupiter, e_h vs. ϕ_d

Fig. (6.16) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Mars and approaches Jupiter. The values for e_h are the values calculated from the R_p from Fig. (6.7). This indicates that as the Jupiter R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

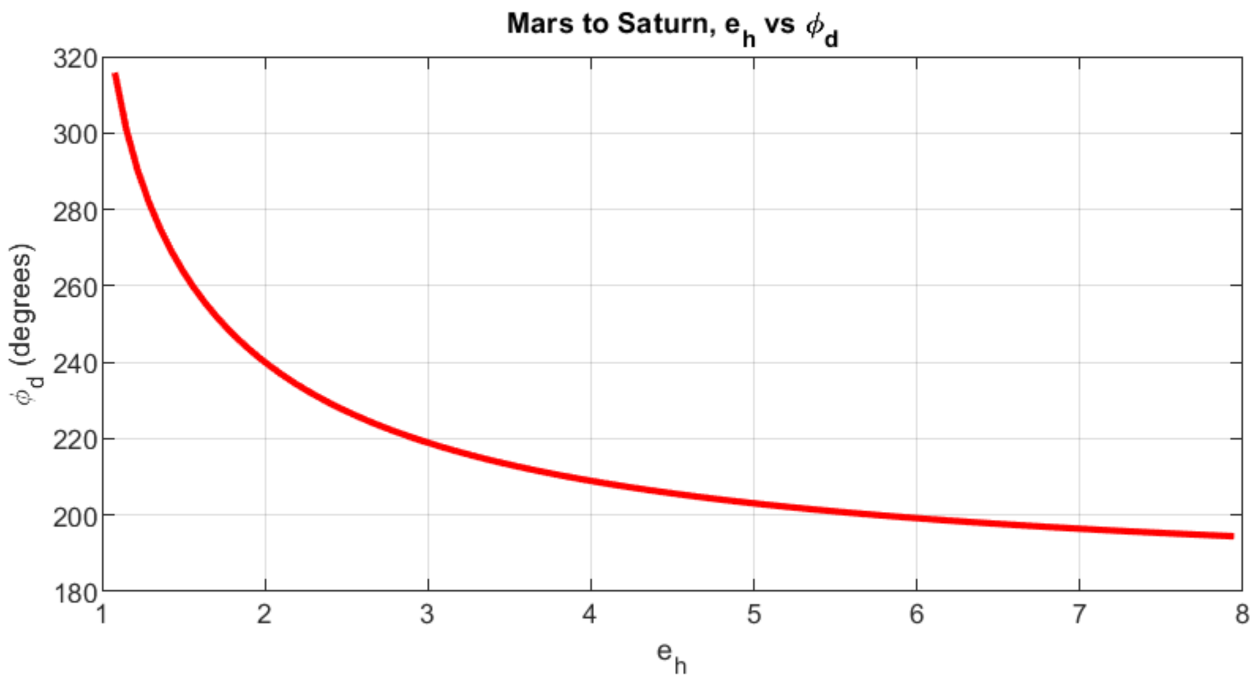

Figure 6.17 Mars to Saturn, e_h vs. ϕ_d

Fig. (6.17) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Mars and approaches Saturn. The values for e_h are the values calculated from the R_p from Fig. (6.8). This indicates that as the Saturn R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

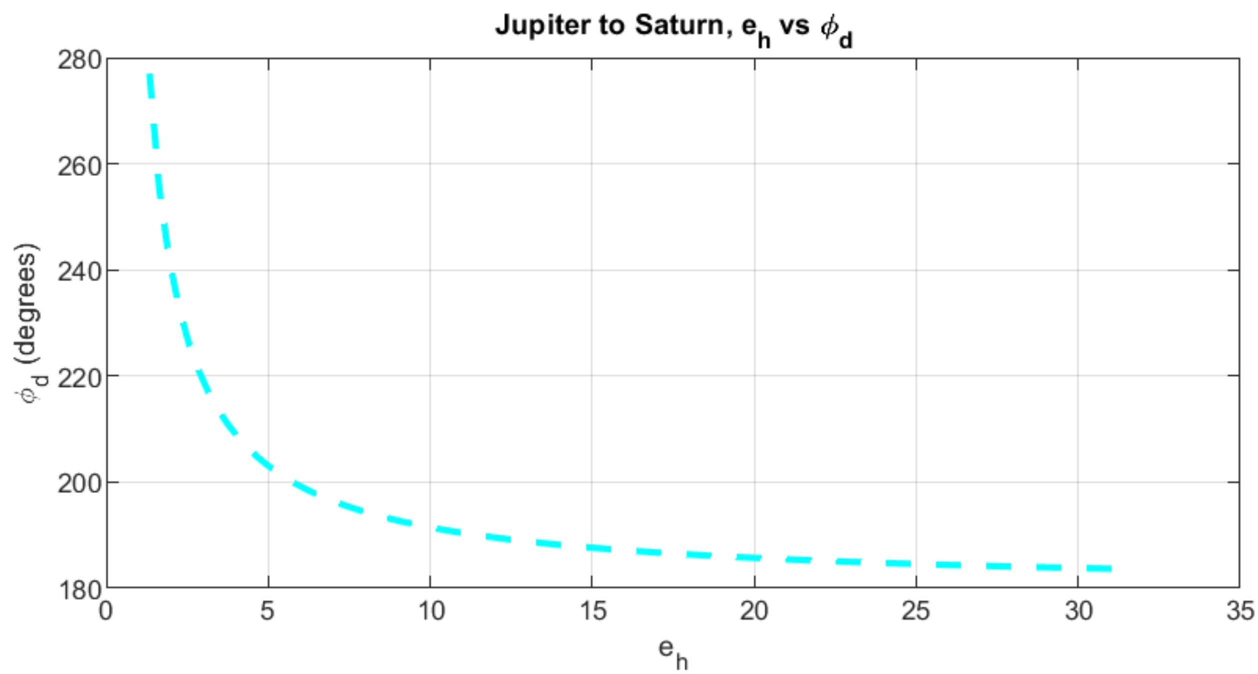

Figure 6.18 Jupiter to Saturn, e_h vs. ϕ_d

Fig. (6.18) displays the relationship between the eccentricity and the angle of the departure velocity, where the spacecraft departs from Jupiter and approaches Saturn. The values for e_h are the values calculated from the R_p from Fig. (6.9). This indicates that as the Saturn R_p increases, the eccentricity increases, which in turn reflects a decrease in the angle of the departure velocity.

