

Electro-optic Devices



Chapter 8

Physics 208, Electro-optics

Peter Beyersdorf

Modulators



- Encode analog or digital signals on an optical wave
- Shift optical power to other spectral frequency bands
- Act as fast switches for laser Q-switching, etc...
- Control power and or phase of a beam

Tunable Lasers

Femtosecond Laser

WDM Laser Sources

Benchtop
Laser Sources

HeNe Lasers

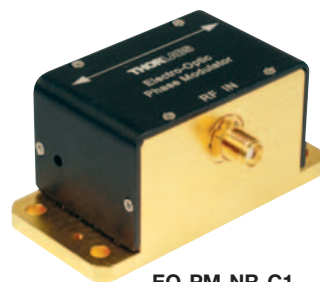
ASE Sources

Terahertz

**Electro-Optic
Modulators**

Laser & ASE Systems

Electro-Optic Modulators



EO-PM-NR-C1



EO-AM-NR-C2
With EO-GTH5M



EO-HVA

Thorlabs' free-space electro-optic (EO) amplitude and phase lithium niobate modulators combine crystal growth and electro-optic materials. Our standard modulators use undoped lithium niobate. For higher-power operation, we offer MgO-doped lithium niobate. The standard EO modulators are broadband DC-coupled, and a high Q resonant model option is available.

Our standard DC-coupled broadband EO modulators consist of an EO crystal packaged in a housing optimized for maximum RF performance. The RF drive signal is connected directly to the EO crystal via the SMA RF input. An external RF driver supplies the drive voltage for the desired modulation. The crystal may be modulated from DC up to the frequency limits of the external RF driver.

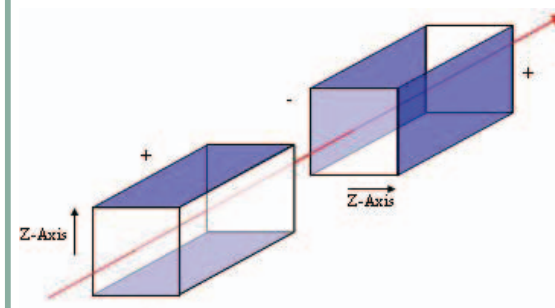
For flexibility, Thorlabs also offers a second modulator option. Resonant frequency modulators simplify the driver requirements for many applications where the modulator is operated at a single frequency. A high Q resonant tank circuit located inside the modulator boosts the low-level RF input voltage from a standard function generator to the high voltage needed to get full depth of modulation. Call our tech support team for details on the specific resonant frequencies available.

EO Amplitude Modulator

The electro-optic amplitude modulator (EO-AM) is a Pockels cell type modulator consisting of two matched lithium niobate crystals packaged in a compact housing with an RF input connector. Applying an electric field to the crystal induces a change in the indices of refraction (both ordinary and extraordinary), giving rise to an electric field-dependent birefringence,

Features

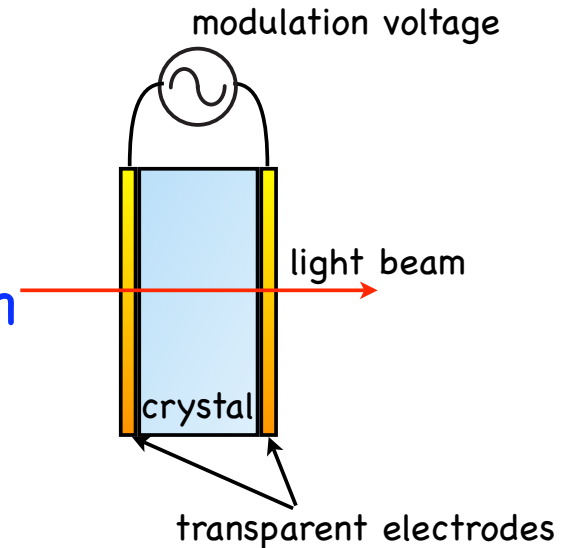
- High Performance in a Compact Package
- Broadband DC Coupled and High Q Resonant Models for Low RF Drive
- Standard Broadband AR and Custom Coatings
- Ø2mm Clear Aperture
- SMA Female Modulation Input Connector
- MgO-Doped Versions for High Power
- DC to 100MHz
- Custom OEM Versions Available



Longitudinal Modulators

- Short crystal lengths
- Large field-of-view
- Phase or amplitude modulation
- EO phase shift is of the form

$$\Delta\phi = \frac{\pi}{\lambda} n^3 r E L$$



Longitudinal LiNbO₃ Modulator

In a z-cut LiNbO₃ crystal

$$x^2\left(\frac{1}{n_0^2} + r_{13}E\right) + y^2\left(\frac{1}{n_0^2} + r_{13}E\right) + z^2\left(\frac{1}{n_e^2} + r_{33}E\right) = 1$$

