Huygen's Principle

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Physics 158

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Class Outline

- Huygen's Principle
- Fermat's Principle
- Snell's Law

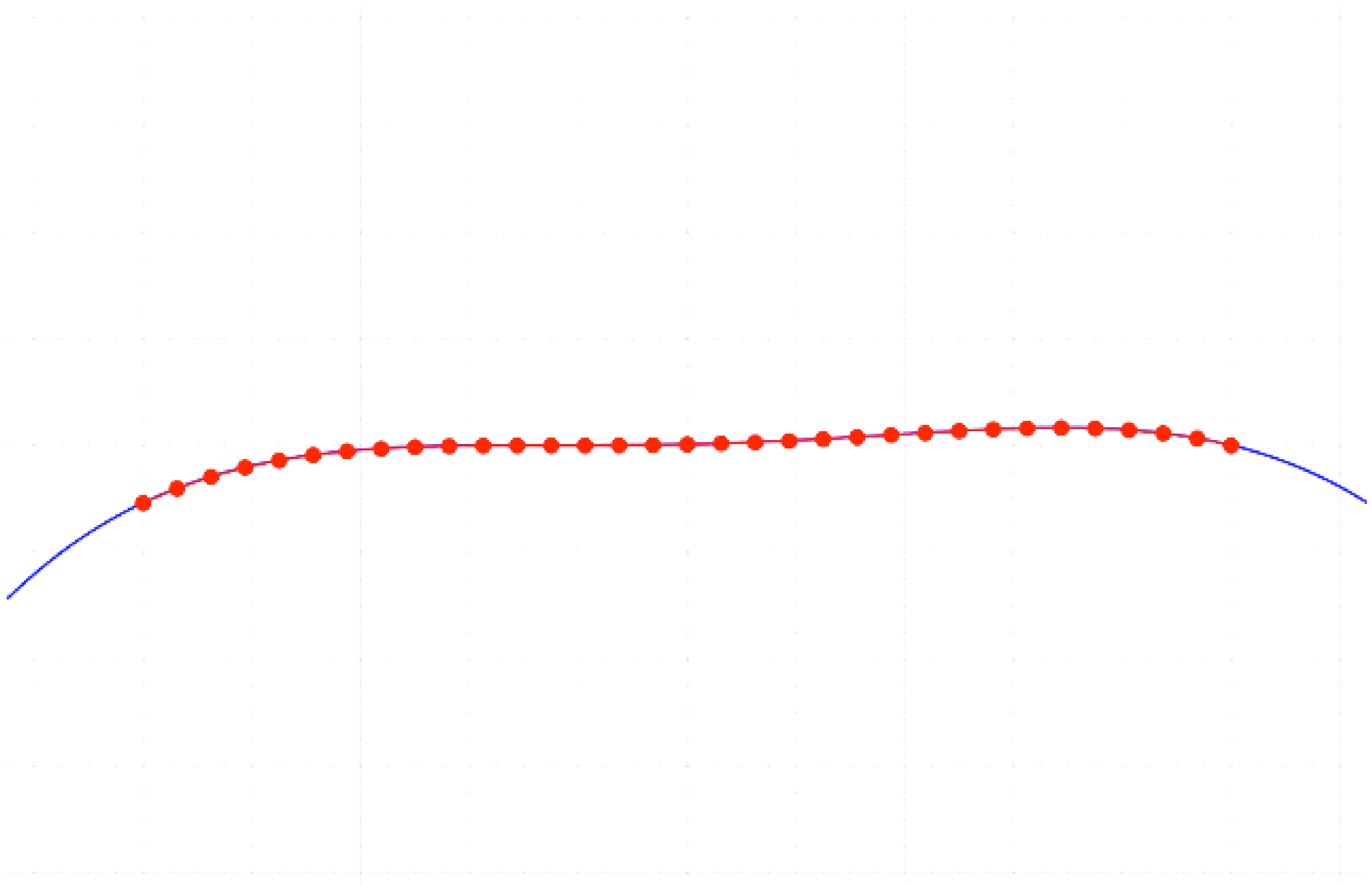
Wave Propagation

- A wavefront is a surface over which the phase of (an electromagnetic) wave is the same
- How do wavefronts propagate through space?
  - Plane waves move in a direction perpendicular to the wavefront
  - Spherical waves move spherically outwards from the center of curvature
  - What about an arbitraty wavefront?

