INSTRUCTIONS:

1. Answer ONLY the specified number of questions from the options provided in each section. Do not answer more than the required number of questions. Each section takes one hour.

2. Your answers must be on the paper provided. No more than one answer per page. Do not answer two questions on the same sheet of paper.

3. If you use more than one sheet of paper for a question, write “Page 1 of 2” and “Page 2 of 2.”

4. Write ONLY on one side of each sheet. Use only pen. Answers in pencil will be disqualified.

5. Write ------ END ------ at the end of each answer.

6. Write your exam identification number in the upper right-hand corner of each sheet of paper.

7. Write the question number in the upper right-hand corner of each sheet of paper.

Section 1: Microeconomic Theory—Answer Any Two Questions.

1A. (Liu) A firm has $L$ units of labor at its disposal. Its output are three different commodities. Producing $x$, $y$, and $z$ units of these commodities requires $\alpha x^2$, $\beta y^2$, and $\gamma z^2$ units of labor, respectively.

   a. Solve the problem:
      \[
      \max ax + by + cz \quad \text{subject to} \quad \alpha x^2 + \beta y^2 + \gamma z^2 = L
      \]
      where $a$, $b$, $c$, $\alpha$, $\beta$, and $\gamma$ are positive constants.

   b. Put $a = 4$, $b = c = 1$, $\alpha = 1$, $\beta = 1/4$, and $\gamma = 1/5$, and solve the problem again in this case.

   c. What happens to the maximum value of $4x + y + z$, when $L$ increases from 100 to 101? Find both the exact change and the appropriate linear approximation based on the interpretation of the Lagrange multiplier.

1B. (Hajikhameneh) Answer the following questions for a consumer with utility function $U(x, y) = x^a y^{1-a}$ and total income $I$, when $p_x$ is the price of good $x$ and $p_y$ is the price of good $y$.

   a. Find the uncompensated demand functions for $x$ and $y$ using Lagrange method.

   b. Derive the indirect utility function $V(p_x, p_y, I)$.

   c. Show that $\frac{\partial V}{\partial I} = \lambda$ the Lagrange multiplier. What is the economic interpretation of $\lambda$?
1C. (Hajikhameneh) Find and describe the Bayesian–Nash equilibrium in the following game. There are two players in this game; Player 1 and 2. Actions available to Player 1 are \{a, b, c, d\} and to Player 2 are \{A, B\}. The top and bottom payoffs belong to Player 1 and Player 2, respectively.