giving

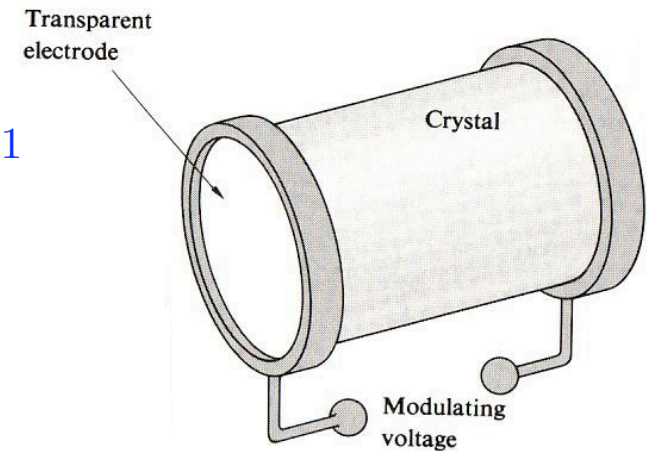
$$n_x = n_0 - \frac{1}{2}n_0^3r_{13}E,$$

$$n_y = n_0 - \frac{1}{2}n_0^3r_{13}E,$$

$$n_z = n_e - \frac{1}{2}n_0^3r_{33}E,$$

Which means there is no birefringence ($n_x=n_y$) induced by the modulation, only a pure phase delay results

$$\phi = kL = \frac{2\pi}{\lambda}n_0L - \frac{\pi}{\lambda}n_0^3r_{13}V$$



Longitudinal LiNbO₃ Modulator

the “half wave” voltage is

$$V_{\pi} = \frac{\lambda}{n_0^3 r_{13}}$$

and if the applied voltage is sinusoidal such that

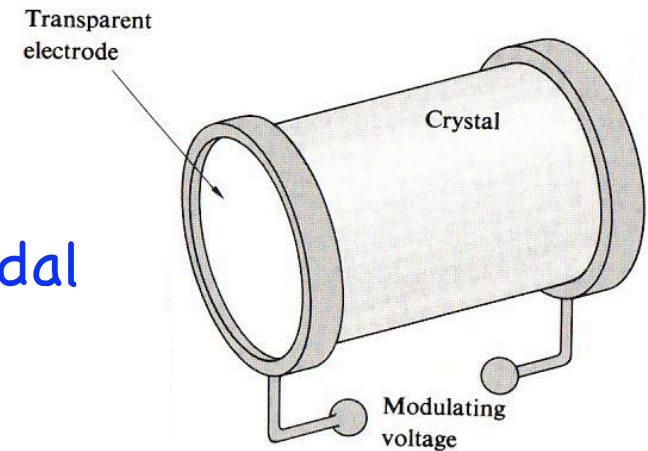
$$V = V_m \sin \omega_t,$$

The transmitted field can be expressed as

$$E(z, t) = E_{in} e^{i(\omega t - kz - \phi_0 + \delta \sin \omega_t)}$$

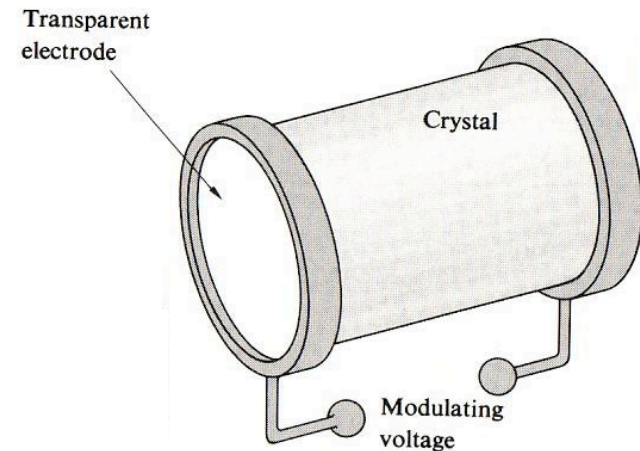
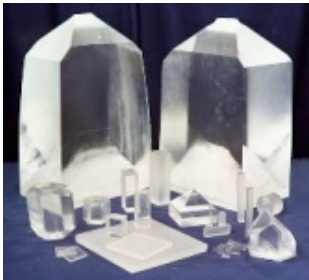
where

$$\delta = \frac{\pi}{\lambda} n_0^3 r_{13} V_m = \pi \frac{V_m}{V_{\pi}} \quad \text{and} \quad \phi_0 = \frac{2\pi}{\lambda} n_0 L$$



KDP Example

Design an amplitude modulator using a KDP crystal with longitudinal modulation (applied electric field in the direction of propagation)
Find an expression for the transmitted power as a function of applied voltage



Recall our previous example calculating the principle axes and indices of refraction in KDP...

