# Acoustooptic Devices

Chapter 10

Physics 208, Electro-optics

Peter Beyersdorf

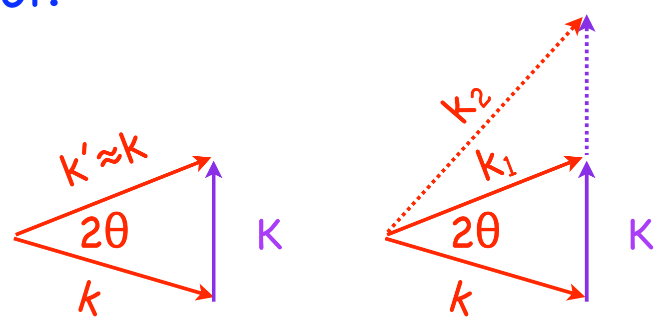
# Overview

- Raman-Nath Diffraction (chapter 9)
- AO Modulators
- AO deflectors
- Bandwidth
- Figures of Merit

# Raman-Nath Diffraction

Our expressions for the optical field diffracted from the acoustic wave assume there is only one diffracted order (m=±1). Is it possible to have |m|>1, corresponding to more than 1 phonon absorbed?

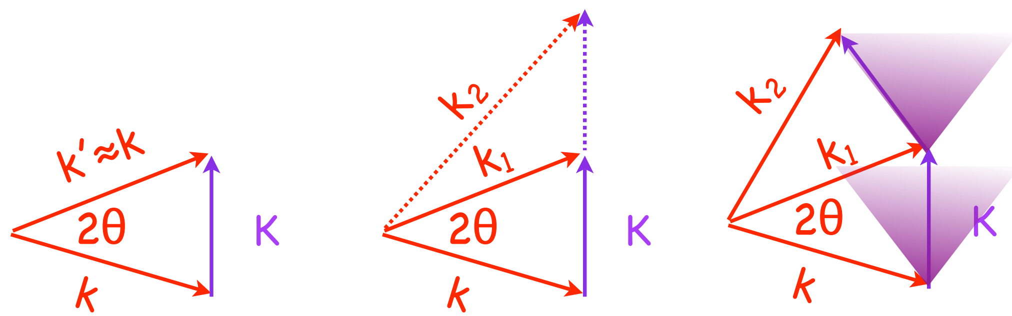
Conservation of energy requires k and k′ be almost the same length (except for the relatively small increase of k′ due to absorbed energy of phonon). This isn't possible for more than one order at a time, with a unique K vector.



# Acoustic Beam Spread

If the acoustic wave is a finite beam with width ΔX, Heisenberg's uncertainty relation $\Delta x \Delta p \geq \hbar/2$ tells us it will have a range of K-vectors $\Delta K = 1/(2\Delta X)$.

If the range of K-vectors is large compared to the Bragg angle, multiple phonos to be absorbed without changing length of k′ relative to k.



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# Raman-Nath Criterion

Acoustic beam spread is $\Delta K = K\Delta\Theta$

$$\Delta\Theta = \frac{\Delta K}{K}$$

$$\Delta\Theta = \frac{\Lambda}{2\pi\Delta X}$$

Thus the Bragg angle ($\theta_b = \lambda/(2n\Lambda)$ )can be expressed in terms of the acoustic beam spread

$$Q = 2\pi\frac{\lambda\Delta X}{n\Lambda^2}$$

Q>1 is called the "Bragg regime" and corresponds to single order diffraction

Q<1 is called the "Raman-Nath regime" and corresponds to multiple order diffraction