Huygen's Principle

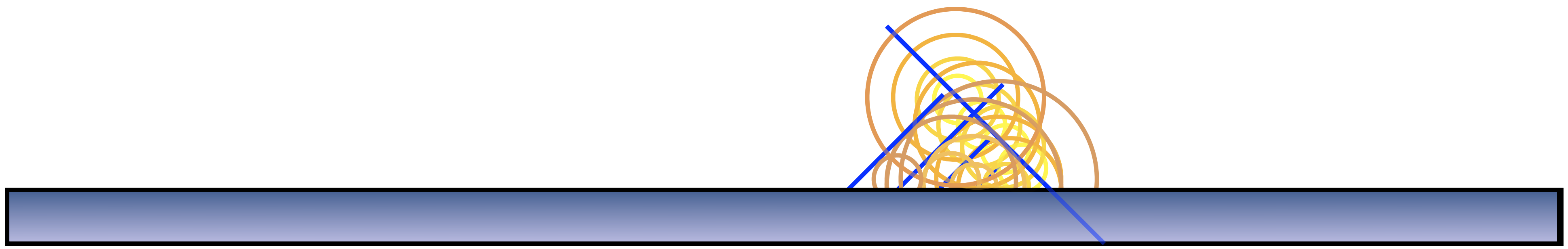
- Consider every point on a wavefront as a "secondary source" of a wavelet
- Add up the field from all the wavelets at a any point in space at a later time to get the field at a later point in time