KDP Example

- Potassium Dihydrogen Phosphate (KH_2PO_4)
- $\bar{4}2m$ crystal group (tetragonal) [table 7.3]
- Form of r_{ik} matrix is found from table 7.2

$$r_{ik} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

- Electrooptic and optical properties [table 7.3]
 - $r_{41}=8 \times 10^{-12}$ m/V, $r_{63}=11 \times 10^{-12}$ m/V @633 nm
 - $n_x=n_y=n_o=1.5115$, $n_z=n_e=1.4698$

KDP Example

equation for index ellipsoid is

$$\left(\frac{1}{n_x^2} + r_1 k E_k\right) x^2 + \left(\frac{1}{n_y^2} + r_2 k E_k\right) y^2 + \left(\frac{1}{n_z^2} + r_3 k E_k\right) z^2 + 2yzr_{4k}E_k + 2x zr_{5k}E_k + 2xyr_{6k}E_k = 1$$

which becomes

$$\left(\frac{1}{n_x^2} + r_1 k E_k\right) x^2 + \left(\frac{1}{n_y^2} + r_2 k E_k\right) y^2 + \left(\frac{1}{n_z^2} + r_3 k E_k\right) z^2 + 2yzr_{41}E_x + 2x zr_{52}E_y + 2xyr_{63}E_z = 1$$

or

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2yzr_{41}E_x + 2x zr_{41}E_y + 2xyr_{63}E_z = 1$$

for a field applied entirely along z:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2xyr_{63}E_z = 1$$

KDP Example

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2xyr_{63}E_z = 1$$

We can choose a new set of principle coordinates x' , y' and z' that recast this equation into the form

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

From inspection $z=z'$, so let $x'y'$ and xy differ by a rotation of θ around the z -axis:

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

$$\frac{(x' \cos \theta - y' \sin \theta)^2}{n_o^2} + \frac{(x' \sin \theta + y' \cos \theta)^2}{n_o^2} + \frac{z^2}{n_e^2} + 2(x' \cos \theta - y' \sin \theta)(x' \sin \theta + y' \cos \theta)r_{63}E_z = 1$$

KDP Example

Evaluating gives

$$\frac{(x' \cos \theta - y' \sin \theta)^2}{n_o^2} + \frac{(x' \sin \theta + y' \cos \theta)^2}{n_o^2} + \frac{z'^2}{n_e^2} + 2(x' \cos \theta - y' \sin \theta)(x' \sin \theta + y' \cos \theta)r_{63}E_z = 1$$

or

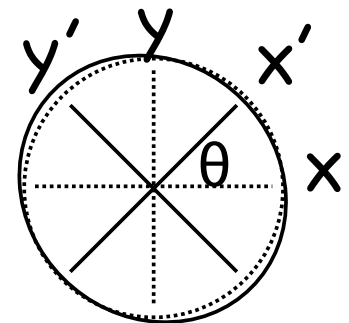
$$\frac{x'^2}{n_o^2} + \frac{y'^2}{n_o^2} + \frac{z'^2}{n_e^2} + \frac{z'^2}{n_e^2} - \frac{x'y' \sin 2\theta}{n_o^2} + \frac{x'y' \sin 2\theta}{n_o^2} + (x'^2 \sin 2\theta + 2x'y' \cos 2\theta - y'^2 \sin 2\theta)r_{63}E_z = 1$$

cross term $2x'y' \cos(2\theta) \rightarrow 0$ for $\theta=45^\circ$ giving

$$\frac{x'^2}{n_o^2} + \frac{y'^2}{n_o^2} + \frac{z'^2}{n_e^2} + (x'^2 - y'^2)r_{63}E_z = 1$$

or

$$\left(\frac{1}{n_o^2} + r_{63}E_z \right) x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z \right) y'^2 + \frac{z'^2}{n_e^2} = 1$$



KDP Example

Equating

$$\left(\frac{1}{n_o^2} + r_{63}E_z\right)x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right)y'^2 + \frac{z'^2}{n_e^2} = 1$$

to

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

gives

$$\left(\frac{1}{n_o^2} + r_{63}E_z\right) = \frac{1}{n_x'^2}$$

$$n_x'^2 = n_o^2 \left(\frac{1}{1 + n_o^2 r_{63} E_z} \right)$$

$$n_x'^2 \approx n_o^2 (1 - n_o^2 r_{63} E_z) \quad \text{for } E_z \ll 1/(n_o^2 r_{63}) = 4 \times 10^{10} \text{ V/m}$$

so

$$n_x' \approx n_o \left(1 - \frac{1}{2} n_o^2 r_{63} E_z \right) \quad n_y' \approx n_o \left(1 + \frac{1}{2} n_o^2 r_{63} E_z \right) \quad n_z' = n_e$$

Longitudinal KDP Modulator

In a KDP Pockel's cell with an electric field along the z-axis

$$\Gamma = \frac{\omega}{c} n_o^3 r_{63} E L = \frac{\omega}{c} n_o^3 r_{63} V$$

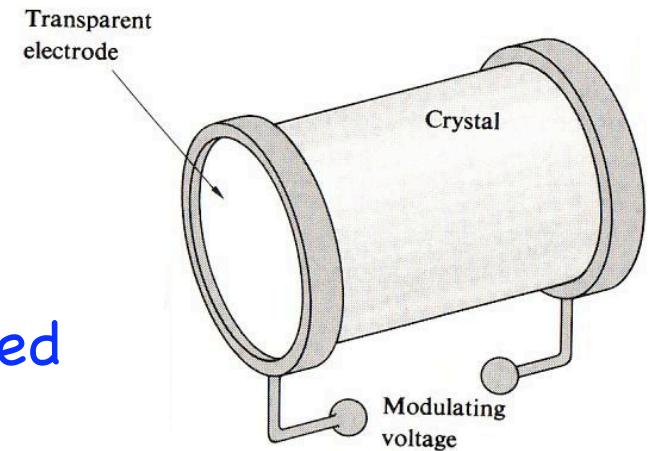
Independent of L. For light polarized in the x' or y'