6.3 Rendezvous

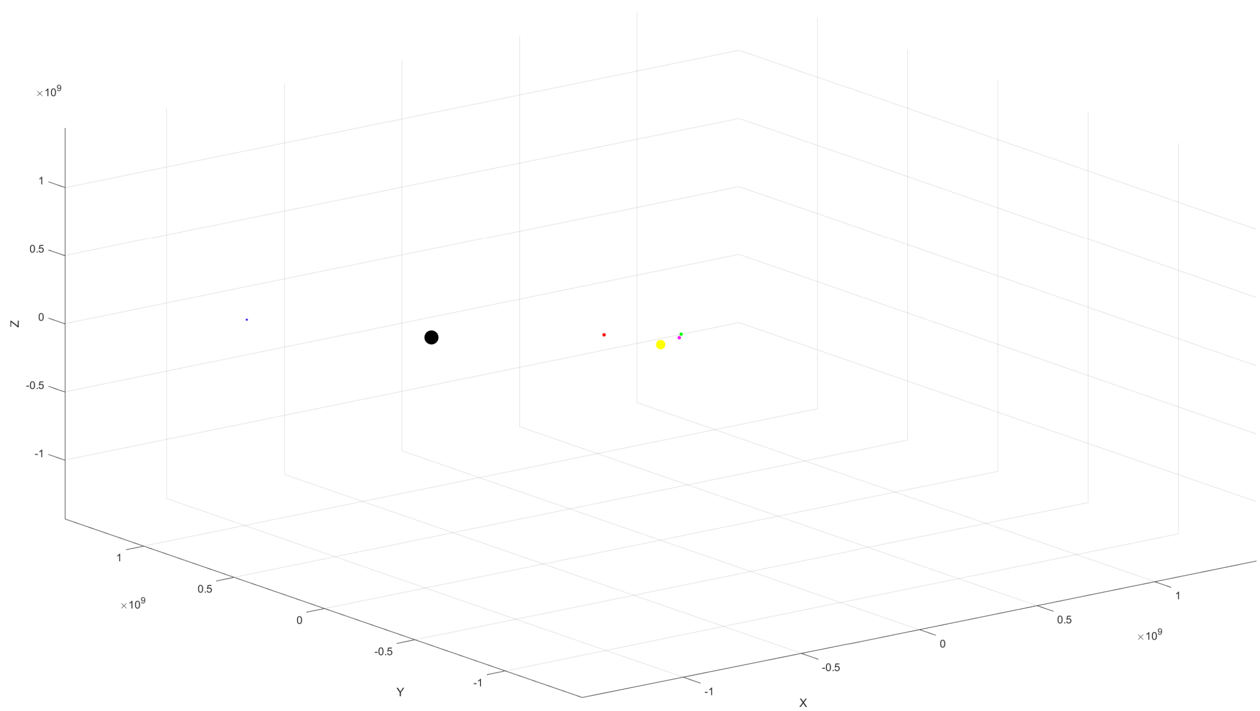

Figure 6.19 Position of planets upon departure

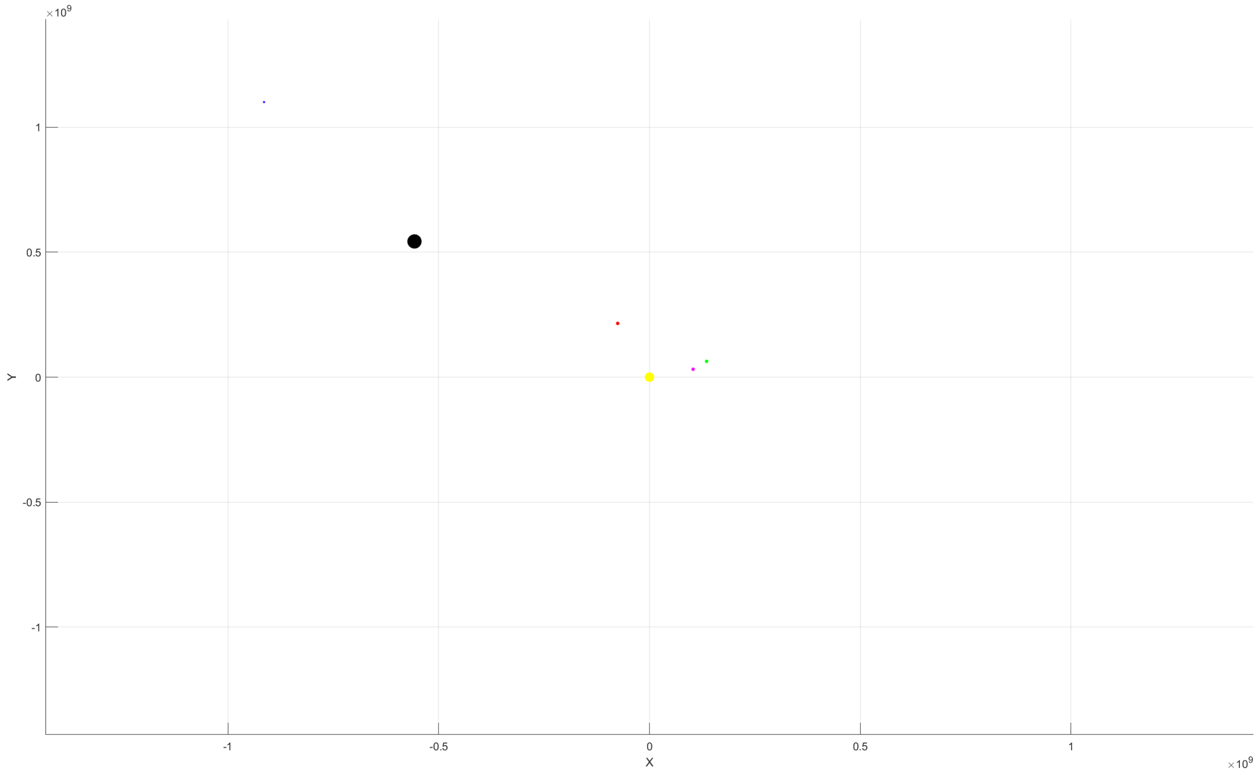

Figure 6.20 Position of planets upon departure as two-dimensional

Fig. (6.19) and (6.20) display the results of where planets are to be positioned when there is an ideal departure for an Earth to Saturn to Titan transfer. Since the most efficient method is to go from Earth to Saturn, the initial phase angle that is required is 105.7 degrees.

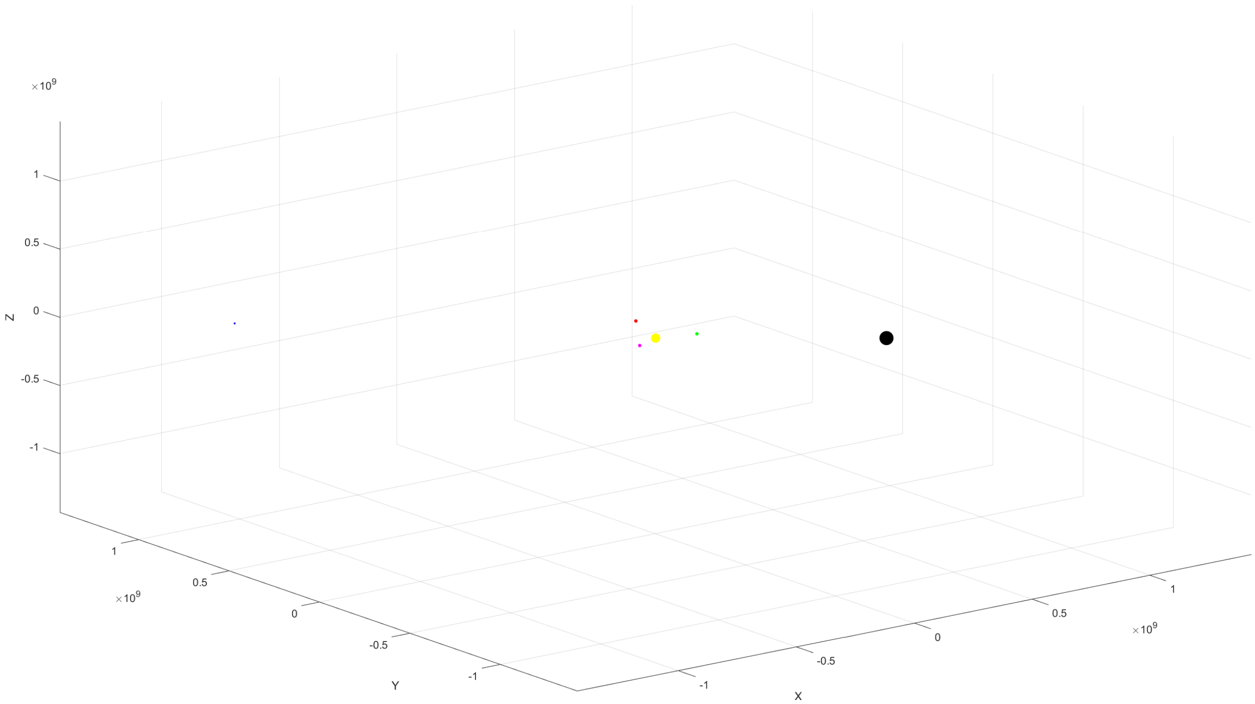

Figure 6.21 Position of planets upon arrival

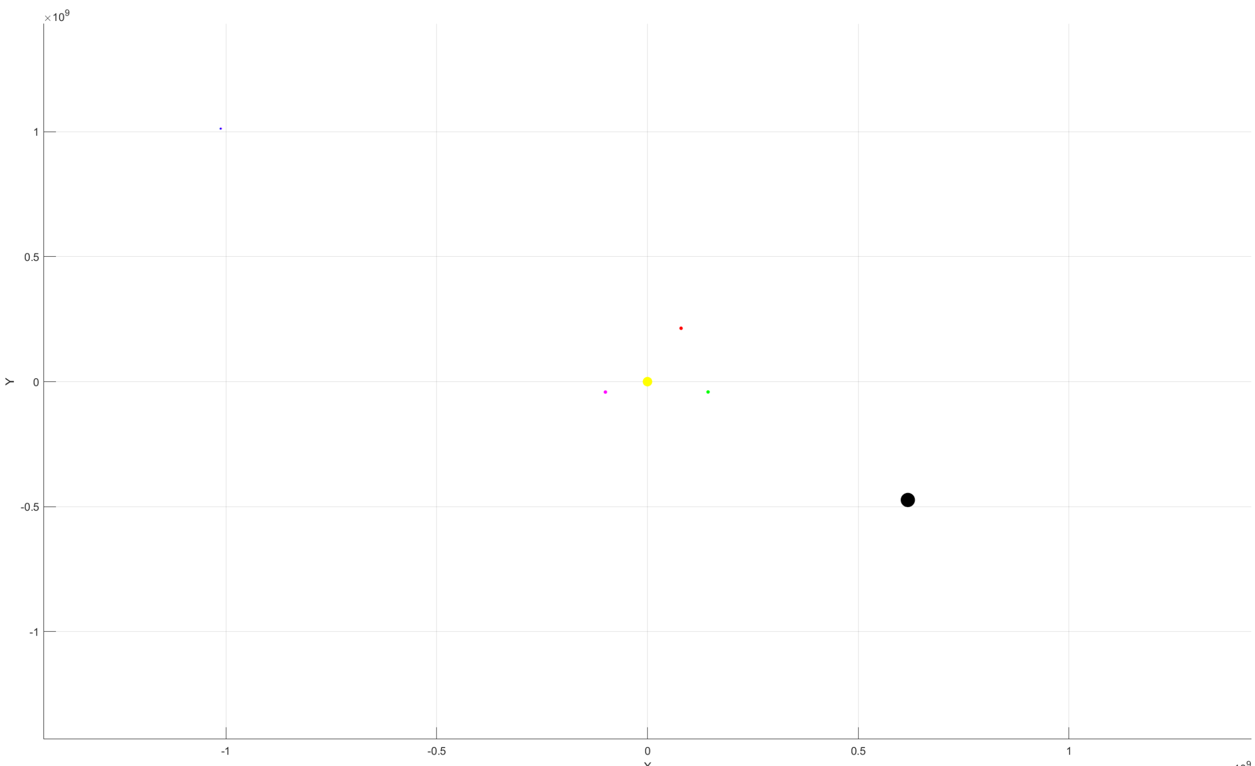

Figure 6.22 Position of planets upon arrival as two-dimensional

Fig. (6.21) and (6.22) display the results of where planets are to be positioned after the departure from Earth and the spacecraft has arrived at Saturn to Titan. The final phase angle that is required is -207.8 degrees.

7.0 Analysis

7.1 Flyby Analysis

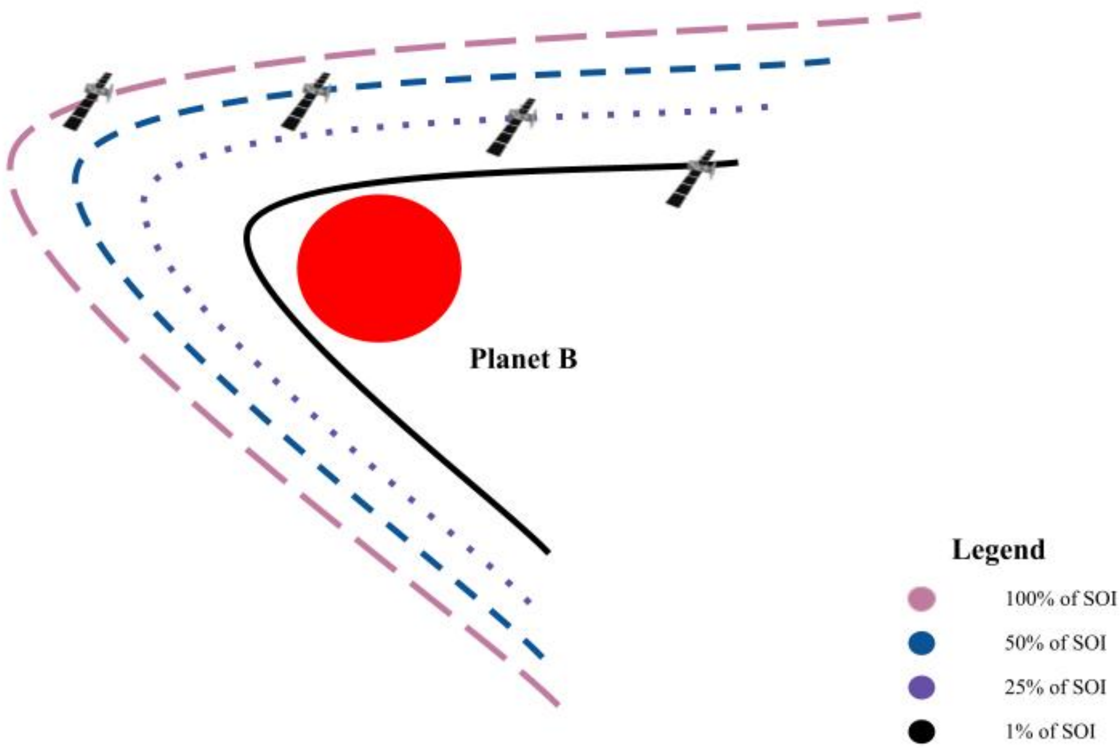

Figure 7.1 Various flyby analysis

In Fig. (6.1) – (6.9), the graphs display how the eccentricity increases as the radius increases. The calculations were completed in MATLAB. The r_p used is a range of the SOI, using one percent of the total SOI, to using 100% of the total SOI. In Fig. (6.10) – Fig. (6.18), the graphs display how the angle of the departure velocity decreases as the eccentricity decreases. The results of Fig. (6.1) – (6.18) can be analyzed and concluded in Fig. (6.19), which shows how an increase in r_p would result in a decrease in eccentricity, which affects the trajectory and angle of the flyby.