Example of Huygen's Principle



Law of Reflection

- Using Huygen's principle we can demonstrate the law of reflection

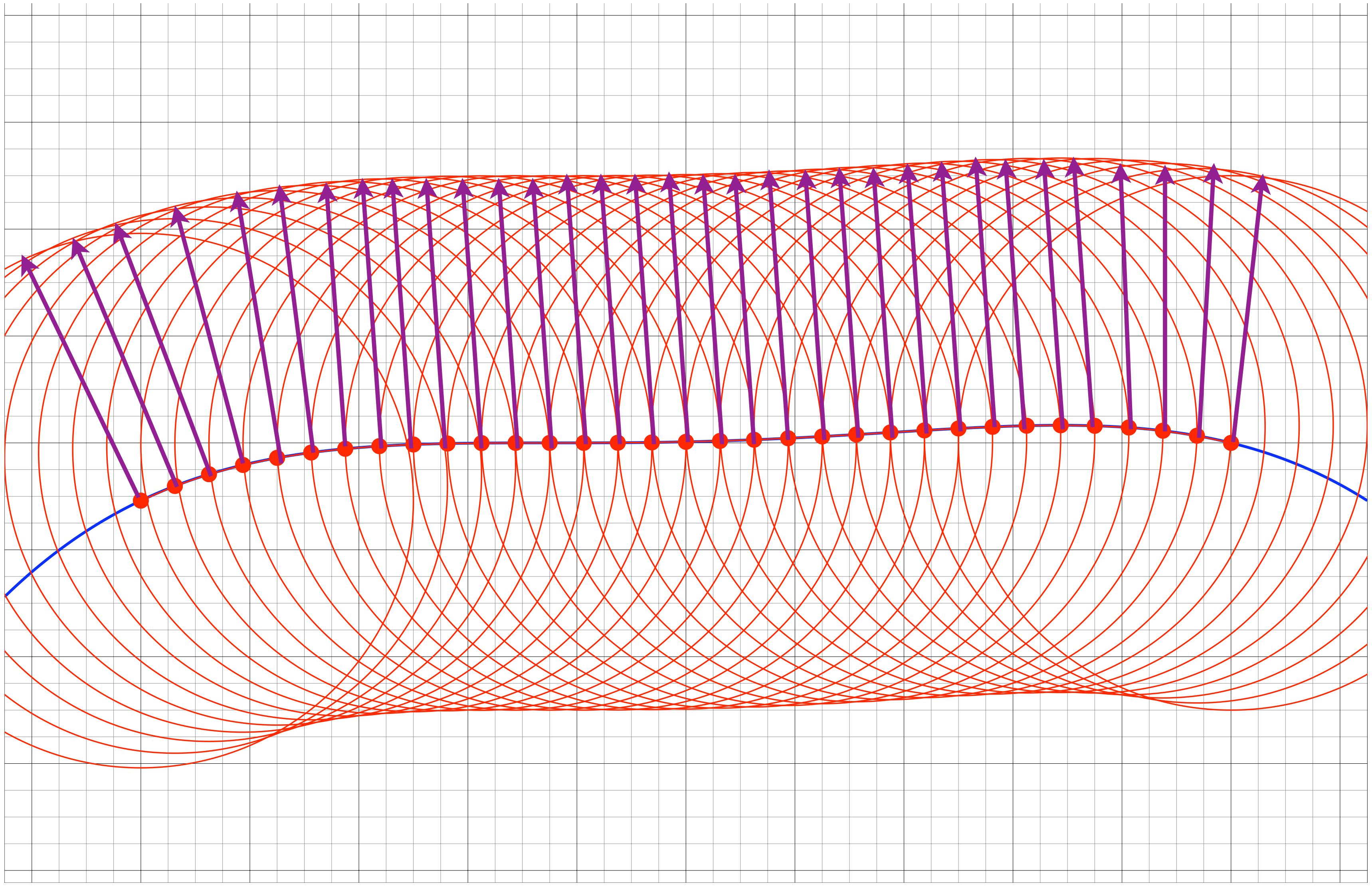


Angle of reflected wavefront is equal to that of incident wavefront

Light Rays

- We can approximate the wave the wavefront propagates by considering "rays" of the light that travel perpendicular to the wavefront (in homogenous media). Analysis the propagation of rays is called **Geometrical Optics**
- A Ray denotes the direction of radiant energy flow
- Later we will consider exact solutions to wavefront propagation, when we study **Physical Optics**

