Geometrical Optics

Thursday, 9/21/2006

Physics 158

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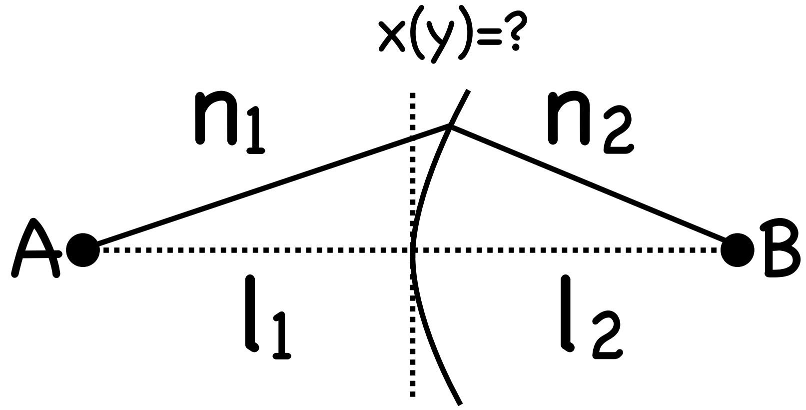
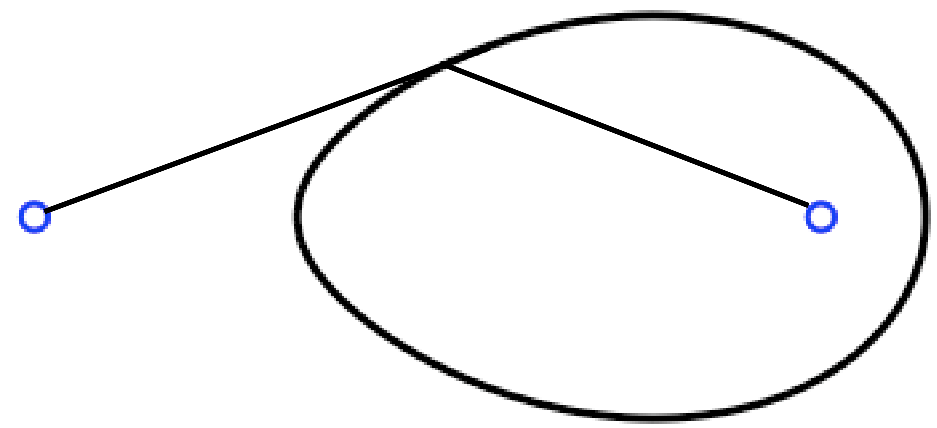
Class Outline

- Refraction at curved surfaces
    - Aspherical surfaces
    - Spherical surfaces
- Thin lenses
    - Thin lens equation
    - Magnification
- Ray Tracing
- Mirrors

Refraction at Aspherical Surfaces

Using Fermat's principle, compute the shape of an interface between two media if a point in one media is to be perfectly imaged to a point in the other media





$$OPL = n_1 \left(\sqrt{(l_1 + x(y))^2 + y^2} \right) + n_2 \left(\sqrt{(l_2 - x(y))^2 + y^2} \right)$$

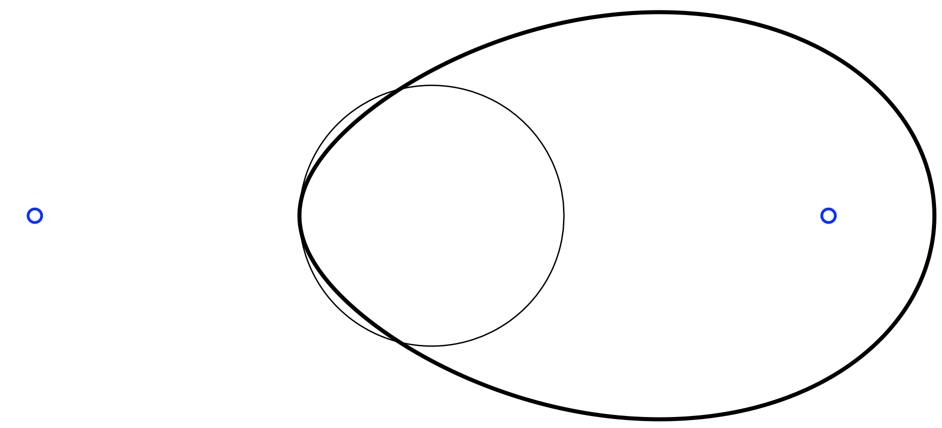
If OPL is a constant, this is the equation for a "Cartesian Oval"