$$\delta n = \frac{1}{2} n_o^3 r_{63} E_{max}$$

giving

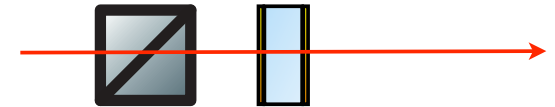
$$\delta n_{eff} = \frac{1}{2} n_o^3 r_{63} V_{max} k_0 \operatorname{sinc} \Delta \kappa L$$

so the modulation depth is maximized for $L \rightarrow 0$



Longitudinal KDP Modulator

For this to be used as a phase modulator the light must be polarized along x' or y' and the phase shift is



$$\Delta\phi = k\Delta nL = \frac{\pi}{\lambda}n_o^3r_{63}V$$

The half-wave voltage is

$$V_\pi = \frac{\lambda}{n^3r_{63}}$$

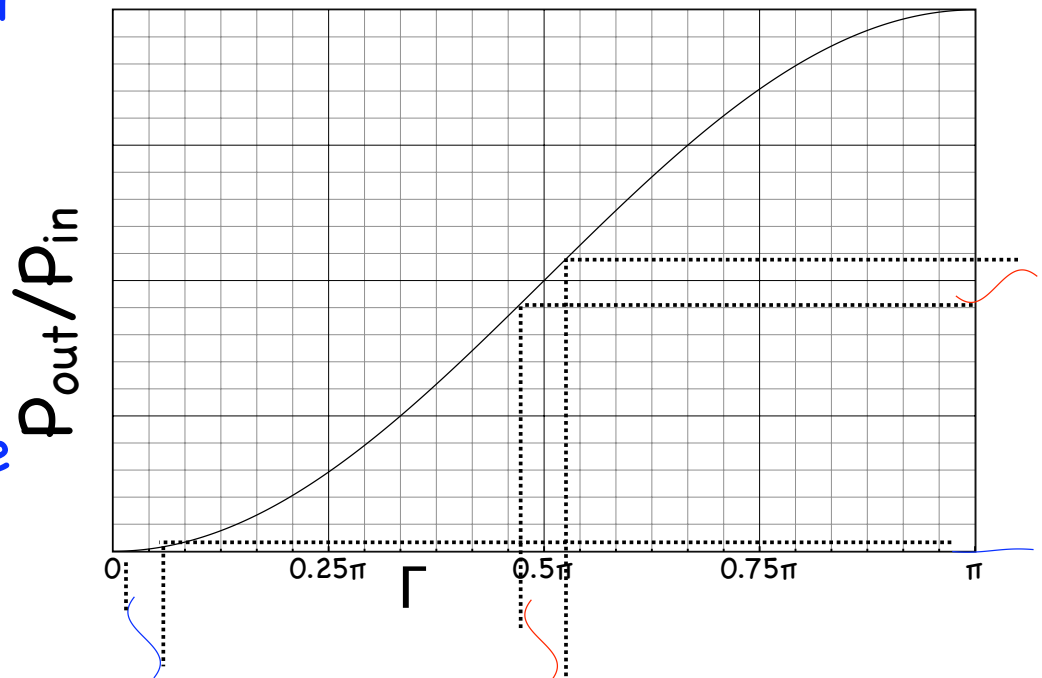
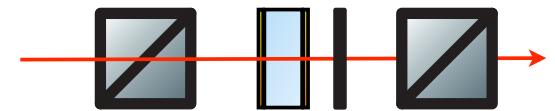
and the modulation depth is

$$\delta = \frac{\pi}{\lambda}n^3r_{63}V_m = \pi\frac{V_m}{V_\pi}$$

Longitudinal KDP Modulator

For this to be used as an amplitude modulator the polarization rotation produced when the input light is along x or y needs to be converted into amplitude change by an output polarizer such that $T = \sin^2(\Gamma/2)$

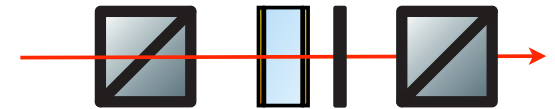
A quarter wave plate can add an additional $\pi/2$ static birefringence to bias the output for maximum modulation



Longitudinal KDP Modulator

The birefringence of this system is

$$\Gamma = \frac{\pi}{2} + \frac{2\pi}{\lambda} n^3 r_{41} V_m \sin \omega_m t = \frac{\pi}{2} + \pi \frac{V_m}{V_\pi} \sin \omega_m t$$



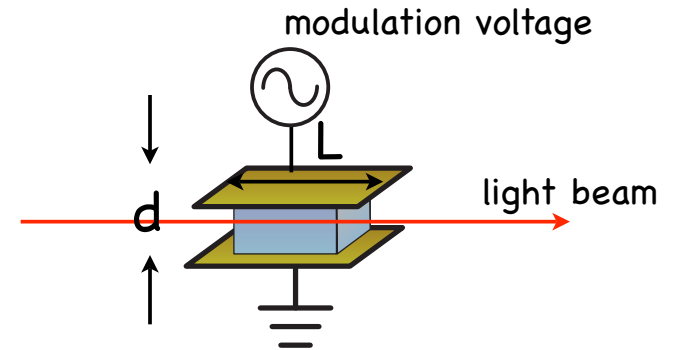
giving a transmission of

$$T = \frac{1}{2} \left(1 + \pi \frac{V_m}{V_\pi} \sin \omega_m t \right)$$

In any such amplitude modulator half of the power in the optical beam is dumped.