Light Rays

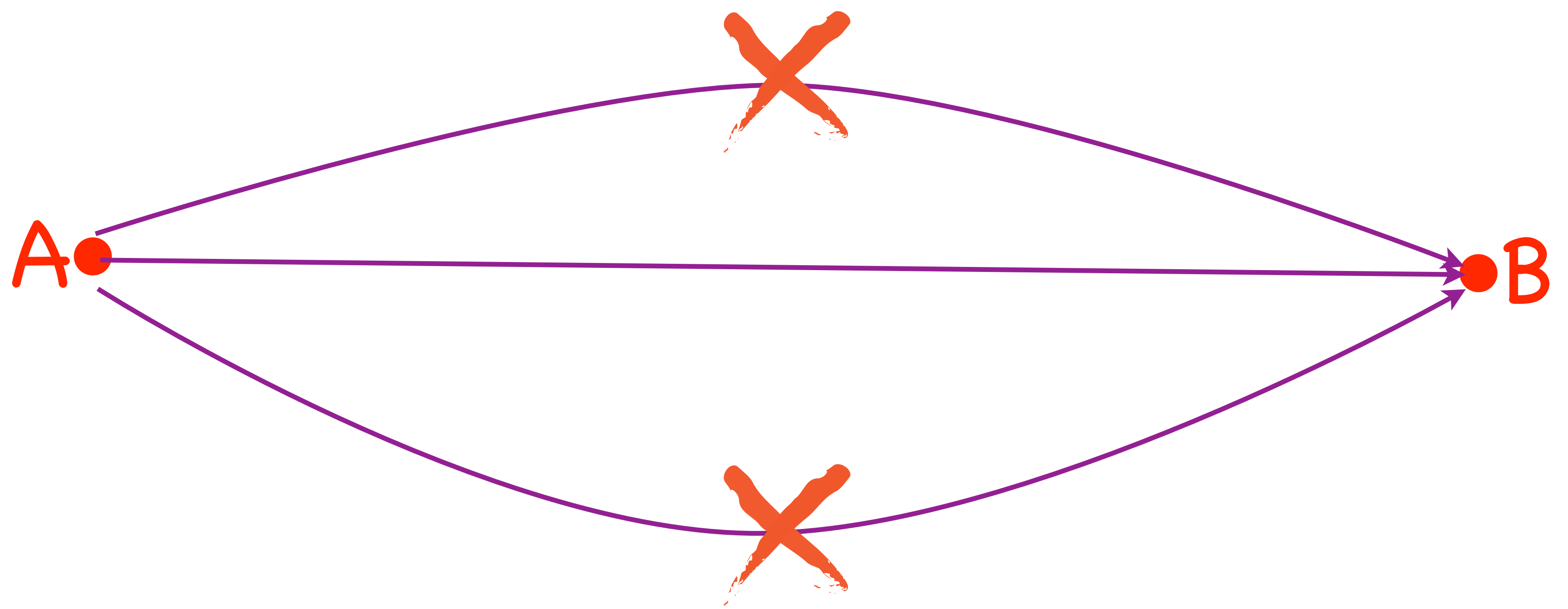


Wavefronts versus Rays

- We can think of the wavefront of a wave propagating through space defining the path of the rays
    - Huygen's prinicple tells us how to calculate this
- We can think of the rays propagating through space defining the wavefronts
    - How can this be calculated?

Fermat's principle

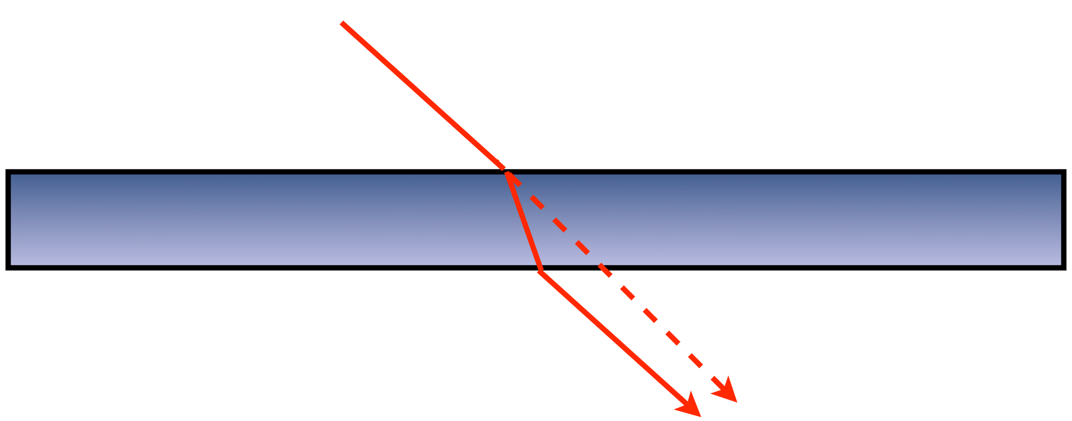
- Light rays traveling from point A to point B will follow paths that represent a (local) minimum in travel time from point A to point B