Understanding the angle of the flyby trajectory can be manipulated by the radius and eccentricity is significant to the calculations. These results prove that an increase in eccentricity results to a decrease in the departure angle. The departure angles must be ideal for the most efficient flybys and rendezvous for Titan.

7.2 Mission Analysis

The most cost-efficient method to get from Earth to Titan is to use a Hohmann transfer to Saturn. This method would take longer, however, the wait time to launch to Saturn is calculated to be 306.96 days. The time to get from Earth to Saturn would be 2.2200 x 10^3 days, or 6.08 years. To use an efficient flyby analysis to get from Earth to Jupiter to Saturn would be more than 100 years. Once at Saturn, the restricted three body problem for the Saturn-Titan system. The total time this mission would take to get to Titan is seven years, with no gravity assistance.

7.3 Discussion

The mission design was guided by the assumptions, which led to an efficient and simple trajectory. The Earth to Saturn to Titan path is the simplest and fastest, neglecting the wait time. If the assumption that of the flight path was removed, the results would be different. This current design uses equations of motion to calculate the velocities required in order to ensure a path without any excessive burns.

8.0 Conclusion

The Earth to Titan mission design is an interest in aerospace for the purpose that Titan has potential to host life. This evidence was provided by the Cassini mission. Previous studies have shown that life is possible because of the atmospheric makeup. The method to calculate an efficient path is the Hohmann transfer. Going step by step, the velocities were calculated through MATLAB and the results would show that an increase in the altitude increases the eccentricity, which would result in a gradual decrease in theta. The assumptions allowed for the design to focus on the velocity that would be ideal and reduce any unnecessary burns, resulting in an Earth to Saturn to Titan path.

Using the N-body simulations, an ideal path could be found using calculations for the synodic period and phase angles. The MATLAB code ran 100 years of iterations, which resulted in finding the most efficient path to be the Earth to Saturn Hohmann transfer. The total time would take seven years for the spacecraft to get into Titan's orbit. This path takes the least amount of time to send a spacecraft to orbit Saturn and is the most cost efficient. The concept of sending a spacecraft to Titan would advance the science in the aerospace industry.

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APPENDIX A. EARTH TO TITAN MATLAB CODE

```
close all;
clear all;
clc;
```

Initial Conditions

Julian Day (JD): 001 2023 UTC: 00:00:00

```
    % Variables
G = (6.67 * 10^(-20)); % (km^3)/(kgs^2)
Msun = 1.989 * (10^30); % kg
year = 2*pi; % Every 2*pi is equal to one year
Years = 200*year;

% Mass (kg)
M_Venus = 4.87 * (10^24);
M_Earth = 5.97 * (10^24);
M_Mars = 6.42 * (10^23);
M_Jupiter = 1.898 * (10^27);
M_Saturn = 5.683 * (10^26);

mu = G*Msun;
muv = G*M_Venus
mue = G*M_Earth
mum = G*M_Mars
muj = G*M_Jupiter
mus = G*M_Saturn

% radius of planet (km)
r_Venus = 6051;
r_Earth = 6378.1;
r_Mars = 3389.5;
r_Jupiter = 69911;
r_Saturn = 58232;

% semi-major axis (km)
a_Venus = 108.2 *(10^6);
a_Earth = 149.6 *(10^6);
a_Mars = 228.0 *(10^6);
a_Jupiter = 778.5 *(10^6);
a_Saturn = 143.2 *(10^7);
```

muv =

3.2483e+05

mue =

398199

mum =

4.2821e+04

muj =

1.2660e+08

mus =

3.7906e+07

Calculations

```
% Escape Velocity
Esc_Venus = sqrt((2*G*M_Venus)/r_Venus);
Esc_Earth = sqrt((2*G*M_Earth)/r_Earth);
Esc_Mars = sqrt((2*G*M_Mars)/r_Mars);
Esc_Jupiter = sqrt((2*G*M_Jupiter)/r_Jupiter);
Esc_Saturn = sqrt((2*G*M_Saturn)/r_Saturn);

% Circular Orbital Velocity of the planet with respect to the Sun (km/sec)
SVV = -sqrt(mu/a_Venus);
SVE = -sqrt(mu/a_Earth);
SVM = -sqrt(mu/a_Mars);
SVJ = -sqrt(mu/a_Jupiter);
SVS = -sqrt(mu/a_Saturn);

% semi-major axis between planets
EAV = (a_Earth + a_Venus)*.5;
EAM = (a_Earth + a_Mars)*.5;
EAJ = (a_Earth + a_Jupiter)*.5;
EAS = (a_Earth + a_Saturn)*.5;
VAM = (a_Venus + a_Mars)*.5;
VAJ = (a_Venus + a_Jupiter)*.5;
VAS = (a_Venus + a_Saturn)*.5;
MAJ = (a_Mars + a_Jupiter)*.5;
MAS = (a_Mars + a_Saturn)*.5;
JAS = (a_Jupiter + a_Saturn)*.5;

% Velocity of Spacecraft on departure of Planet w/ respect to the Sun
vd_EAV = sqrt(mu)*sqrt((2/a_Earth)-(1/EAV));
vd_EAM = sqrt(mu)*sqrt((2/a_Earth)-(1/EAM));
vd_EAJ = sqrt(mu)*sqrt((2/a_Earth)-(1/EAJ));
vd_EAS = sqrt(mu)*sqrt((2/a_Earth)-(1/EAS));
vd_VAM = sqrt(mu)*sqrt((2/a_Venus)-(1/VAM));
vd_VAJ = sqrt(mu)*sqrt((2/a_Venus)-(1/VAJ));
vd_VAS = sqrt(mu)*sqrt((2/a_Venus)-(1/VAS));
vd_MAJ = sqrt(mu)*sqrt((2/a_Mars)-(1/MAJ));
vd_MAS = sqrt(mu)*sqrt((2/a_Mars)-(1/MAS));
vd_JAS = sqrt(mu)*sqrt((2/a_Jupiter)-(1/JAS));

% Velocity of Spacecraft on arrival to Planet w/ respect to the Sun
v_EAV = -sqrt(mu)*sqrt((2/a_Venus)-(1/EAV));
v_EAM = -sqrt(mu)*sqrt((2/a_Mars)-(1/EAM));
v_EAJ = -sqrt(mu)*sqrt((2/a_Jupiter)-(1/EAJ));
v_EAS = -sqrt(mu)*sqrt((2/a_Saturn)-(1/EAS));
v_VAM = -sqrt(mu)*sqrt((2/a_Mars)-(1/VAM));
v_VAJ = -sqrt(mu)*sqrt((2/a_Jupiter)-(1/VAJ));
v_VAS = -sqrt(mu)*sqrt((2/a_Saturn)-(1/VAS));
v_MAJ = -sqrt(mu)*sqrt((2/a_Jupiter)-(1/MAJ));
v_MAS = -sqrt(mu)*sqrt((2/a_Saturn)-(1/MAS));
v_JAS = -sqrt(mu)*sqrt((2/a_Saturn)-(1/JAS));
```

```
% Velocity of Spacecraft on arrival to Planet w/respect to the Planet
% +/- dependent on Sun coordinate system
v_iEV = v_EAV - SVV;
v_iEM = v_EAM - SVM;
v_iEJ = v_EAJ - SVJ;
v_iES = v_EAS - SVS;
v_iVM = v_VAM - SVM;
v_iVJ = v_VAJ - SVJ;
v_iVS = v_VAS - SVS;
v_iMJ = v_MAJ - SVJ;
v_iMS = v_MAS - SVS;
v_iJS = v_JAS - SVS;
```