# Raman-Nath Diffraction

Consider a narrow acoustic beam introducing a change in the index of refraction in a material of

$$\Delta n(0 < x < \Delta X) = \Delta n_0 \sin(\Omega t - \vec{K} \cdot \vec{r})$$

an optical wave expressed by

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

is incident at x=0, upon exiting the acoutic beam at x=ΔX it can be written as

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k} \cdot \vec{r} - \phi)}$$

with

$$\phi = \int_0^{\Delta X} \frac{1}{\cos\theta} \frac{\omega}{c} \Delta n dx$$

where θ is the direciton of the optical beam relative to the x-axis

# Raman-Nath Diffraction

If ΔX is small (i.e. in the Rama-Nath regime), index n (x,t) seen by optical beam can be considered constant across beam width

$$\phi = \frac{\Delta X}{\cos\theta}\frac{\omega}{c}\Delta n = \frac{\Delta X}{\cos\theta}\frac{\omega}{c}\Delta n_0 \sin(\Omega t - \vec{K}\cdot\vec{r})$$

giving

$$\vec{E} = \vec{E}_0 e^{i\left(\omega t - \vec{k}\cdot\vec{r} - \delta \sin(\Omega t - \vec{K}\cdot\vec{r})\right)}$$

where

$$\delta \equiv \frac{\Delta X}{\cos\theta}\frac{\omega}{c}\Delta n_0$$

is called the modulation index. This can be expanded using a form of the Jacobi-Anger identity

$$e^{iz\cos\phi} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\phi}$$

# Bessel Function Expansion

Expanding

$$\vec{E} = \vec{E}_0 e^{i\left(\omega t - \vec{k}\cdot\vec{r} - \delta \sin(\Omega t - \vec{K}\cdot\vec{r})\right)}$$

into Bessel functions gives

$$\vec{E} = \vec{E}_0 \sum_{m=-\infty}^{\infty} J_m(\delta) e^{i\left((\omega - m\Omega)t - (\vec{k} - m\vec{K})\cdot\vec{r}\right)}$$

and the diffraction efficiency for the m$^{th}$ order diffracted beam is

$$\eta_m = J_m^2(\delta) = J_m^2\left(\frac{\Delta X}{\cos\theta}\frac{\omega}{c}\Delta n_0\right)$$

# Bessel Function Expansion

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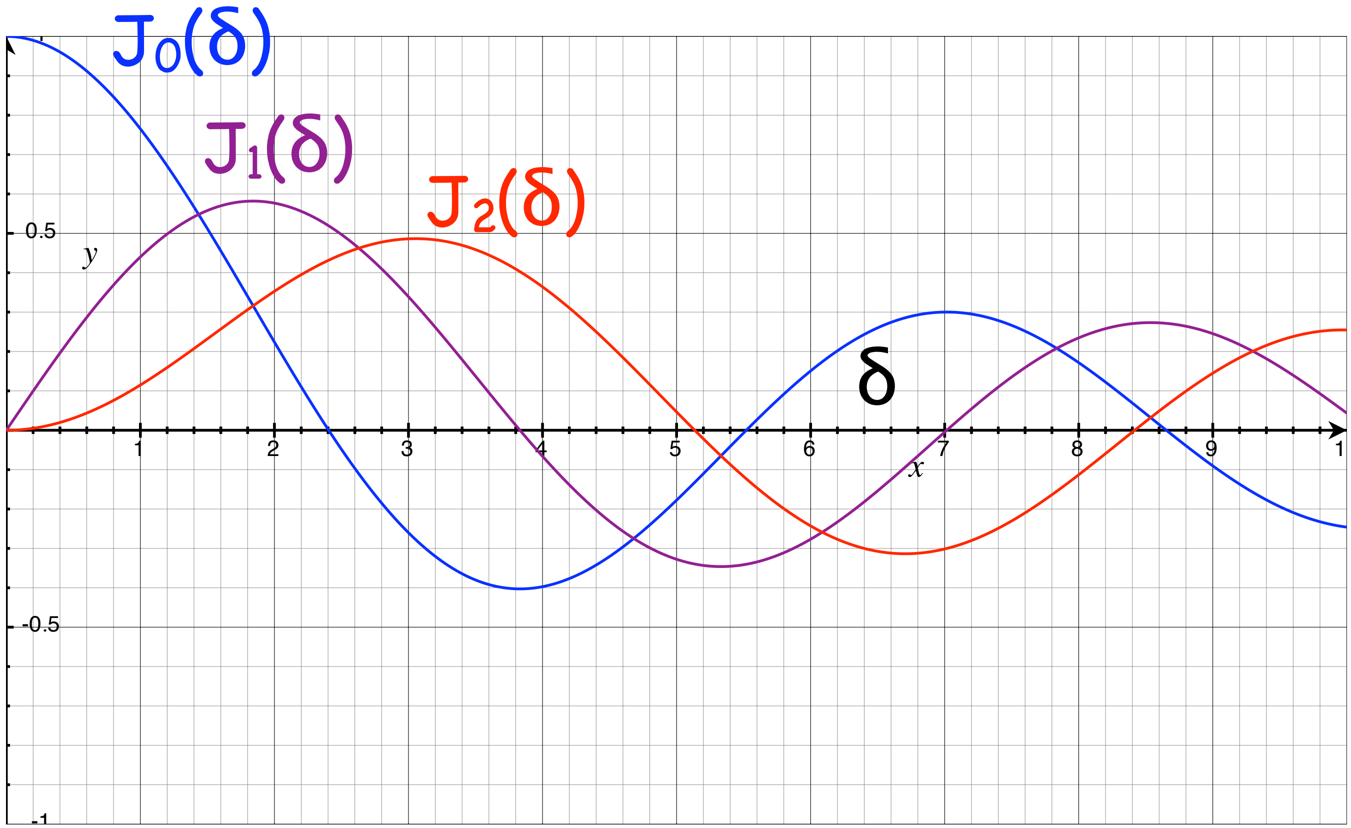