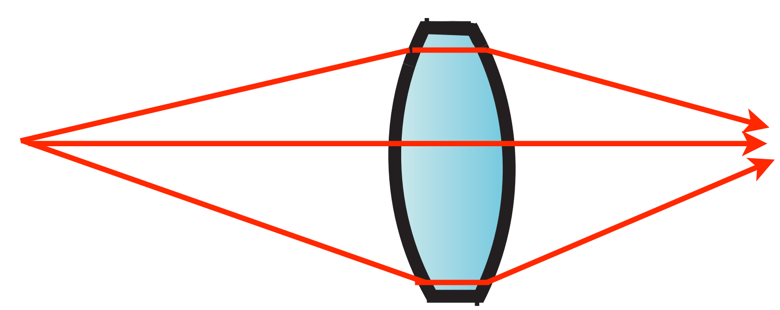
Examples of Fermat's Principle

Mirage from a hot road surface

Apparent position of sun above horizon after actual sunset



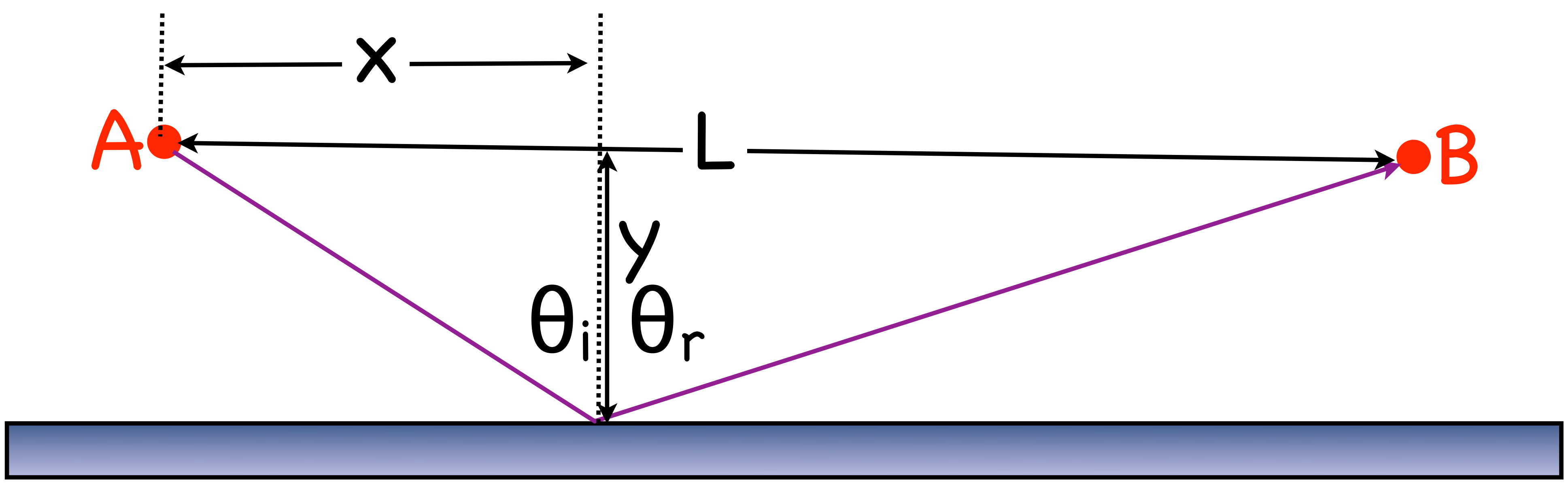
displacement of a ray through a plane of glass



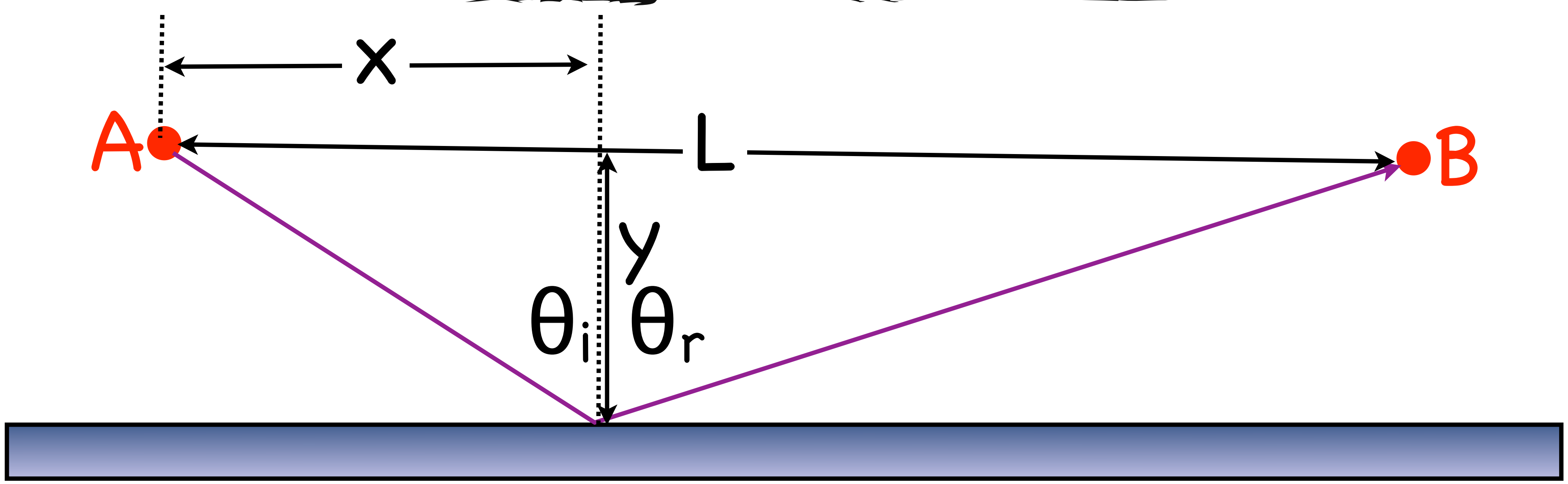
focusing by a lens

Law of Reflection

- Light rays traveling from point A to point B will follow paths that represent a (local) minimum in travel time from point A to point B