Transverse Modulators

- Long interaction region
- Low half-wave voltages
- Limited acceptance angle
- Modulation depth proportional to VL/d



LiNbO₃ Example

LiNbO₃ is a trigonal crystal with 3m symmetry, therefore it is uniaxial with an electrooptic tensor of the form

$$r_{ik} = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$$

For propagation along y, and an electric field applied along z

$$x^2 \left(\frac{1}{n_o^2} + r_{13}E \right) + y^2 \left(\frac{1}{n_o^2} + r_{13}E \right) + z^2 \left(\frac{1}{n_e^2} + r_{33}E \right) = 1$$

giving

$$n_x = n_o - \frac{1}{2}n_o^3 r_{13}E, \quad n_z = n_e - \frac{1}{2}n_e^3 r_{33}E$$

LiNbO₃ Example

Giving

$$n_z - n_x = (n_e - n_o) - \frac{1}{2} (n_e^3 r_{33} - n_o^3 r_{13}) E$$

for a birefringence of

$$\Gamma = \frac{2\pi}{\lambda} (n_e - n_o) L - \frac{\pi}{\lambda} (n_e^3 r_{33} - n_o^3 r_{13}) \frac{L}{d} V$$

The half-wave voltage is

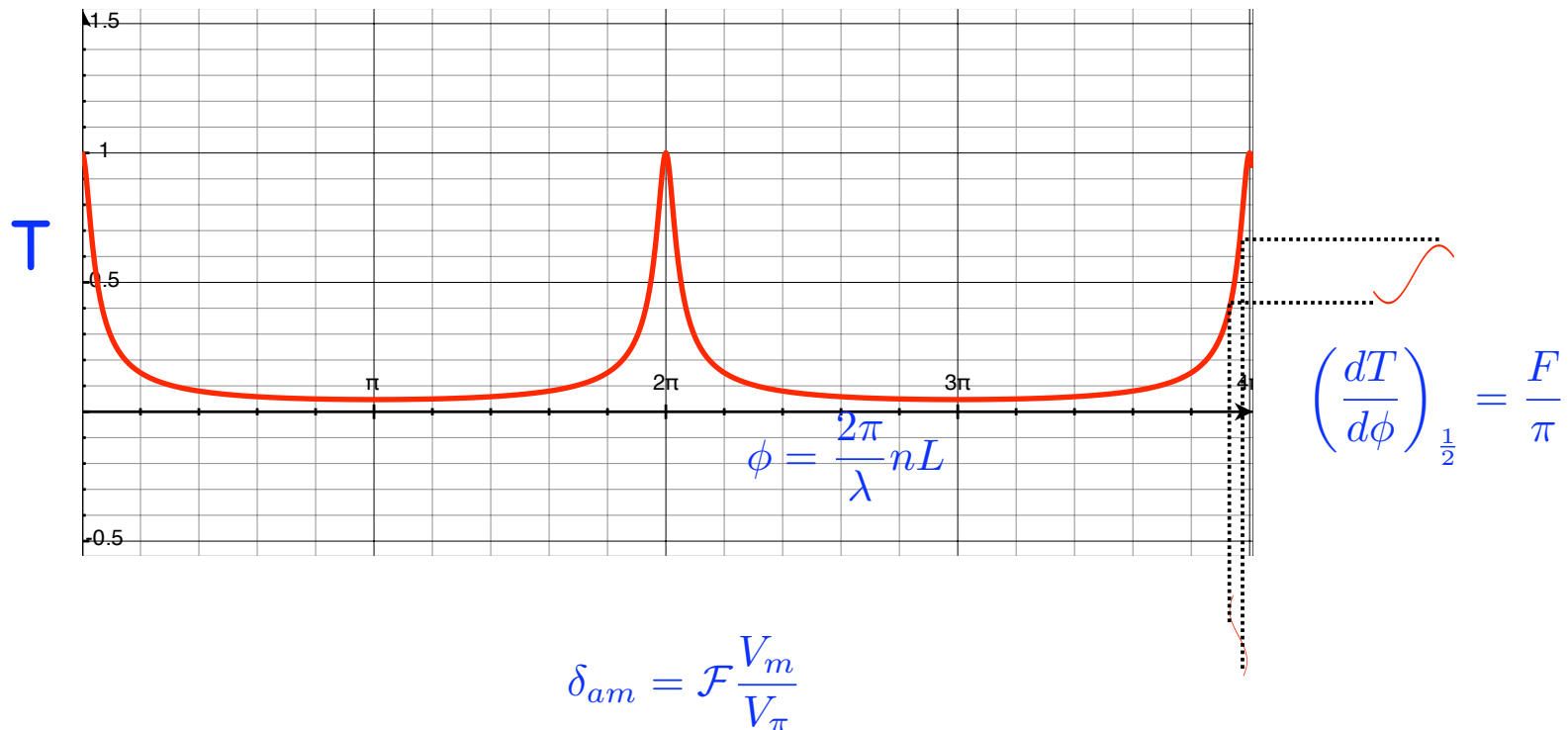
$$V_\pi = \frac{d}{L} \frac{\lambda}{n_e^3 r_{33} - n_o^3 r_{13}}$$