Aspheric Lenses

- The action of imaging one point onto another is the task of a **lens**.
- The cartesian oval shape calculated on the last slide describes the shape of an optimal lens called an aspheric lens
- How might one manufacture such a lens?

Spherical Lenses

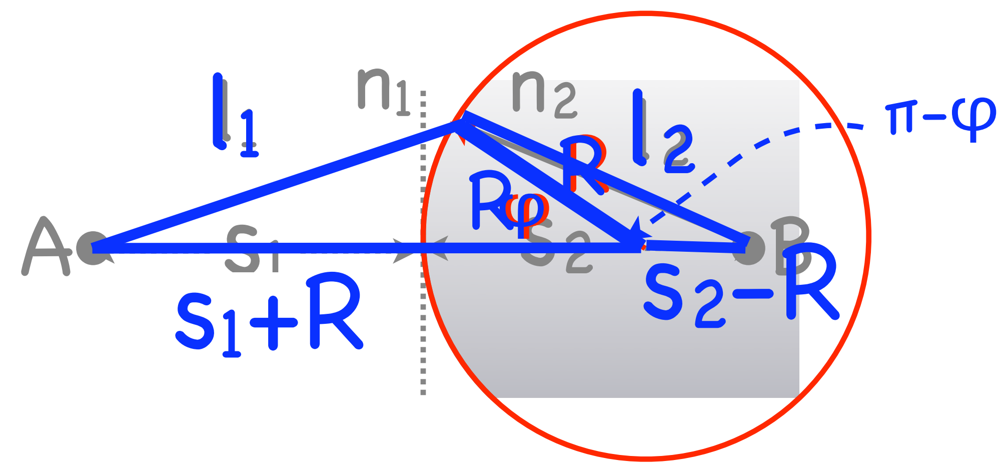
For sufficiently small deviations from the **optical axis** the cartesian oval can be approximated by a sphere with the same radius of curvature



Spherical surfaces are particularly easy to machine because of the spherical symmetry – thus "spherical lenses" tend to be much cheaper than aspherical lenses

Refraction at Spherical Surfaces

Using Fermat's principle, compute the shape of an interface between two media if a point in one media is to be imaged to a point in the other media



$$l_1^2 = R^2 + (s_1 + R)^2 - 2R(s_1 + R)\cos\phi$$

$$l_2^2 = R^2 + (s_2 - R)^2 - 2R(s_2 - R)\cos(\pi - \phi)$$

$$OPL = n_1 l_1 + n_2 l_2$$

$$\frac{dOPL}{d\phi} = -n_1 \frac{R(s_1 + R)\sin\phi}{l_1} + n_2 \frac{R(s_2 - R)\sin\phi}{l_2} \rightarrow 0$$

Refraction at Spherical Surfaces

$$\frac{dOPL}{d\phi} = -n_1 \frac{R(s_1 + R)\sin\phi}{l_1} + n_2 \frac{R(s_2 - R)\sin\phi}{l_2} \to 0$$