SOI Radius (km)

```
% Sphere of Influence (SOI)
VSOI = 616289.732;
ESOI = 924415.913;
MSOI = 577424.152;
JSOI = 48208452.07;
SSOI = 54743849.22;

% Time of Flight (seconds)

tof_VE = (pi/sqrt(mu))*(EAV)^(3/2);
tof_VM = (pi/sqrt(mu))*(VAM)^(3/2);
tof_VJ = (pi/sqrt(mu))*(VAJ)^(3/2);
tof_VS = (pi/sqrt(mu))*(VAS)^(3/2);
tof_EM = (pi/sqrt(mu))*(EAM)^(3/2);
tof_EJ = (pi/sqrt(mu))*(EAJ)^(3/2);
tof_ES = (pi/sqrt(mu))*(EAS)^(3/2);
tof_MJ = (pi/sqrt(mu))*(MAJ)^(3/2);
tof_MS = (pi/sqrt(mu))*(MAS)^(3/2);
tof_JS = (pi/sqrt(mu))*(JAS)^(3/2);

% Convert Time of Flight to days
dof_VE = tof_VE/86400;
dof_VM = tof_VM/86400;
dof_VJ = tof_VJ/86400;
dof_VS = tof_VS/86400;
dof_EM = tof_EM/86400;
dof_EJ = tof_EJ/86400;
dof_ES = tof_ES/86400;
dof_MJ = tof_MJ/86400;
dof_MS = tof_MS/86400;
dof_JS = tof_JS/86400;

% Flight Paths
%Earth to Saturn
fp_ES = tof_ES;
%Earth to Venus to Saturn
fp_EVS = tof_VE + tof_VS;
%Earth to Venus to Mars to Saturn
fp_EVMS = tof_VE + tof_VM + tof_MS;
%Earth to Venus to Jupiter to Saturn
fp_EVJS = tof_VE + tof_VJ + tof_JS;
%Earth to Venus to Mars to Jupiter to Saturn
fp_EVMJS = tof_VE + tof_VM + tof_MJ + tof_JS;
%Earth to Mars to Saturn
fp_EMS = tof_EM + tof_MS;
%Earth to Mars to Jupiter to Saturn
```

```
fp_EMJS = tof_EM + tof_MJ + tof_JS;
%Earth to Jupiter to Saturn
fp_EJS = tof_EJ + tof_JS;

%Flight Paths conversion to days (1 day = 86400 s)
%Earth to Saturn
dfp_ES = fp_ES/86400;
%Earth to Venus to Saturn
dfp_EVS = fp_EVS/86400;
%Earth to Venus to Mars to Saturn
dfp_EVMS = fp_EVMS/86400;
%Earth to Venus to Jupiter to Saturn
dfp_EVJS = fp_EVJS/86400;
%Earth to Venus to Mars to Jupiter to Saturn
dfp_EVMJS = fp_EVMJS/86400;
%Earth to Mars to Saturn
dfp_EMS = fp_EMS/86400;
%Earth to Mars to Jupiter to Saturn
dfp_EMJS = fp_EMJS/86400;
%Earth to Jupiter to Saturn
dfp_EJS = fp_EJS/86400;
```

Rp

```
%this is the radius of the planet plus the altitude
%the altitude is a range of the planet's SOI

dx = .01; %step
%Venus
hv = (.01*VSOI):dx*VSOI:(1.0*VSOI);
vrp = r_Venus + hv;
%Earth
he = (.01*ESOI):dx*ESOI:(1.0*ESOI);
erp = r_Earth + he;
%Mars
hm = (.01*MSOI):dx*MSOI:(1.0*MSOI);
mrp = r_Mars + hm;
%Jupiter
hj = (.01*JSOI):dx*JSOI:(1.0*JSOI);
jrp = r_Jupiter + hj;
%Saturn
hs = (.01*SSOI):dx*SSOI:(1.0*SSOI);
srp = r_Saturn + hs;
```

The Eccentricity of the flyby hyperbola

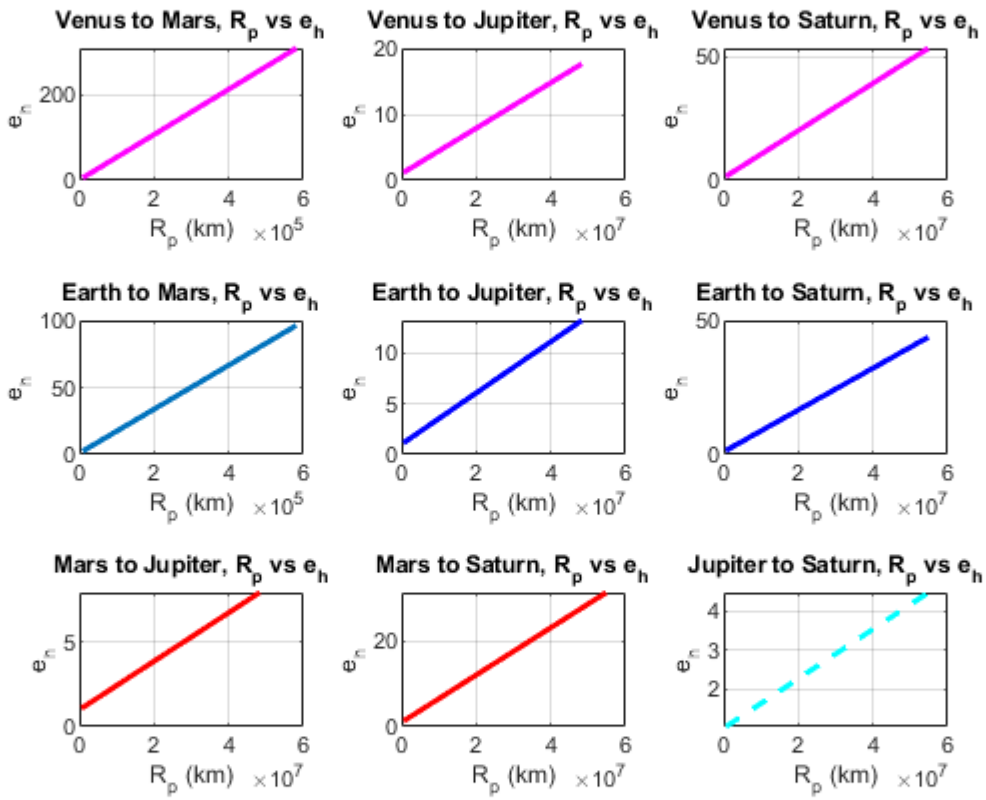
```
% Venus to Mars
VMeh = 1 + ((mrp*(v_iVM)^2)/(G*M_Mars));
% Venus to Jupiter
VJeh = 1 + ((jrp*(v_iVJ)^2)/(G*M_Jupiter));
% Venus to Saturn
VSeh = 1 + ((srp*(v_iVS)^2)/(G*M_Saturn));
% Earth to Mars
EMeh = 1 + ((mrp*(v_iEM)^2)/(G*M_Mars));
% Earth to Jupiter
EJeh = 1 + ((jrp*(v_iEJ)^2)/(G*M_Jupiter));
% Earth to Saturn
ESeh = 1 + ((srp*(v_iES)^2)/(G*M_Saturn));
% Mars to Jupiter
MJeh = 1 + ((jrp*(v_iMJ)^2)/(G*M_Jupiter));
```

```
% Mars to Saturn
MSeh = 1 + ((srp*(v_iMS)^2)/(G*M_Saturn));
% Jupiter to Saturn
JSeh = 1 + ((srp*(v_iJS)^2)/(G*M_Saturn));
```

```
figure('Name','R_p vs e_h','NumberTitle','off'),
t = tiledlayout(3, 3)
nexttile
plot(mrp, VMeh, 'm')
title('Venus to Mars, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(jrp, VJeh, 'm')
title('Venus to Jupiter, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(srp, VSeh, 'm')
title('Venus to Saturn, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(mrp, EMeh)
title('Earth to Mars, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(jrp, EJeh,'b')
title('Earth to Jupiter, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(srp, ESeh,'b')
title('Earth to Saturn, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on
```

```
nexttile
plot(jrp, MJeh,'r')
title('Mars to Jupiter, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(srp, MSeh,'r')
title('Mars to Saturn, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(srp, JSeh,'c--')
title('Jupiter to Saturn, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on
```

```
t = 

  TiledChartLayout with properties:

    TileArrangement: 'fixed'
           GridSize: [3 3]
            Padding: 'loose'
        TileSpacing: 'loose'

  Use GET to show all properties
```




```
one = 1;

phi_a = 180



% Venus to Mars
VMdelta = 2*asind(one./VMeh);
% Venus to Jupiter
VJdelta = 2*asind(one./VJeh);
% Venus to Saturn
VSdelta = 2*asind(one./VSeh);
% Earth to Mars
EMdelta = 2*asind(one./EMeh);
% Earth to Jupiter
EJdelta = 2*asind(one./EJeh);
% Earth to Saturn
ESdelta = 2*asind(one./ESeh);
% Mars to Jupiter
MJdelta = 2*asind(one./MJeh);
% Mars to Saturn
MSdelta = 2*asind(one./MSeh);
% Jupiter to Saturn
JSdelta = 2*asind(one./JSeh);
```

```
phi_a =

   180
```

```
VMphi_d = phi_a + VMdelta;

% Venus to Jupiter
VJphi_d = phi_a + VJdelta;
% Venus to Saturn
VSphi_d = phi_a + VSdelta;
% Earth to Mars
EMphi_d = phi_a + EMdelta;
% Earth to Jupiter
EJphi_d = phi_a + EJdelta;
% Earth to Saturn
ESphi_d = phi_a + ESdelta;
% Mars to Jupiter
MJphi_d = phi_a + MJdelta;
% Mars to Saturn
MSphi_d = phi_a + MSdelta;
% Jupiter to Saturn
JSphi_d = phi_a + JSdelta;
```

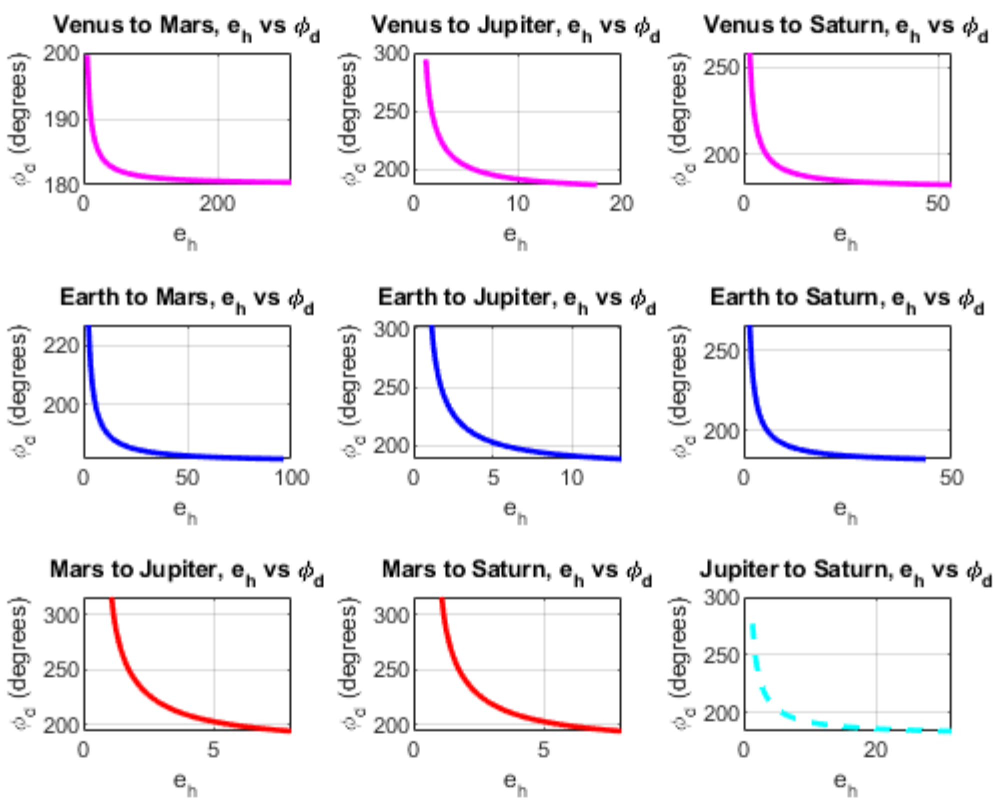
Eccentricity vs Delta Phi plot

```
figure('Name','e_h vs. \phi_d','NumberTitle','off'),
t = tiledlayout(3, 3)
nexttile
plot(VMeh,VMphi_d,'m')
title('Venus to Mars, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(VJeh,VJphi_d,'m')
title('Venus to Jupiter, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(VSeh,VSphi_d,'m')
title('Venus to Saturn, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on
```

```
nexttile
plot(EMeh,EMphi_d,'b')
title('Earth to Mars, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(EJeh,EJphi_d,'b')
title('Earth to Jupiter, e_h vs \phi_d')
xlabel('e_h')
    ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(ESeh,ESphi_d,'b')
title('Earth to Saturn, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(MJeh,MJphi_d,'r')
title('Mars to Jupiter, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(MJeh,MJphi_d,'r')
title('Mars to Saturn, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

nexttile
plot(MSeh,MSphi_d,'c--')
title('Jupiter to Saturn, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on
```

```
t = 

  TiledChartLayout with properties:
```

```
TileArrangement: 'fixed'

       GridSize: [3 3]

        Padding: 'loose'

    TileSpacing: 'loose'


Use GET to show all properties
```