Law of Reflection



$$OPL = \sqrt{x^2 + y^2} + \sqrt{(L-x)^2 + y^2}$$

$$\frac{d(OPL)}{dx} = \frac{x}{\sqrt{x^2+y^2}} - \frac{L-x}{\sqrt{(L-x)^2+y^2}}$$

$$\frac{d(OPL)}{dx} = \sin\theta_i - \sin\theta_r \to 0$$

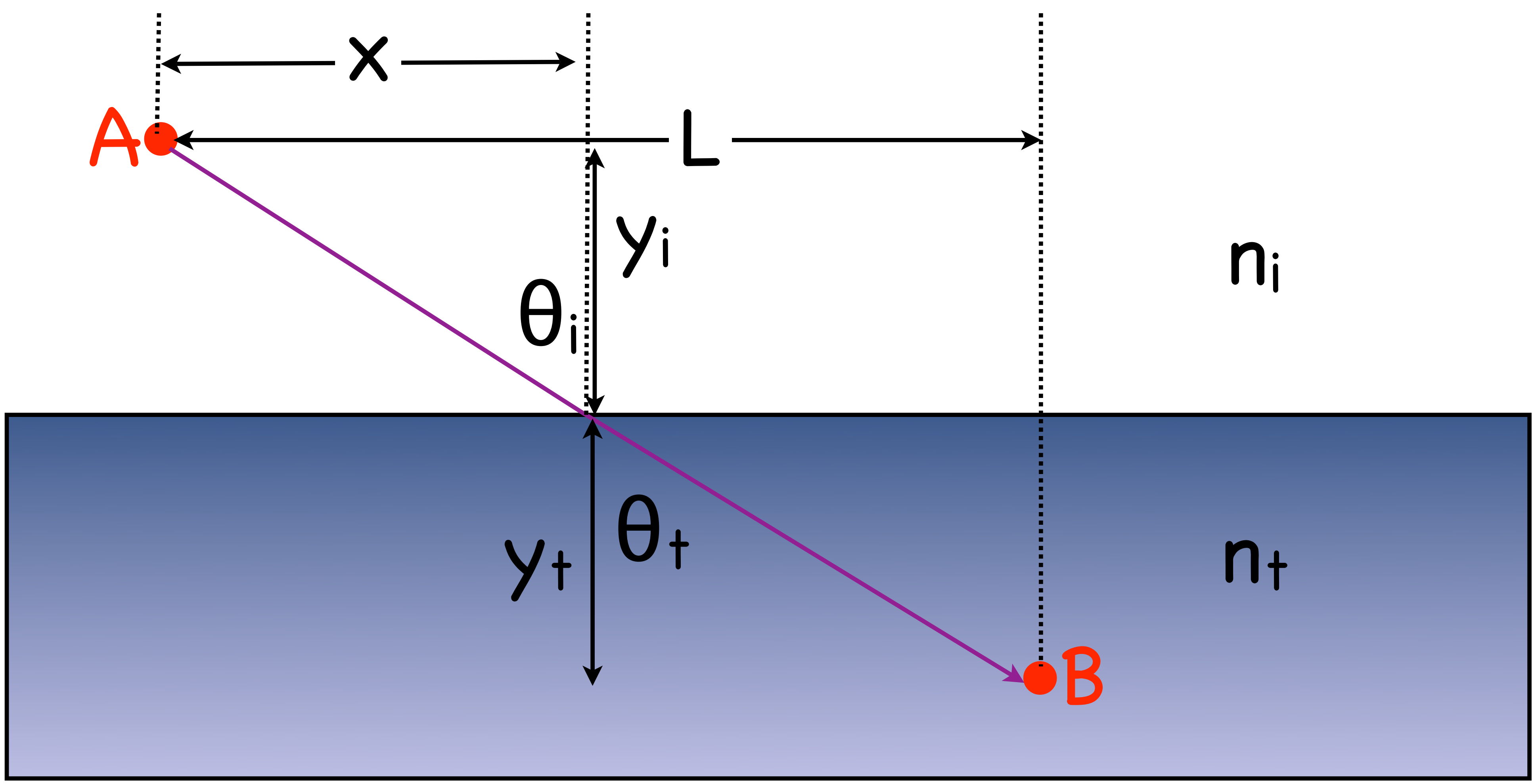
$\sin\theta_i = \sin\theta_r$ **Law of Reflection**