# Example

Calculate the acoustic beam width $\Delta X$ to maximize diffraction into the m=1 order for $\Delta n_0=0.0001$, $\theta=0^0$ and $\lambda=633$ nm. What fraction of the power gets diffracted into the m=1 beam?

$J_1(\delta)$ is max at $\delta\approx1.85$, solving for $\Delta X$ we get $\Delta X=1.9$ mm

Evaluating $\eta=J_1^2(1.85)\approx0.33$

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# Acoustooptic Devices

Like the electrooptic effect, the acoustooptic effect can be used to construct Modulators, beam deflectors, frequency shifters, tunable filters.

Unlike the electrooptic effect, the electrical driving signal needs to be encoded on an RF carrier, rather than applied directly.

# AO Modulators

The diffraction efficiency in an acoustooptic interaction is

$$\eta = \frac{I_{\text{diffracted}}}{I_{\text{incident}}} = \sin^2\left(\frac{\pi L}{\lambda\sqrt{2}\cos\theta_B}\sqrt{MI_a}\right)$$

where L is the interaction length, $\theta_b$ is the Bragg angle, $I_a$ is the acoustooptic intensity and

$$M = \frac{n^6\bar{p}^2}{\rho v^3}$$

is a figure of merit for the material that depends on polarization and angle, the interaction configuration and the direction of propagation. At low acoustic intensity

$$\eta \approx \frac{\pi^2 L^2}{2\lambda^2\cos^2\theta_B} M I_a$$

any amplitude modulation of the acoustic signal can be used to modulate the intensity of the diffracted beam

# AO Deflectors

The Bragg angle determines the direction of the diffracted beam and obeys

$$2\Lambda \sin\theta = \lambda/n$$

where $\Lambda = v/f$ is the wavelength of the acoustic wave of frequency f with a speed of propagation v. Thus the Bragg angle can be written

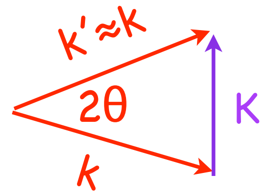
$$\theta_B = \sin^{-1}\frac{\lambda f}{2n\nu} \approx \frac{\lambda f}{2n\nu}$$

since it is a small angle.

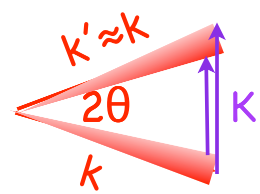
The angle of diffraction $2\theta_b$ is linearly proportional to f. Thus frequency modulation of the acoustic signal can modulate the deflection angle of the diffracted beam

# Bandwidth

The analysis so far has assumed monochromatic optical and acoustic radiation (i.e. plane waves, not beams) and requires a unique acoustic K vector to meet Bragg condition




Modulation on acoustic wave is equivalent to introducing frequency components with different K-vectors. For these to produce diffraction the optical and/or acoustic beam has to have a range of k-vectors (i.e. be a beam not a plane wave)