Static birefringence term can be tuned to provide $\pi/2$ phase shift of bias for use as an amplitude modulator

Optical Feedback Amplitude Modulators

Fabry-Perot cavity transmission with identical reflectivity mirrors is given by

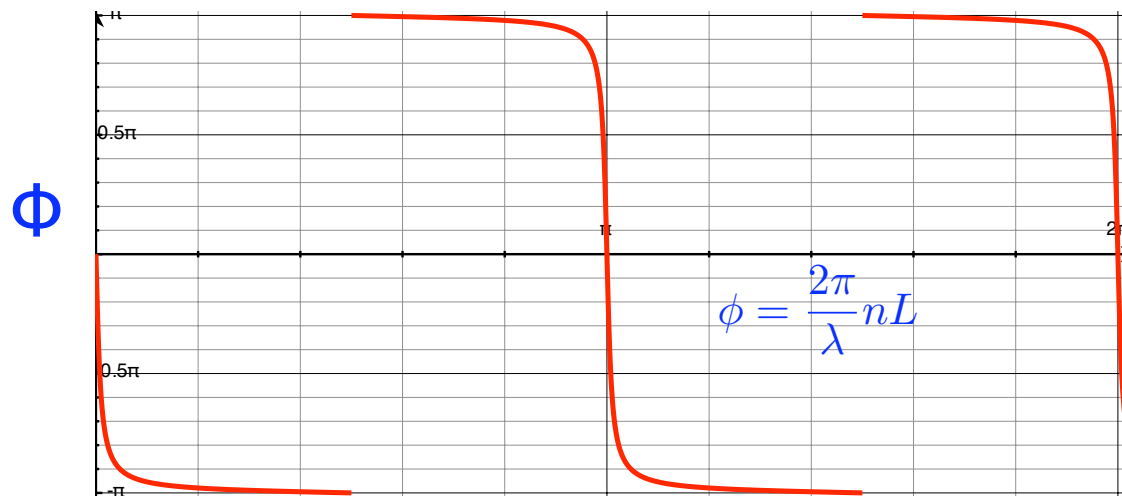
$$T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2 \phi}$$



Optical Feedback Phase Modulators

Fabry-Perot cavity reflectivity with 100% reflectivity end mirror is given by

$$r = e^{i\phi} = \frac{-\sqrt{R} + e^{-2i\phi}}{1 - \sqrt{R}e^{-2i\phi}} \quad \Phi = -2 \tan^{-1} \left(\frac{1 + \sqrt{R}}{1 - \sqrt{R}} \tan \phi \right)$$



For small deviations from resonance

$$\Phi = -2 \tan^{-1} \left(\frac{1 + \sqrt{R}}{1 - \sqrt{R}} \tan \pi \frac{V}{V_\pi} \right) \approx 2\pi \frac{1 + \sqrt{R}}{1 - \sqrt{R}} \frac{V}{V_\pi}$$

High Frequency Design Considerations

Electrodes on crystal form a capacitor that can limits the speed of electrical modulation when connected to a source with finite output impedance ($R_s=50\Omega$ is typical)



$$V_{eom} = \frac{\frac{1}{i\omega C}}{R_s + \frac{1}{i\omega C}} V_m$$

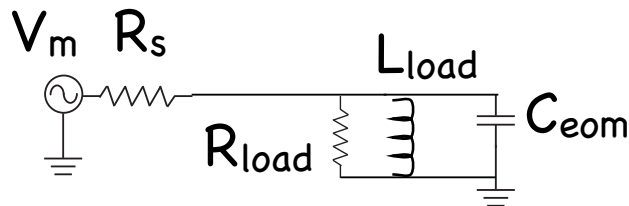
The characteristic frequency of this low-pass filter is $f_c=1/(2\pi R_s C_{eom})$. For a typical EOM $C_{eom}=20$ pF giving 160 MHz. The EOM can not operate efficiently above this frequency in this configuration.