Variational Principle

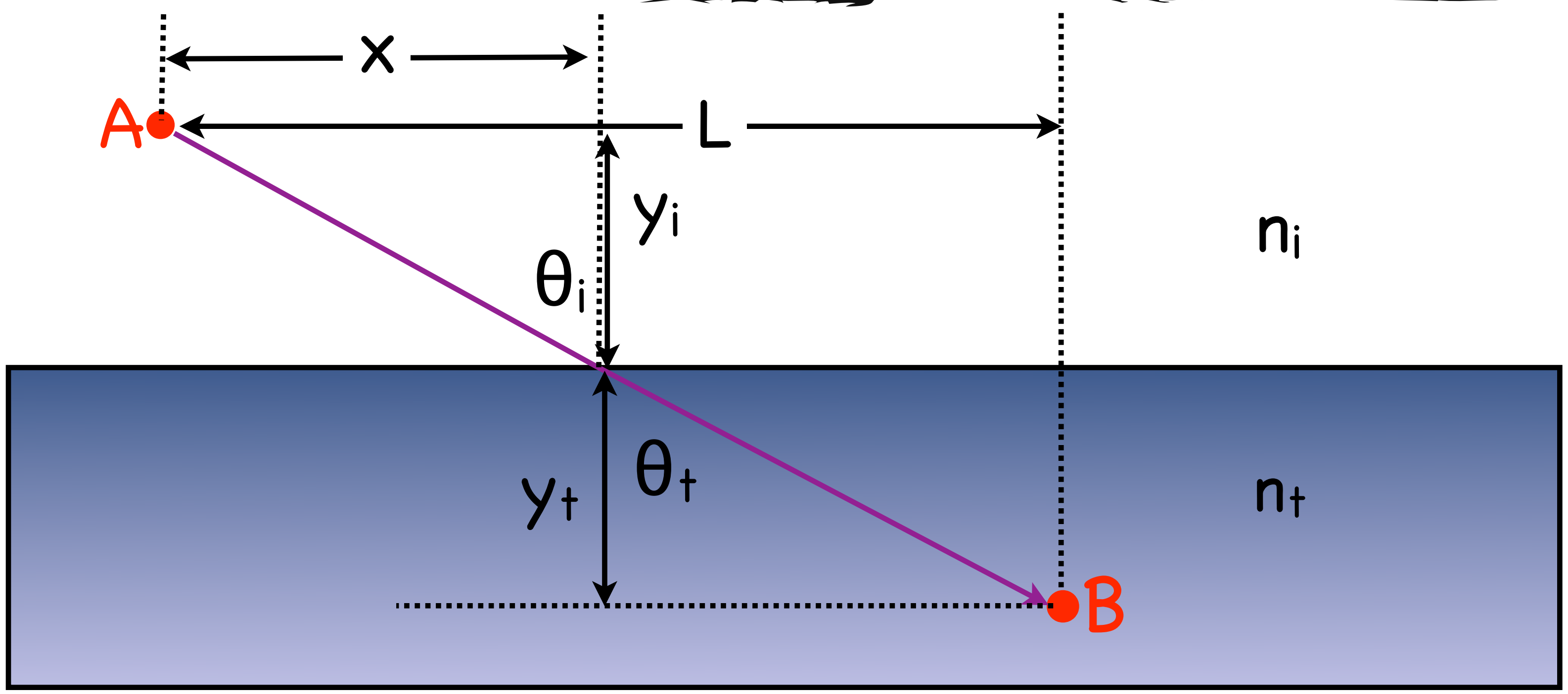
- Restatement of Fermat's Principle
  - Define the "optical path length" (OPL) to be the effective distance traveled by a light ray (if it travels through a dense material of index n the effective distance is n times greater than the actual distance)
  - Small variations in the path taken by a light ray must not affect the optical path length
  - d(OPL)=0 for the path taken by a light ray
    (notice that the optical path length does **not** have to be a minimum)
- Analogous to methods used in other areas of physics
  - Lagrange's method in classical mechanics
  - Feynman's method in quantum mechanics