in the **paraxial wave** approximation $\cos\phi \approx 1$

$$l_1 \to s_1 + \mathcal{O}^2(\phi)$$
$$l_2 \to s_2 + \mathcal{O}^2(\phi)$$

$$n_1 \frac{(s_1 + R)}{s_1} = n_2 \frac{(s_2 - R)}{s_2}$$

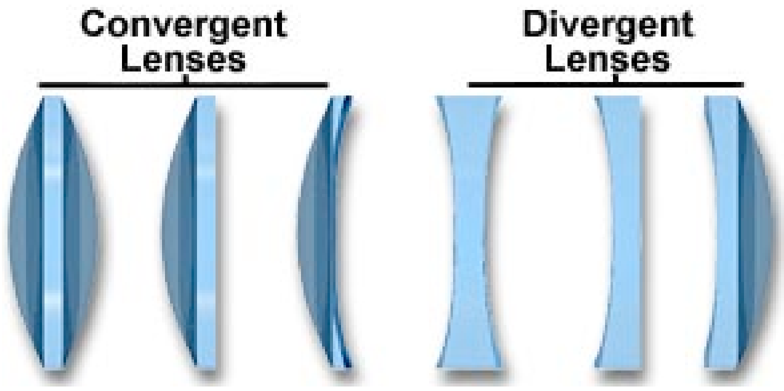
$$\boxed{\frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R}}$$

imaging equation for a spherical interface

Thin Lenses

Most practical applications require that a lens image a point on one side of the lens to a point on the other (i.e. not a point inside the lens material). These lenses must have at least two refracting interfaces.

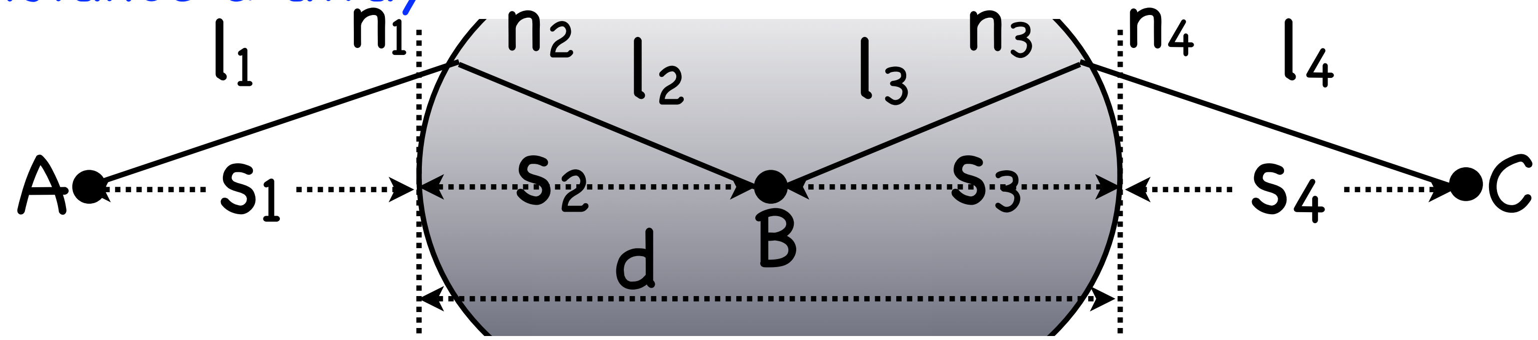
Because of cost considerations, typically each side is either spherical or planer



plano-convex lens

Thin-Lens Equation

Consider the image forming equation for a single spherical interface of radius R_1, then consider the image formed by that interface as the object being imaged by a second spherical interface of radius $-R_2$ i.e. curved in the opposite direction, a distance d away



$$\frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R_1} \qquad \frac{n_3}{s_3} + \frac{n_4}{s_4} = \frac{n_4 - n_3}{R_2}$$