Delta Velocity

```
% Venus to Mars
vd_iVM = [sin(VMdelta); cos(VMdelta)].*v_iVM; %V_INF * COS(DELTA) SY -
V_INF*SIN(DELTA) SX
vdVM = [0; SVM] - vd_iVM; %Flyby Departure Velocity of the Spacecraft w/respect to the
Sun
VMgradvel = vdVM - [0; v_VAM]; % Flyby Departure Velocity - Flyby Arrival Velocity
w/respect to the Sun (Change in Vel.)
VMgradvelfin = sqrt(sum(VMgradvel.^2)); %magnitude of the change in velocity (km/sec)
VMvsubinf = vdVM(2,:); %Isolate second row (SY)
h_VM = a_Mars * VMvsubinf; %specific angular momentum
vdVMtwo = sqrt(sum(vdVM.^2)); %magnitude of departure velocity
a_VM = -(((vdVMtwo.^2)/mu) - (2/a_Mars));
newaVM = a_VM.^(-1);
```

```
VMetwo = sqrt(1-((h_VM.^2)/(newaVM*mu)));
VMnumtheta = ((h_VM.^2)-a_Mars*mu);
VMdentheta = (a_Mars*mu*VMetwo);
VMnewtheta = acosd(VMnumtheta/VMdentheta);

% Venus to Jupiter
vd_iVJ = [sin(VJdelta); cos(VJdelta)].*v_iVJ;
vdVJ = [0; SVJ] - vd_iVJ;
VJgradvel = vdVJ - [0; v_VAJ];
VJgradvelfin = sqrt(sum(VJgradvel.^2));
VJvsubinf = vdVJ(2,:);
h_VJ = a_Jupiter * VJvsubinf;
vdVJtwo = sqrt(sum(vdVJ.^2));
a_VJ = -(((vdVJtwo.^2)/mu) - (2/a_Jupiter));
newaVJ = a_VJ.^(-1);
VJetwo = sqrt(1-((h_VJ.^2)/(newaVJ*mu)));
VJnumtheta = ((h_VJ.^2)-a_Jupiter*mu);
VJdentheta = (a_Jupiter*mu*VJetwo);
VJnewtheta = acosd(VJnumtheta/VJdentheta);

% Venus to Saturn
vd_iVS = [sin(VSdelta); cos(VSdelta)].*v_iVS;
vdVS = [0; SVS] - vd_iVS;
VSgradvel = vdVS - [0; v_VAS];
VSgradvelfin = sqrt(sum(VSgradvel.^2));
VSvsubinf = vdVS(2,:);
h_VS = a_Saturn * VSvsubinf;
vdVStwo = sqrt(sum(vdVS.^2));
a_VS = -(((vdVStwo.^2)/mu) - (2/a_Saturn));
newaVS = a_VS.^(-1);
VSetwo = sqrt(1-((h_VS.^2)/(newaVS*mu)));
VSnumtheta = ((h_VS.^2)-a_Saturn*mu);
VSdentheta = (a_Saturn*mu*VSetwo);
VSnewtheta = acosd(VSnumtheta/VSdentheta);

% Earth to Mars
vd_iEM = [sin(EMdelta); cos(EMdelta)].*v_iEM;
vdEM = [0; SVM] - vd_iEM;
EMgradvel = vdEM - [0; v_EAM];
EMgradvelfin = sqrt(sum(EMgradvel.^2));
EMvsubinf = vdEM(2,:);
h_EM = a_Mars * EMvsubinf;
vdEMtwo = sqrt(sum(vdEM.^2));
a_EM = -(((vdEMtwo.^2)/mu) - (2/a_Mars));
newaEM = a_EM.^(-1);
EMetwo = sqrt(1-((h_EM.^2)/(newaEM*mu)));
EMnumtheta = ((h_EM.^2)-a_Mars*mu);
EMdentheta = (a_Mars*mu*EMetwo);
EMnewtheta = acosd(EMnumtheta/EMdentheta);

% Earth to Jupiter
vd_iEJ = [sin(EJdelta); cos(EJdelta)].*v_iEJ;
vdEJ = [0; SVJ] - vd_iEJ;
EJgradvel = vdEJ - [0; v_EAJ];
EJgradvelfin = sqrt(sum(EJgradvel.^2));
EJvsubinf = vdEJ(2,:);
h_EJ = a_Jupiter * EJvsubinf;
vdEJtwo = sqrt(sum(vdEJ.^2));
a_EJ = -(((vdEJtwo.^2)/mu) - (2/a_Jupiter));
newaEJ = a_EJ.^(-1);
EJetwo = sqrt(1-((h_EJ.^2)/(newaEJ*mu)));
EJnumtheta = ((h_EJ.^2)-a_Jupiter*mu);
EJdentheta = (a_Jupiter*mu*EJetwo);
```

```
EJnewtheta = acosd(EJnumtheta/EJdentheta);

% Earth to Saturn
vd_iES = [sin(ESdelta); cos(ESdelta)].*v_iES;
vdES = [0; SVS] - vd_iES;
ESgradvel = vdES - [0; v_EAS];
ESgradvelfin = sqrt(sum(ESgradvel.^2));
ESvsubinf = vdES(2,:);
h_ES = a_Saturn * ESvsubinf;
vdEStwo = sqrt(sum(vdES.^2));
a_ES = -(((vdEStwo.^2)/mu) - (2/a_Saturn));
newaES = a_ES.^(-1);
ESetwo = sqrt(1-((h_ES.^2)/(newaES*mu)));
ESnumtheta = ((h_ES.^2)-a_Saturn*mu);
ESdentheta = (a_Saturn*mu*ESetwo);
ESnewtheta = acosd(ESnumtheta/ESdentheta);

% Mars to Jupiter
vd_iMJ = [sin(MJdelta); cos(MJdelta)].*v_iMJ;
vdMJ = [0; SVJ] - vd_iMJ;
MJgradvel = vdMJ - [0; v_MAJ];
MJgradvelfin = sqrt(sum(MJgradvel.^2));
MJvsubinf = vdMJ(2,:);
h_MJ = a_Jupiter * MJvsubinf;
vdMJtwo = sqrt(sum(vdMJ.^2));
a_MJ = -(((vdMJtwo.^2)/mu) - (2/a_Jupiter));
newaMJ = a_MJ.^(-1);
MJetwo = sqrt(1-((h_MJ.^2)/(newaMJ*mu)));
MJnumtheta = ((h_MJ.^2)-a_Jupiter*mu);
MJdentheta = (a_Jupiter*mu*MJetwo);
MJnewtheta = acosd(MJnumtheta/MJdentheta);

% Mars to Saturn
vd_iMS = [sin(MSdelta); cos(MSdelta)].*v_iMS;
vdMS = [0; SVS] - vd_iMS;
MSgradvel = vdMS - [0; v_MAS];
MSgradvelfin = sqrt(sum(MSgradvel.^2));
MSvsubinf = vdMS(2,:);
h_MS = a_Saturn * MSvsubinf;
vdMStwo = sqrt(sum(vdMS.^2));
a_MS = -(((vdMStwo.^2)/mu) - (2/a_Saturn));
newaMS = a_MS.^(-1);
MSetwo = sqrt(1-((h_MS.^2)/(newaMS*mu)));
MSnumtheta = ((h_MS.^2)-a_Saturn*mu);
MSdentheta = (a_Saturn*mu*MSetwo);
MSnewtheta = acosd(MSnumtheta/MSdentheta);

% Jupiter to Saturn
vd_iJS = [sin(JSdelta); cos(JSdelta)].*v_iJS;
vdJS = [0; SVS] - vd_iJS;
JSgradvel = vdJS - [0; v_JAS];
JSgradvelfin = sqrt(sum(JSgradvel.^2));
JSvsubinf = vdJS(2,:);
h_JS = a_Saturn * JSvsubinf;
vdJStwo = norm(vdJS);
a_JS = -(((vdJStwo.^2)/mu) - (2/a_Saturn));
newaJS = a_JS.^(-1);
JSetwo = sqrt(1-((h_JS.^2)/(newaJS*mu)));
JSnumtheta = ((h_JS.^2)-a_Saturn*mu);
JSdentheta = (a_Saturn*mu*JSetwo);
JSnewtheta = acosd(JSnumtheta/JSdentheta);
```