The angular spread in the beams thus determines the useful modulation bandwidth of the device

# Modulation Bandwidth

Differentiating expression for Bragg angle

$$\theta_B \approx \frac{\lambda f}{2n\nu}$$

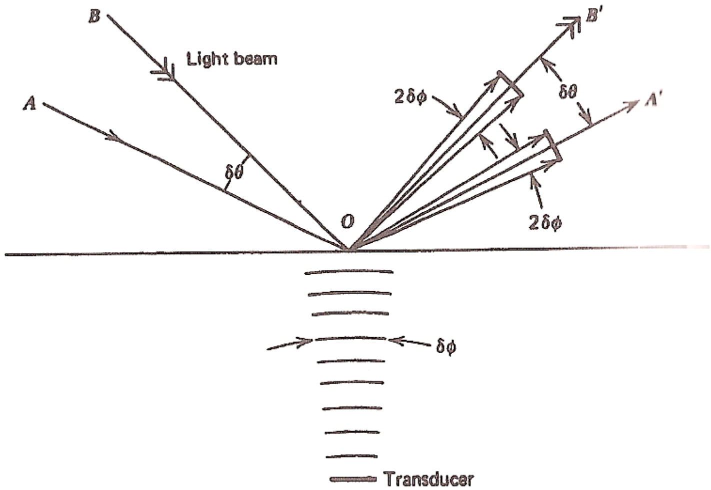
gives a relation between the bandwidth $\Delta f$ and the spread in the incident angle

$$\Delta f = \frac{2n\nu \cos\theta}{\lambda}\Delta\theta$$

but the spread in incident angle contains a contribution $\delta\theta \approx 2\lambda/(\pi n \omega_0)$ from the optical beam with Gaussian waist $\omega_0$, and $\delta\varphi \approx \Lambda/L$ from the acoustic beam of width L

$$\Delta\theta = \delta\theta + \delta\phi$$

# Modulation Bandwidth




The modulation sidebands on the acoustic beam couple to different output angles for the optical beam. For the optical modulation sidebands to overlap the angular spread in the acoustic beam must equal or exceed that of the optical beam $\delta\varphi \geq \delta\theta = \lambda / \pi n \omega_0$ giving a bandwdith

$$(\Delta f)_m = \frac{1}{2}\Delta f = \frac{2\nu}{\pi\omega_0}\cos\theta$$

which is roughly equal to the reciprocal of the transit time of the acoustic wave across the optical beam

# Modulation Bandwidth

Requiring the spread in optical beam not exceed the diffraction angle requires $\Delta\theta \leq \theta_b$ so that

$$\frac{(\Delta f)_m}{f} \approx \frac{\Delta f}{2f} < \frac{1}{2}$$

where $\Delta f_m$ is the bandwidth of the signal that produces a spread in the acoustic frequency from $f \pm \Delta f$.

This means if the RF carrier frequency f can be modulated at a frequency up to f/2. Clearly if it were modulated at a frequency of f, there would be DC components that would fail to deflect the diffracted beam causing it to overlap with the undiffracted beam.

# Material Figures of Merit

Efficiency and Bandwidth of a modulator are important properties that depend on the material and configuration geometry of the modulator. Various figures of merit relate these quantities of material independent of

$$M_1 = \frac{n^7 p^2}{\rho v}$$

Proportional to the diffraction efficiency and the modulation bandwidth in a modulator of length L and heigh h where

$$2\eta f_0 \Delta f = \left(\frac{n^2 p^2}{\rho \nu}\right) \frac{2\pi^2}{\lambda^3 \cos\theta_B} \left(\frac{P_\alpha}{h}\right)$$

$$M_2 = \frac{n^6 p^2}{\rho v^3}$$

Proportional to the diffraction efficiency in a modulator of length L and heigh h at acoustic power $P_a$ where

$$\eta = \frac{\pi^2}{2\lambda^2 \cos^2\theta_B} \left(\frac{L}{h}\right) M_2 P_a$$

# References

- Yariv & Yeh "Optical Waves in Crystals" chapter 10