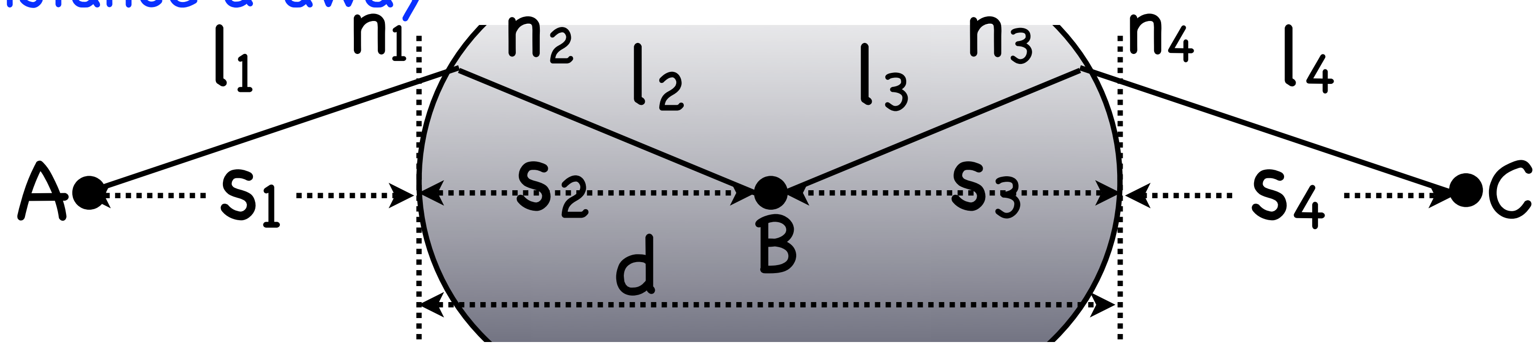
Using $\quad n_4 = n_1 \quad n_3 = n_2 \quad s_3 = d - s_2$

gives

$$-\frac{n_2}{d - s_2} + \frac{n_1}{s_4} = \frac{n_1 - n_2}{R_2}$$

Thin-Lens Equation

Consider the image forming equation for a single spherical interface of radius R_1, then consider the image formed by that interface as the object being imaged by a second spherical interface of radius $-R_2$ i.e. curved in the opposite direction, a distance d away



$$\frac{n_1}{s_1} + \frac{n_2}{s_2} \qquad\qquad R_1$$

$$+\ \frac{n_2}{d - s_2} + \frac{n_1}{s_4} = \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{s_1} + \frac{d n_2}{s_2(d - s_2)} + \frac{n_1}{s_4} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

0

$$\boxed{\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}}$$

Gaussian lens formula)

$$\boxed{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

lens makers formula

Thin Lenses

The sign convention is as follows

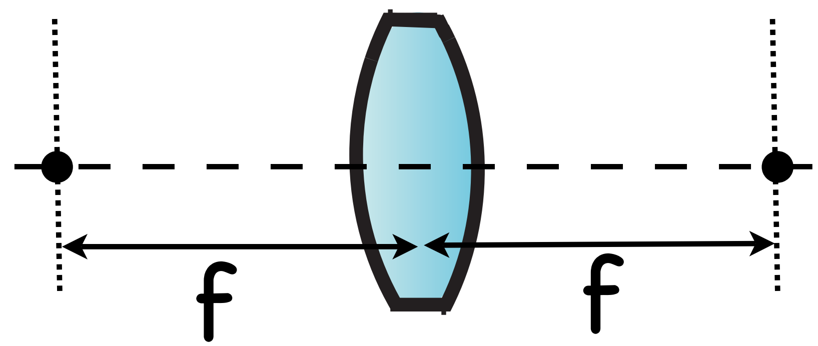
> $f>0$ if the lens is a converging lens (i.e. if the center is thicker than the edges and it has a higher index than the surrounding medium)
>
> $R>0$ if the center of curvature is after the surface (i.e. the light rays are incident on a convex surface)
>
> $s_o>0$ and $s_i>0$ if they are on opposite sides of the mirror

At what distance from a lens, will an object located at ∞ be located?

At what distance from a lens must a light source be located for the lens to **collimate** the light emanating from the object?

What is the closest an object can be to its image?

Thin Lenses



f	focal length
•	focal points
- - -	optical axis
………	focal planes

The optical axis connects the center of the radius of curvature of the two surfaces (i.e. it is the axis of symmetry of the lens)