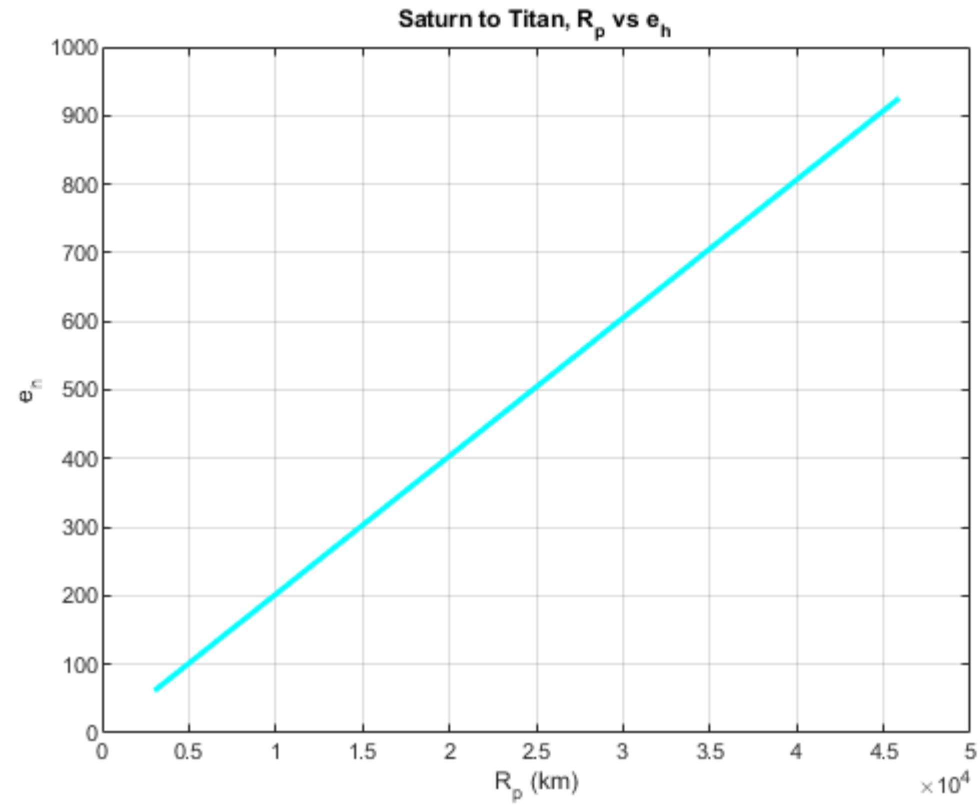
```
% Titan

M_Titan = 1.345 * (10^23); % Mass of Titan (kg)
r_Titan = 2575 % radius of Titan (km)
a_Titan = 1221870 % semi-major axis of Titan (km)
mut = G*M_Titan

% Barycenter
mass_planet = M_Saturn
mass_moon = M_Titan

pi_one = mass_planet/(mass_planet+mass_moon)
pi_two = mass_moon/(mass_moon+mass_planet)

mr_one = pi_two*a_Titan % mass ratio of planet
mr_two = pi_one*a_Titan % mass ratio of moon

% Angular Velocity
av = (sqrt(G*(mass_planet+mass_moon)))/a_Titan^(3/2) ;
```

```
r_Titan =

        2575

a_Titan =

     1221870

mut =

   8.9711e+03

mass_planet =
```

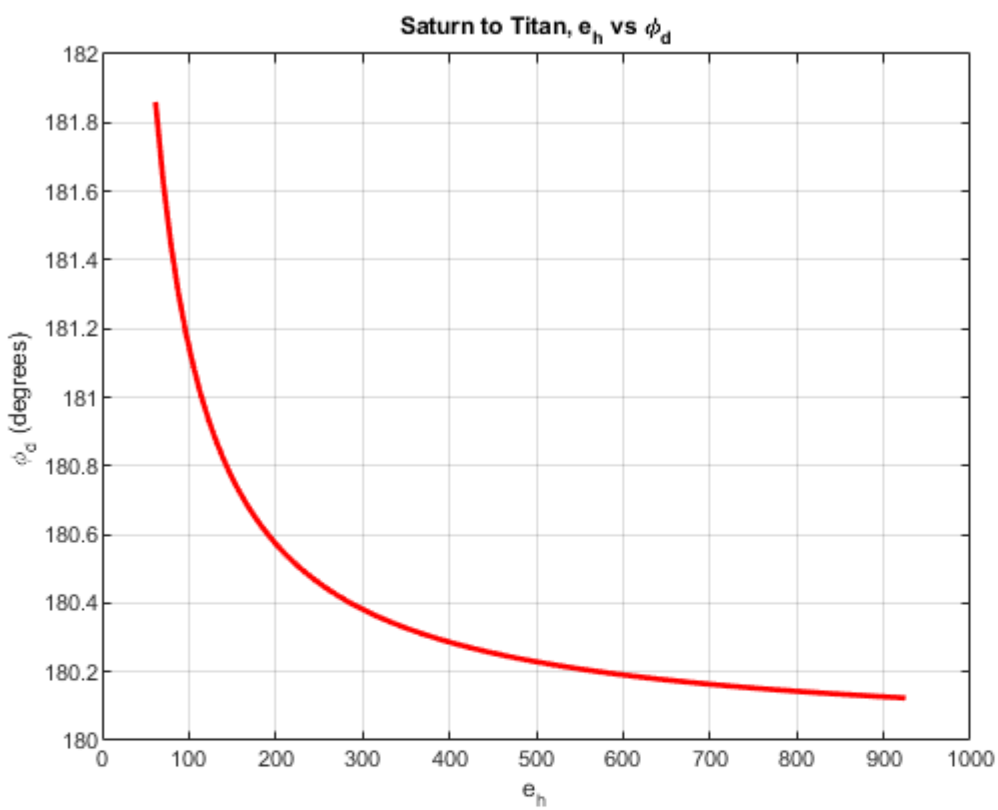
```
   5.6830e+26


mass_moon =

   1.3450e+23


pi_one =

    0.9998


pi_two =

   2.3661e-04


mr_one =

  289.1125


mr_two =

   1.2216e+06
```

Titan Calculations

```
% Circular Orbital Velocity of Titan with respect to Saturn (km/sec)
SVT = -sqrt((G*M_Saturn)/a_Titan);

% semi-major axis between Saturn & Titan
SAT = (a_Saturn + a_Titan)*.5;

% Velocity of Spacecraft on arrival to Titan w/ respect to Saturn
v_SAT = -sqrt(G*M_Saturn)*sqrt((2/a_Titan)-(1/SAT));

% Velocity of Spacecraft on arrival to Titan w/respect to Saturn
% +/- dependent on Saturn coordinate system
v_iST = v_SAT + SVT;

% Titan SOI (km)
TSOI = 43306.04056;

% Rp
ht = (.01*TSOI):dx*TSOI:(1.0*TSOI);
trp = r_Titan + ht;

% Saturn to Titan
STeh = 1 + ((trp*(v_iST)^2)/(G*M_Titan));

figure('Name','R_p vs e_h','NumberTitle','off'),
plot(trp, STeh, 'c')
title('Saturn to Titan, R_p vs e_h')
xlabel('R_p (km)')
ylabel('e_h')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on

% Saturn to Titan delta
STdelta = 2*asind(one./STeh);
STphi_d = phi_a + STdelta;

figure('Name','e_h vs. \phi_d','NumberTitle','off'),
plot(STeh,STphi_d,'r')
title('Saturn to Titan, e_h vs \phi_d')
xlabel('e_h')
ylabel('\phi_d (degrees)')
set(findall(gcf,'type','line'),'linewidth',2)
set(gca,'fontsize',8)
grid on
```



Saturn to Titan, R_p vs e_h
e_h
R_p (km)
×10^4



Saturn to Titan, e_h vs φ_d
φ_d (degrees)
e_h

Synodic Period

Amount of days for specified planet to orbit the sun

```
V_days = 225.0 % Venus
E_days = 365.256 % Earth
M_days = 687.00 % Mars
J_days = 4333.80 % Jupiter
S_days = 10759.00 % Saturn

% Earth Days Ratio
per_V = (E_days/V_days)
per_M = (E_days/M_days)
per_J = (E_days/J_days)
per_S = (E_days/S_days)
per_T = 0.04357674

% Position of Titan
pos_Titan = a_Saturn + a_Titan

% Syndodic Period (days)
% Venus
T_VE = (V_days*E_days)/abs(V_days - E_days); % Earth relative to Venus
T_VM = (V_days*M_days)/abs(V_days - M_days); % Mars relative to Venus
T_VJ = (V_days*J_days)/abs(V_days - J_days); % Jupiter relative to Venus
T_VS = (V_days*S_days)/abs(V_days - S_days); % Saturn relative to Venus

% Earth
T_EM = (E_days*M_days)/abs(E_days - M_days); % Mars relative to Earth
T_EJ = (E_days*J_days)/abs(E_days - J_days); % Jupiter relative to Earth
T_ES = (E_days*S_days)/abs(E_days - S_days); % Saturn relative to Earth

% Mars
T_MJ = (M_days*J_days)/abs(M_days - J_days); % Jupiter relative to Mars
T_MS = (M_days*S_days)/abs(M_days - S_days); % Saturn relative to Mars

% Jupiter
T_JS = (J_days*S_days)/abs(J_days - S_days); % Saturn relative to Jupiter

% mean motions (rad/day)
n_Venus = (2*pi)/V_days;
n_Earth = (2*pi)/E_days;
n_Mars = (2*pi)/M_days;
n_Jupiter = (2*pi)/J_days;
n_Saturn = (2*pi)/S_days;

% Initial Phase Angle (degrees)
% Venus
IPA_VE = (pi - (n_Earth*dof_VE))*(180/pi); % Venus to Earth
IPA_VM = (pi - (n_Mars*dof_VM))*(180/pi); % Venus to Mars
IPA_VJ = (pi - (n_Jupiter*dof_VJ))*(180/pi); % Venus to Jupiter
IPA_VS = (pi - (n_Saturn*dof_VS))*(180/pi); % Venus to Saturn

% Earth
IPA_EM = (pi - (n_Mars*dof_EM))*(180/pi); % Earth to Mars
IPA_EJ = (pi - (n_Jupiter*dof_EJ))*(180/pi); % Earth to Jupiter
IPA_ES = (pi - (n_Saturn*dof_ES))*(180/pi); % Earth to Saturn

% Mars
IPA_MJ = (pi - (n_Jupiter*dof_MJ))*(180/pi); % Mars to Jupiter
```

```
IPA_MS = (pi - (n_Saturn*dof_MS))*(180/pi); % Mars to Saturn

% Jupiter
IPA_JS = (pi - (n_Saturn*dof_JS))*(180/pi); % Jupiter to Saturn

% Final Phase Angle
% Venus
FPA_VE = (pi - (n_Venus*dof_VE))*(180/pi); % Venus to Earth
FPA_VM = (pi - (n_Venus*dof_VM))*(180/pi); % Venus to Mars
FPA_VJ = (pi - (n_Venus*dof_VJ))*(180/pi); % Venus to Jupiter
FPA_VS = (pi - (n_Venus*dof_VS))*(180/pi); % Venus to Saturn

% Earth
FPA_EM = (pi - (n_Earth*dof_EM))*(180/pi); % Earth to Mars
FPA_EJ = (pi - (n_Earth*dof_EJ))*(180/pi); % Earth to Jupiter
FPA_ES = (pi - (n_Earth*dof_ES))*(180/pi); % Earth to Saturn

% Mars
FPA_MJ = (pi - (n_Mars*dof_MJ))*(180/pi); % Mars to Jupiter
FPA_MS = (pi - (n_Mars*dof_MS))*(180/pi); % Mars to Saturn

% Jupiter
FPA_JS = (pi - (n_Jupiter*dof_JS))*(180/pi); % Jupiter to Saturn

% Wait time (assuming n1 < n2) (days)
NN = 0

% Venus
twait_VE = (-2*FPA_VE - (2*pi*NN))/(n_Earth - n_Venus);
twait_VM = (-2*FPA_VM - (2*pi*NN))/(n_Mars - n_Venus);
twait_VJ = (-2*FPA_VJ - (2*pi*NN))/(n_Jupiter - n_Venus);
twait_VS = (-2*FPA_VS - (2*pi*NN))/(n_Saturn - n_Venus);

% Earth
twait_EM = (-2*FPA_EM - (2*pi*NN))/(n_Mars - n_Earth);
twait_EJ = (-2*FPA_EJ - (2*pi*NN))/(n_Jupiter - n_Earth);
twait_ES = (-2*FPA_ES - (2*pi*NN))/(n_Saturn - n_Earth);

% Jupiter
twait_MJ = (-2*FPA_MJ - (2*pi*NN))/(n_Jupiter - n_Mars);
twait_MS = (-2*FPA_MS - (2*pi*NN))/(n_Saturn - n_Mars);

% Saturn
twait_JS = (-2*FPA_JS - (2*pi*NN))/(n_Saturn - n_Jupiter);
```

```
V_days =

   225

E_days =

  365.2560
```

```
M_days =

   687


J_days =

   4.3338e+03


S_days =

       10759


per_V =

    1.6234


per_M =

    0.5317
```

```
per_J =

    0.0843


per_S =

    0.0339


per_T =

    0.0436


pos_Titan =

   1.4332e+09


NN =

     0
```