Snell's Law

- Light rays traveling from point A to point B will follow paths that represent a (local) extremum in optical path length from point A to point B



Snell's Law



$$OPL = n_i\sqrt{x^2+y^2} + n_t\sqrt{(L-x)^2+y_t^2}$$

$$\frac{d(OPL)}{dx} = \frac{x n_i}{\sqrt{y_i^2+x^2}} - \frac{(L-x)n_t}{\sqrt{y_t^2+(L-x)^2}}$$

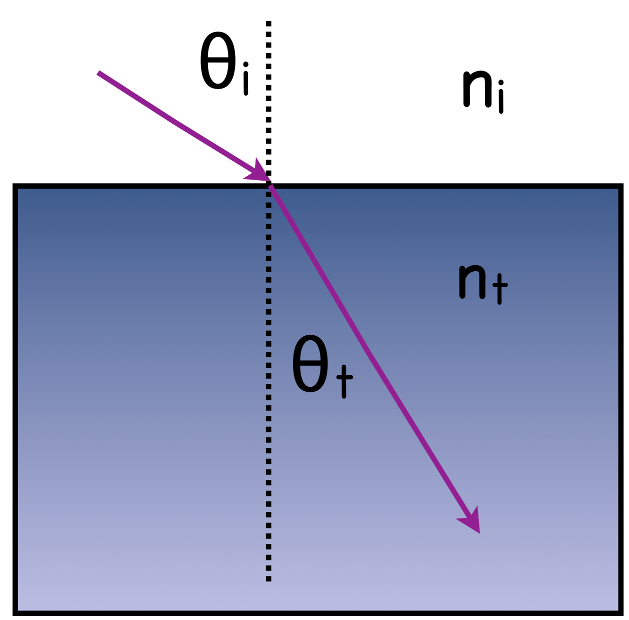
$$\frac{d(OPL)}{dx} = n_i \sin\theta_i - n_t \sin\theta_t \rightarrow 0$$

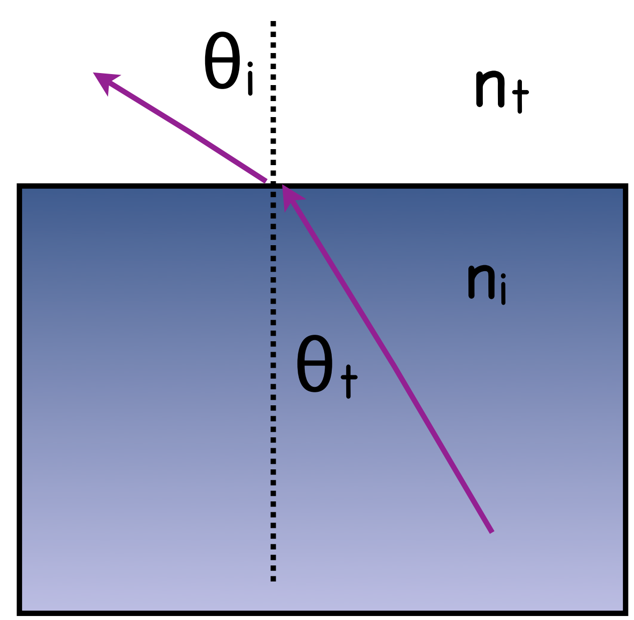
$n_i \sin\theta_i = n_t \sin\theta_t$ **Snell's Law**

Snell's law

- Light traveling from a low-index material to a high-index material will bend towards the normal

$$n_i \sin\theta_i = n_t \sin\theta_t$$





Summary

- We have several mechanisms to understand how wavefronts propagate through space
  - Huygen's principle considers points along a wavefront as secondary sources that create the propagated wavefront
  - Fermat's principle considers the direction of light rays and states that they travel a path where the optical path length is an extremum
- From these principles we can derive the law or reflection and the law of refraction (Snell's law)