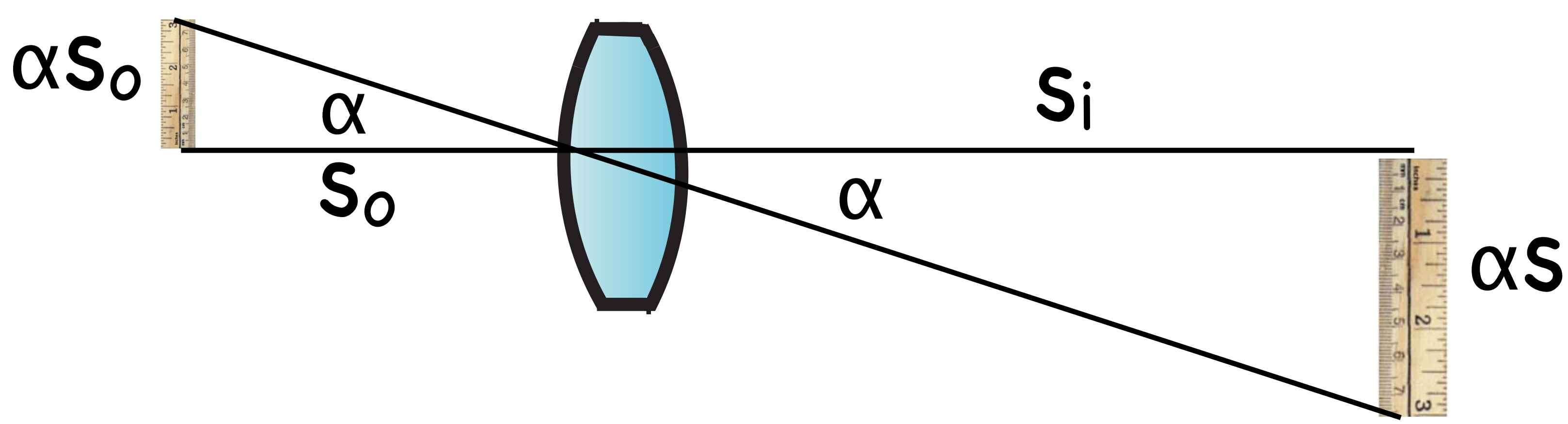
The focal points are one focal length away from the center of the lens on the optical axis

The focal planes are normal to the optical axis and contain the focal points

Magnification

The image formed by a lens has a transverse magnification of

$$M_T = -\frac{s_i}{s_0}$$

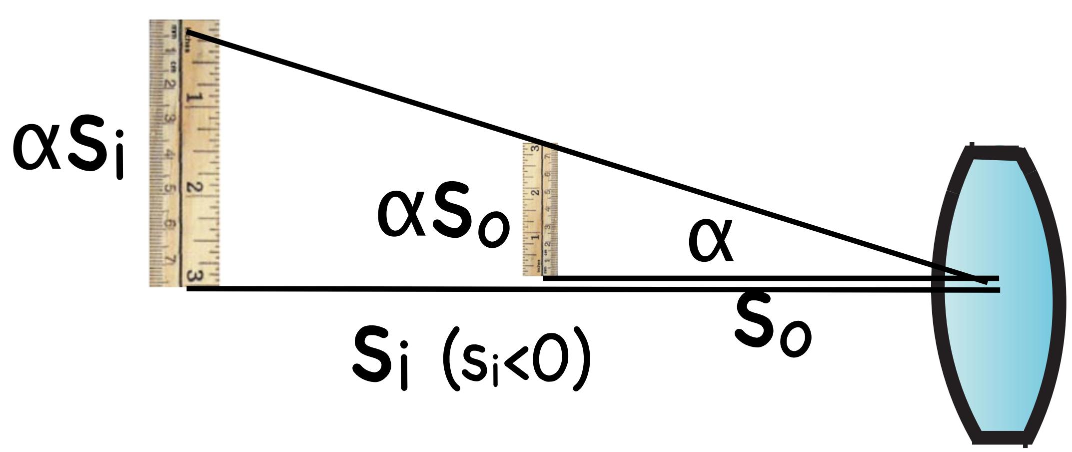


If $s_i>0$ (i.e. if the image and object are on opposite sides of the lens) the image is said to be **real** and the image will be inverted ($M_T<0$)