Parallel Resonant circuits

An inductor in parallel with the EOM can raise the impedance of the EOM

$$V_{eom} = \frac{R_{load}}{R_s + R_{load}} V_m \quad \text{to } \infty \text{ at a frequency}$$

$$f = \frac{1}{2\pi\sqrt{L_{load}C_{eom}}}$$



A parallel resistor R_{load} is added to limit the impedance to that of the transmission line (i.e. the cable) to prevent reflections that would interfere destructively at the EOM giving a usable voltage on the EOM at the resonant frequency

$$V_{eom} = \frac{R_{load}}{R_s + R_{load}} V_m$$

Parallel Resonant circuits



The resonant circuit has a bandwidth

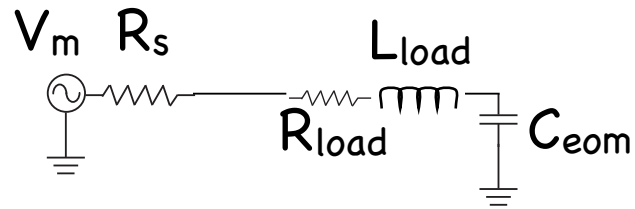
$$\Delta\omega \approx \frac{1}{2\pi R_{load}C_{eom}}$$

and requires a power of the form

$$P = \frac{\Gamma_m^2 \lambda^2 A \epsilon \Delta\omega}{8\pi^2 L (n^3 r)^2}$$

to reach a modulation depth of Γ_m in a crystal of cross sectional area A , dielectric constant ϵ , length L

Series Resonant circuits



An inductor in series with the EOM can allow amplification of the modulation voltage at the resonant frequency $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{L_{load}C_{eom}}}$

The frequency dependent voltage divider gives

$$V_{eom} = \frac{X_c(\omega)}{X_c(\omega) + X_l(\omega) + R_{load} + R_s} V_m \quad \text{at the resonant}$$

frequency $|V_{eom}| = \frac{1}{\omega_0(R_s + R_{load})C_{eom}} V_m$

for $R_s=R_{load}=50\Omega$, $C=20$ pF and $f_0=10$ MHz

$$V_{eom}/V_m \approx 80$$

Transit Time Limitations

We've seen that high frequency modulation can get "averaged out" when a wave spends a large fraction of a modulation period in the crystal

$$\frac{\delta n_{eff}}{\delta n} = \text{sinc}(\Delta\kappa L) \quad \Delta\kappa \equiv \frac{\Omega}{2} \left[\frac{1}{v_p} - \frac{1}{v_m} \right]$$

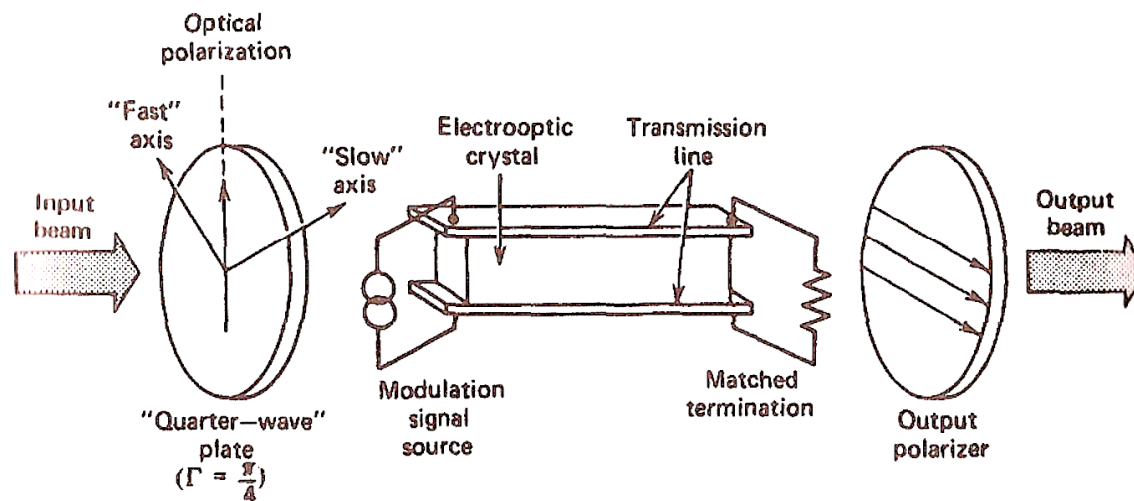
For a value of $\Delta\kappa L = \pi/2$ the modulation is reduced to 0.64 of the unaveraged modulation depth. Calling the frequency at which this occurs f_c , the cutoff frequency for useful modulation, we get for $v_m = \infty$ (i.e. a "lumped modulator")

$$f_c = \frac{\Omega_c}{2\pi} = \frac{c}{2nL}$$

for $L=3\text{cm}$, $n=2$, we find $f_c=2.5\text{ GHz}$

Traveling Wave Modulators

If the EOM electrodes are made into a transmission line the phase mismatch can be greatly improved. For $v_p = v_m$ the transit time limitation can be removed and modulation depth increased by increasing crystal length



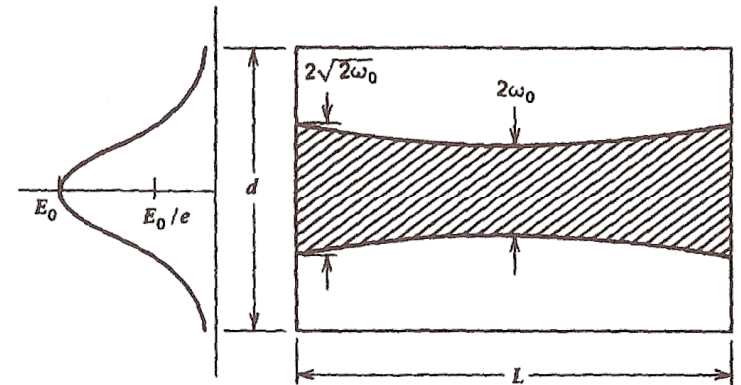
Geometrical Considerations

For maximum modulation depth in a transverse, free-space modulator it is desirable to have a long length L and a small width d .