Plotting the Solar System

```
Radius = a_Saturn + a_Titan + 2*r_Titan % Maximum graph dimensions
```

```
% Assume Circular Orbits for each planet
Et = 0:pi/180:Years; % Earth time span(1.0149*pi)/180
EarthX = a_Earth * cos(Et);
EarthY = a_Earth * sin(Et);
EarthZ = 0*tan(Et);

Vt = 0:(per_V*pi)/180:Years; % Venus time span
VenusX = a_Venus * cos(Vt);
VenusY = a_Venus *sin(Vt);
VenusZ = 0*tan(Vt);

Mt = 0:(per_M*pi)/180:Years; % Mars time span
MarsX = (a_Mars * cos(Mt));
MarsY = (a_Mars *sin(Mt));
MarsZ = 0*tan(Mt);

Jt = 0:(per_J*pi)/180:Years; % Jupiter time span
JupiterX = a_Jupiter * cos(Jt);
JupiterY = a_Jupiter *sin(Jt);
JupiterZ = 0*tan(Jt);

St = 0:(per_S*pi)/180:Years; % Saturn time span
SaturnX = a_Saturn * cos(St);
SaturnY = a_Saturn * sin(St);
SaturnZ = 0*tan(St);

Tt = -Years*pi:(2*pi)/180:Years; % Titan time span
TitanX = a_Titan*cos(Tt);
TitanY = a_Titan*sin(Tt);
TitanZ = 0*tan(Tt);

% Satellite (Q)
%syms QX QY;
%fimplicit(1.34364042*(10)*QX^2+2.6714264*(10)*QX*QY+1.33754593*(10)*QY^2==1);
qt = 0:(per_S*pi)/180:Years;
QX = 1.34364042.*cos(qt).^2 - 2.6714264.*cos(qt).*sin(qt);
QY = 1.33754593.*sin(qt).^2 + 2.6714264.*cos(qt).*sin(qt);
QZ = 0

x0 = [0];
y0 = [0];
z0 = [0];

% Plotting the planets initial position
figure('Name','N-Body','NumberTitle','off')
Sun = plot3(x0, y0, z0, '.y', 'MarkerSize',30') % Sun coordinates
hold on
xlabel('X')
ylabel('Y')
zlabel('Z')
grid on
%Venus = plot3(-108051715.7,-5662750.465,z0,'.m','MarkerSize',10) % Venus coordinates
Earth = plot3(x0,a_Earth,z0, '.g','MarkerSize',10) ;% Earth coordinates
%Q = plot3(a_Earth+r_Earth,y0,z0,'.y','Markersize',5) % Starting Satellite coordinate
Mars = plot3(-59010742.28,-220231088.4,z0,'.r','MarkerSize',10) ;% Mars coordinates
Jupiter = plot3(a_Jupiter/sqrt(2),a_Jupiter/sqrt(2),z0,'.k','MarkerSize',45); %
Jupiter coordinates
Saturn = plot3((a_Saturn/2)*sqrt(3),-a_Saturn/2,z0,'.m','MarkerSize',5) ;% Saturn
coordinates
Titan = plot3(a_Titan+(a_Saturn/2)*sqrt(3),a_Titan+(-
a_Saturn/2),z0,'.b','Markersize',5); % Titan coordinates
```

```
axis([-Radius, +Radius, -Radius, +Radius, -Radius, +Radius]); % make sure the axis is
fixed;

for n = 9273:14426;

    set(Earth, 'XData', EarthX(n), 'YData', EarthY(n), 'ZData', EarthZ(n)); %// update
Earth position
    %set(Venus, 'XData', VenusX(n), 'YData', VenusY(n),'ZData', VenusZ(n)); % Update
Venus position
    set(Mars, 'XData', MarsX(n), 'YData', MarsY(n),'ZData', MarsZ(n)); % Update Mars
position
    set(Jupiter, 'XData', JupiterX(n), 'YData', JupiterY(n),'ZData', JupiterZ(n)); %
Update Jupiter position
    set(Saturn, 'XData', SaturnX(n), 'YData', SaturnY(n),'ZData', SaturnZ(n)); %
Update Saturn position

    set(Titan, 'XData', TitanX(n) + SaturnX(n), 'YData', TitanY(n)+
SaturnY(n),'ZData', TitanZ(n) + SaturnZ(n)); % Update Titan position
    drawnow %// refresh figure
end

% Plotting the planets Final position
figure('Name','N-Body Final','NumberTitle','off')
Sun = plot3(x0, y0, z0, '.y', 'MarkerSize',30') % Sun coordinates
hold on
xlabel('X')
ylabel('Y')
zlabel('Z')
grid on
%Venus = plot3(-108051715.7,-5662750.465,z0,'.m','MarkerSize',10) % Venus coordinates
Earth = plot3(-1.4733e+08,-2.5978e+07,z0, '.g','MarkerSize',10) % Earth coordinates
%Q = fplot3(a_Earth,y0,z0,'.b','Markersize',20) % Starting Satellite coordinate
Mars = plot3(-1.3515e+08,-1.8363e+08,z0,'.r','MarkerSize',10) % Mars coordinates
Jupiter = plot3(7.7404e+08,8.3225e+07,z0,'.k','MarkerSize',45) % Jupiter coordinates
Saturn = plot3(1.4197e+09,1.8752e+08,z0,'.m','MarkerSize',5) % Saturn coordinates
Titan = plot3(a_Titan+(1.4197e+09),a_Titan+(1.8752e+08),z0,'.b','Markersize',5); %
Titan coordinates
axis([-Radius, +Radius, -Radius, +Radius, -Radius, +Radius]); % make sure the axis is
fixed;

for n = 14426:25185;

    set(Earth, 'XData', EarthX(n), 'YData', EarthY(n), 'ZData', EarthZ(n)); %// update
Earth position
    %set(Q, 'XData', QX(n), 'YData', QY(n), 'ZData', QZ(n)); %// update Satellite
position
    %set(Venus, 'XData', VenusX(n), 'YData', VenusY(n),'ZData', VenusZ(n)); % Update
Venus position
    set(Mars, 'XData', MarsX(n), 'YData', MarsY(n),'ZData', MarsZ(n)); % Update Mars
position
    set(Jupiter, 'XData', JupiterX(n), 'YData', JupiterY(n),'ZData', JupiterZ(n)); %
Update Jupiter position
    set(Saturn, 'XData', SaturnX(n), 'YData', SaturnY(n),'ZData', SaturnZ(n)); %
Update Saturn position

    set(Titan, 'XData', TitanX(n) + SaturnX(n), 'YData', TitanY(n)+
SaturnY(n),'ZData', TitanZ(n) + SaturnZ(n)); % Update Titan position
    drawnow %// refresh figure
end
```

```
Radius =
```

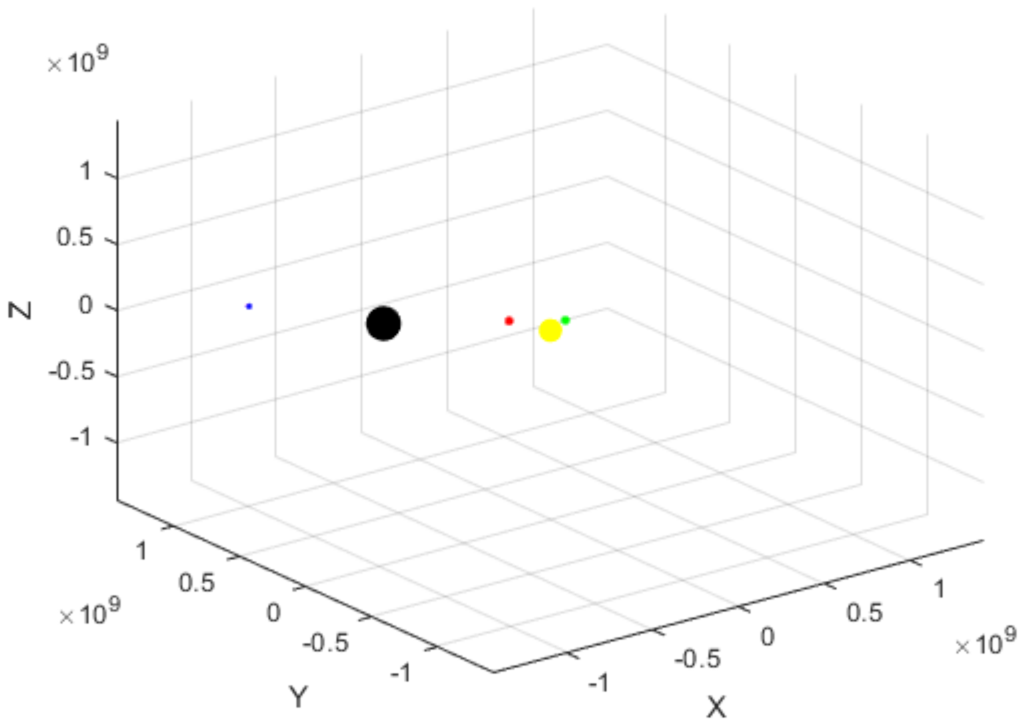
```
   1.4332e+09


QZ =

     0



Sun =

  Line with properties:

              Color: [1 1 0]
          LineStyle: 'none'
          LineWidth: 0.5000
             Marker: '.'
         MarkerSize: 30
    MarkerFaceColor: 'none'
              XData: 0
              YData: 0
              ZData: 0

  Use GET to show all properties


Sun =
```