Magnification

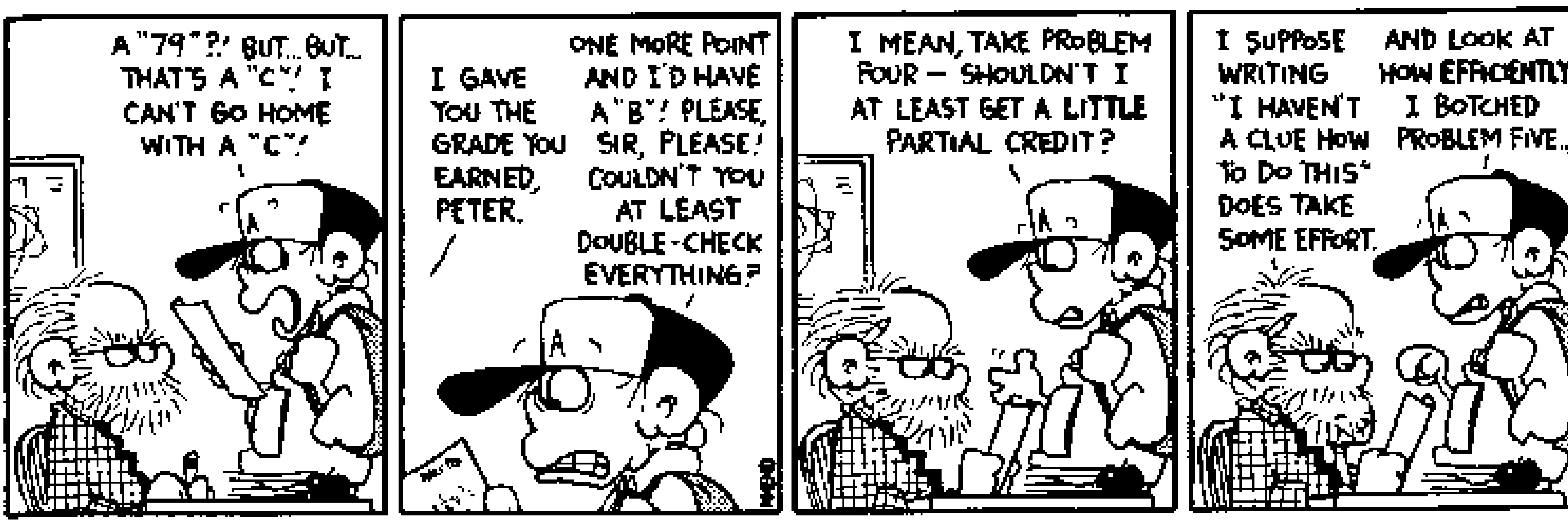
The image formed by a lens has a transverse magnification of

$$M_T = -\frac{s_i}{s_0} > 0$$



If $s_i<0$ (i.e. if the image and object are on the same sides of the lens) the image is said to be **virtual** and will not be inverted ($M_T>0$)

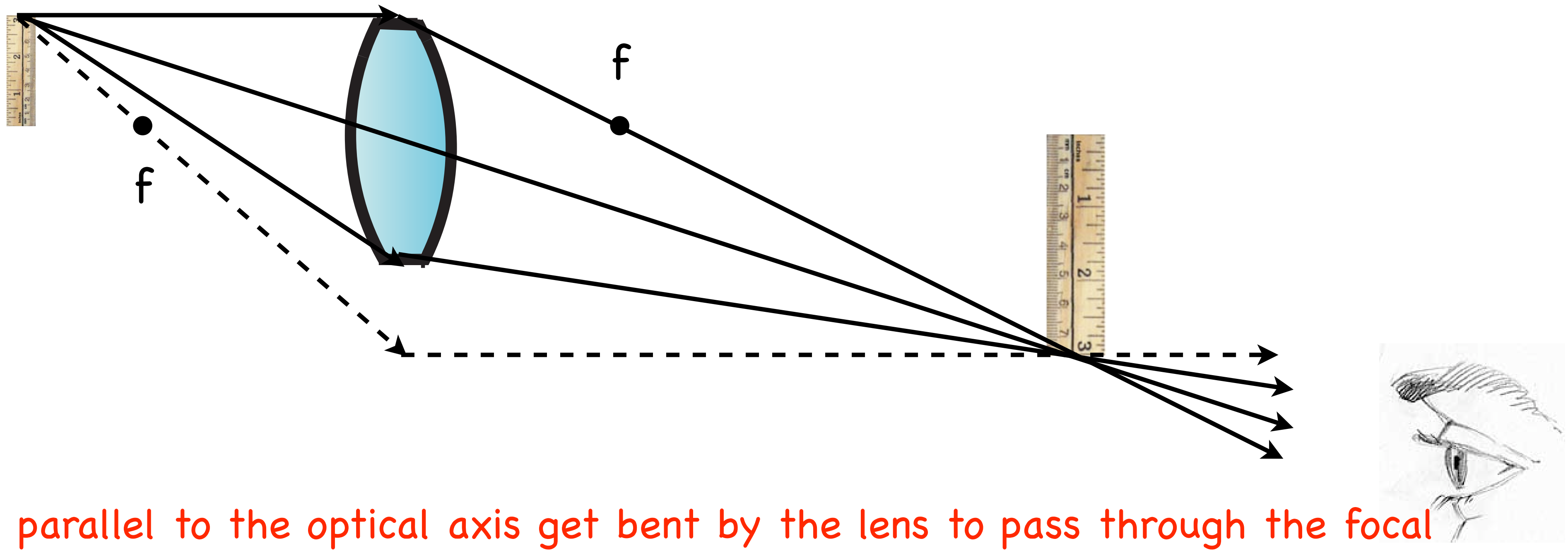
Break



A "79"?! BUT...BUT... THAT'S A "C"! I CAN'T GO HOME WITH A "C"!
I GAVE YOU THE GRADE YOU EARNED, PETER.
ONE MORE POINT AND I'D HAVE A "B"! PLEASE, SIR, PLEASE! COULDN'T YOU AT LEAST DOUBLE-CHECK EVERYTHING?
I MEAN, TAKE PROBLEM FOUR – SHOULDN'T I AT LEAST GET A LITTLE PARTIAL CREDIT?
I SUPPOSE WRITING "I HAVEN'T A CLUE HOW TO DO THIS" DOES TAKE SOME EFFORT.
AND LOOK AT HOW EFFICIENTLY I BOTCHED PROBLEM FIVE...

Ray Tracing

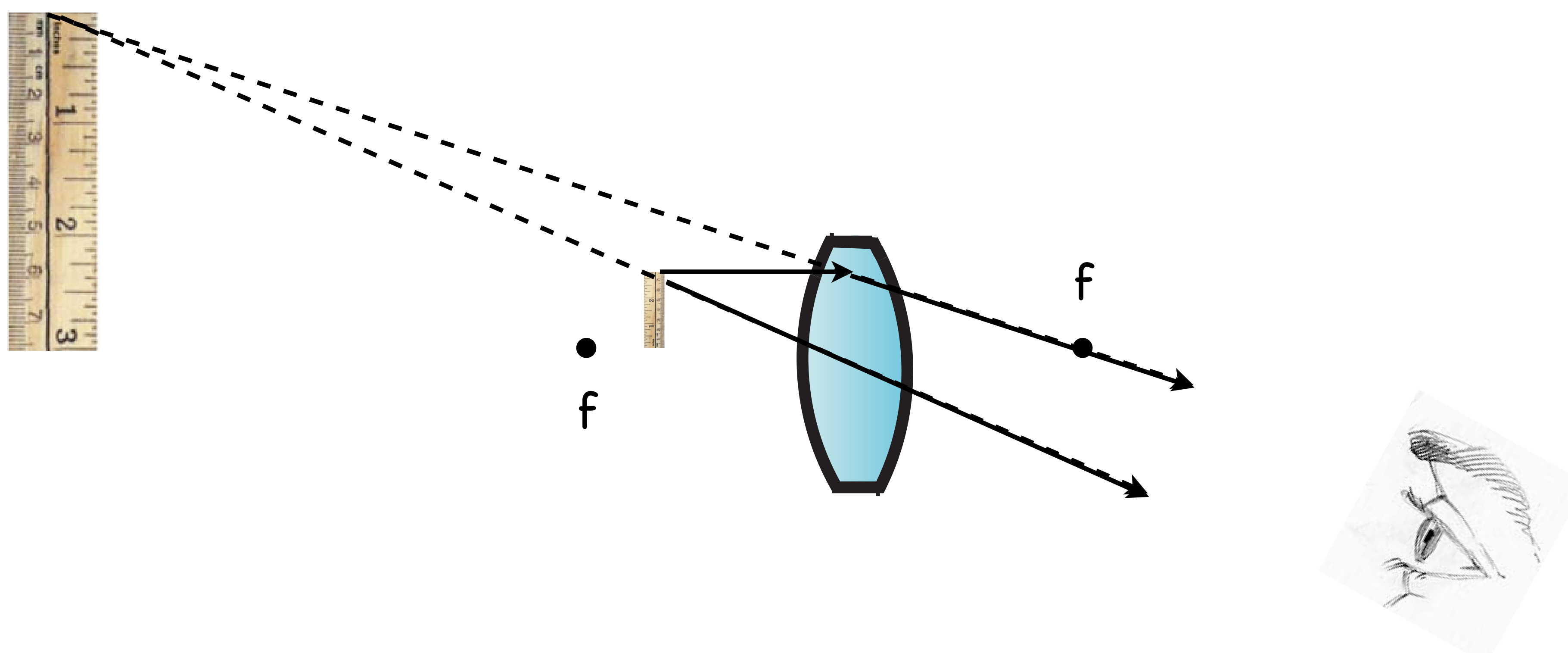
The meaning of real images and imaginary images can be found by tracing the path of rays through the imaging system