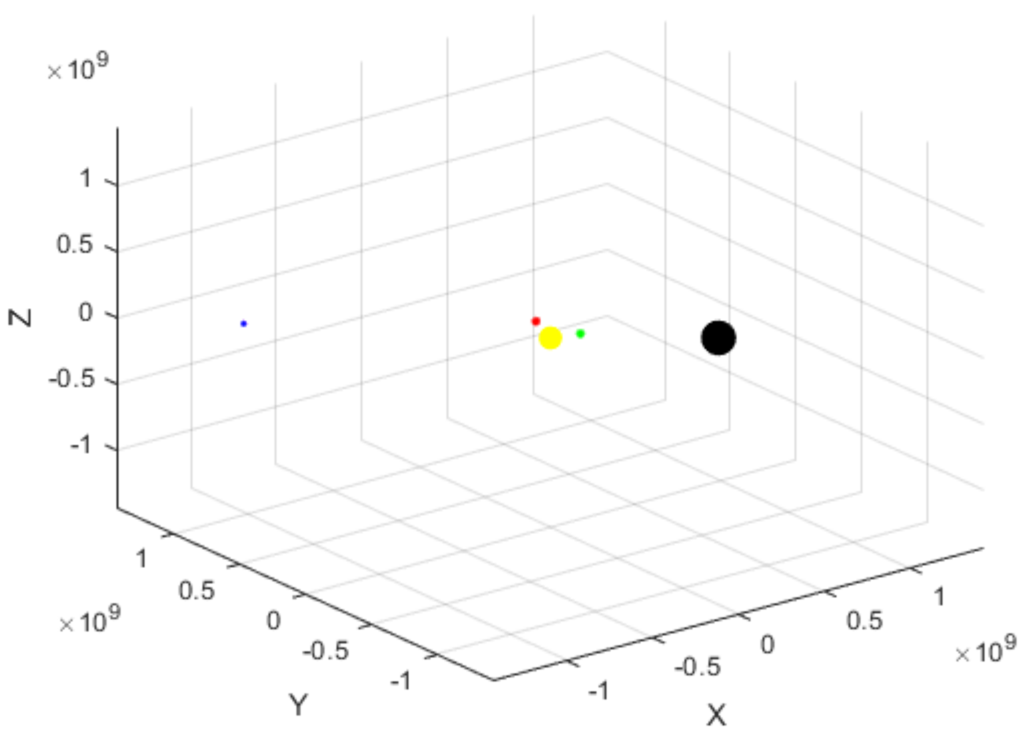
```
  Line with properties:

              Color: [1 1 0]
          LineStyle: 'none'
          LineWidth: 0.5000
             Marker: '.'
         MarkerSize: 30
    MarkerFaceColor: 'none'
              XData: 0
              YData: 0
              ZData: 0

  Use GET to show all properties


Earth =

  Line with properties:

              Color: [0 1 0]
          LineStyle: 'none'
          LineWidth: 0.5000
             Marker: '.'
         MarkerSize: 10
    MarkerFaceColor: 'none'
```

```
             XData: -147330000
             YData: -25978000
             ZData: 0

  Use GET to show all properties


Mars =

  Line with properties:

             Color: [1 0 0]
         LineStyle: 'none'
         LineWidth: 0.5000
            Marker: '.'
        MarkerSize: 10
   MarkerFaceColor: 'none'
             XData: -135150000
             YData: -183630000
             ZData: 0

  Use GET to show all properties


Jupiter =
```

```
  Line with properties:

              Color: [0 0 0]
          LineStyle: 'none'
          LineWidth: 0.5000
             Marker: '.'
         MarkerSize: 45
    MarkerFaceColor: 'none'
              XData: 774040000
              YData: 83225000
              ZData: 0

  Use GET to show all properties


Saturn =

  Line with properties:

              Color: [1 0 1]
          LineStyle: 'none'
          LineWidth: 0.5000
             Marker: '.'
         MarkerSize: 5
    MarkerFaceColor: 'none'
              XData: 1.4197e+09
```

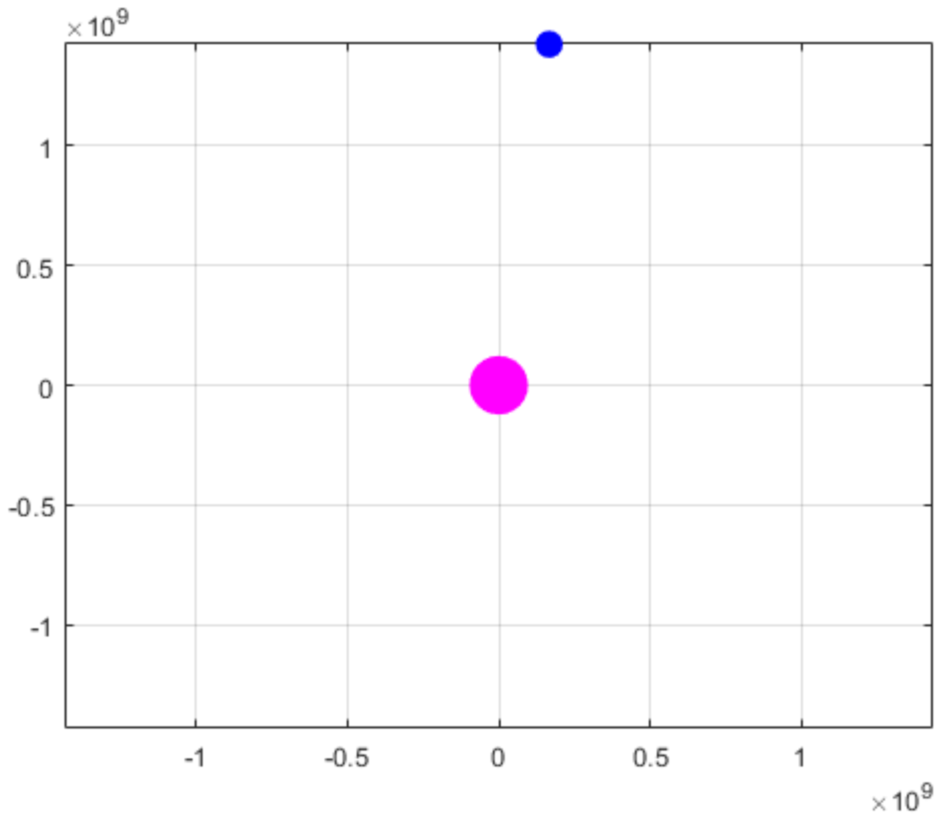
```
        YData: 187520000

        ZData: 0


Use GET to show all properties
```


×10^9
1
0.5
Z 0
-0.5
-1
1
0.5
×10^9
0
-0.5
-1
Y
-1
-0.5
0
0.5
1
×10^9
X


×10^9
1
0.5
Z 0
-0.5
-1
1
0.5
×10^9
0
-0.5
-1
Y
-1
-0.5
0
0.5
1
×10^9
X

```
Titan_sysX = (a_Saturn + a_Titan)*cos(Tt);

Titan_sysY = (a_Saturn + a_Titan)*sin(Tt);

Titan_sysZ = 0*tan(Tt);



figure('Name','Saturn-Titan System','NumberTitle','off')
Saturn_sys = plot(x0,y0,'.m','MarkerSize',75) % Saturn coordinates
hold on
grid on
Titan_sys = plot(a_Titan,y0,'.b','Markersize',35); % Titan coordinates
axis([-Radius, +Radius, -Radius, +Radius]); % make sure the axis is fixed;

for n = 1:100;
    set(Titan_sys, 'XData', Titan_sysX(n), 'YData', Titan_sysY(n)); % Update Titan
position
    drawnow %// refresh figure
end
```

```
Saturn_sys =


  Line with properties:


              Color: [1 0 1]

          LineStyle: 'none'

          LineWidth: 0.5000

             Marker: '.'

         MarkerSize: 75

    MarkerFaceColor: 'none'

              XData: 0

              YData: 0

              ZData: [1×0 double]


  Use GET to show all properties
```



Transposing

```
tp_EarthX = transpose(EarthX);

tp_EarthY = transpose(EarthY);

Earth_Coordinates = [tp_EarthX tp_EarthY];



tp_MarsX = transpose(MarsX);

tp_MarsY = transpose(MarsY);

Mars_Coordinates = [tp_MarsX tp_MarsY];



tp_JupiterX = transpose(JupiterX);

tp_JupiterY = transpose(JupiterY);

Jupiter_Coordinates = [tp_JupiterX tp_JupiterY];
```

```
tp_SaturnX = transpose(SaturnX);

tp_SaturnY = transpose(SaturnY);

Saturn_Coordinates = [tp_SaturnX tp_SaturnY];

%syms QX QY;
%fimplicit3(1.34364042*(10)*QX^2+2.6714264*(10)*QX*QY+1.33754593*(10)*QY^2+0*QZ^2==1);
%QX = @(qt) 1.34364042*cos(qt)^2 - 2.6714264*cos(qt)*sin(qt);
%QY = @(qt) 1.33754593*sin(qt)^2 + 2.6714264*cos(qt)*sin(qt);
%QZ = @(qt) 0
%fplot3(QX,QY,QZ)
```

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