- Rays parallel to the optical axis get bent by the lens to pass through the focal point
- Rays through the center of the lens are undeviated
- Rays that pass through the focus are bent by the lens to be parallel to the optical axis
- All rays from a single point on the object converge to (or diverge from) a single point on the image (in an ideal imaging system)

Ray Tracing

The meaning of real images and virtual images can be found by tracing the path of rays through the imaging system



Examples

- Complete the tables

Converging Lens				
Object	Image			
Location	Type	Location	Orientation	Relative Size
$2f<s_o<\infty$	real	$f<s_i<2f$	inverted	smaller
$s_o=2f$	real	$s_i=2f$	inverted	same
$f<s_o<2f$	real	$2f<s_i<\infty$	inverted	bigger
$s_o=f$	N/A	∞	N/A	N/A
$s_o<f$	virtual	$f<s_i<2f$	upright	bigger

Examples

- Complete the tables

Diverging Lens				
Object	Image			
Location	Type	Location	Orientation	Relative Size
$2f<s_0<\infty$	virtual	$-2f/3<s_i<-f$	upright	smaller
$s_0=2f$	virtual	$s_i=-2f/3$	upright	smaller
$f<s_0<2f$	virtual	$-f/2<s_i<-2f/3$	upright	smaller
$s_0=f$	virtual	$s_i=-f/2$	upright	smaller
$s_0<f$	virtual	$s_i<-f/2$	upright	smaller

Mirrors

Like lenses, curved mirrors can focus or defocus light. Fresnel's principle can be applied to give the relation

$$\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

Thus a mirror is like a lens with a focal length of f=R/2. The sign convention is as follows

R>0 if the center of curvature comes before the surface (i.e. if the mirror is concave as seen by the source)

$s_o>0$ and $s_i>0$ if they are on the same side of the mirror

Summary

- Fermat's principle can be used to derive the shape of a boundary that images light from one side of the boundary to another. This shape is a cartesian oval and is used for aspheric lenses

- Spherical lenses can well approximate the ideal shape of an aspherical lens for paraxial beams and are usually much cheaper to produce and are therefore more common than aspheric lenses

- The thin lens equation is $$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

- A mirror can produce images much as a lens does. The equivalent focal length of a mirror is $f=R/2$.

- The transverse magnification of an imaging system is $M_T=-s_i/s_o$