Beam diffraction limits the maximum value for L/d^2 . At the maximum value $L/d^2 = \pi n / (4S^2 \lambda)$ where S is a numerical factor of order unity that expresses how much larger the crystal aperture must be than the beam to avoid unwanted clipping.

The beam waist should be in the center of the crystal to minimize

$$w_0 = \sqrt{\frac{\lambda L}{2\pi n}}$$



Gaussian Beam Optics



A Gaussian beam has a transverse spatial profile is of the form

$$E(r) = E_0 e^{-r^2/w(z)^2}$$

where the Gaussian beam width $w(z)$ evolves from a minimum of w_0 at the waist according to

$$w^2(z) = w_0^2 \left[1 + \left(\frac{z}{z_r} \right)^2 \right]$$

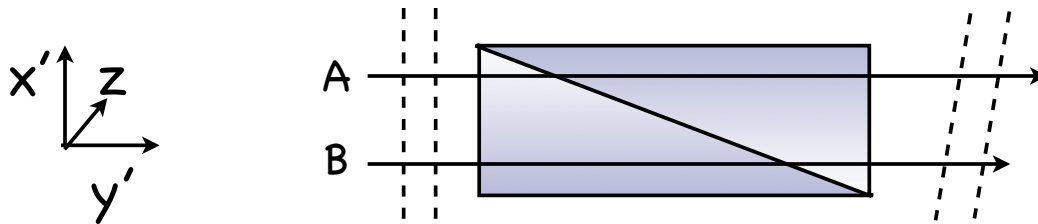
for propagation in the z -direction where

$$z_r = \frac{\pi w_0^2}{\lambda}$$

is called the Rayleigh range

Beam Deflection

A pair of prisms cut from an electro-optic material oriented such that a suitably applied electric field increases the index of refraction in one prism, while decreasing it in the other will produce a differential phase shift between rays going through the top and bottom of the device



The position of a wavefront at the output face is $y' = n(x')L$ giving a wavefront tilt of $\theta = -dy'/dx' = -L \, dn/dx'$

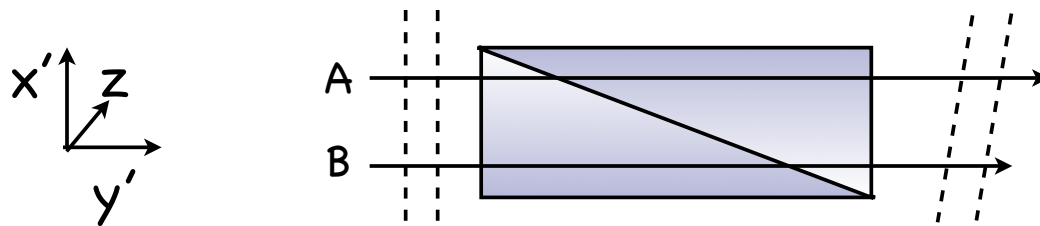
This tilt can be measured in terms of the beam divergence angle $\theta_{\text{beam}} = \lambda / \pi w_0$ to give the number of resolvable spots that the beam deflector can produce when $d = 2w_0$ (confocal configuration) is

$$N = \frac{\pi d L}{2\lambda} \frac{dn}{dx'}$$

Beam Deflection

In a KDP beam deflector where the crystals have their z-axis anti-aligned the indices seen by ray A in the top crystal and for ray B in the bottom crystal are

$$n_A = n_o - \frac{n_o^3}{2} r_{63} E_z \quad n_B = n_o + \frac{n_o^3}{2} r_{63} E_z$$



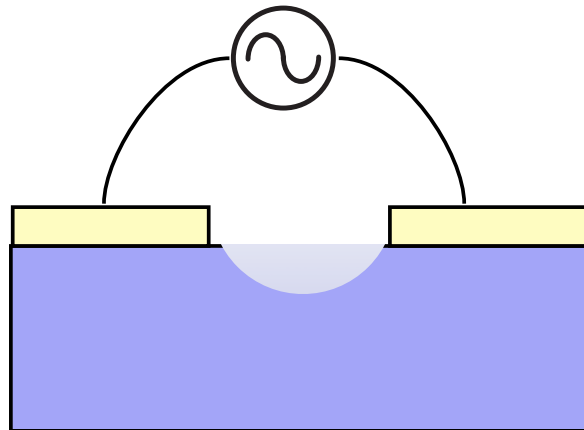
giving

$$N = \frac{\pi L}{2\lambda} n_o^3 r_{63} E_z$$

For $N=1$, $\Gamma=1/2$ $n_o^3 r_{63} E_z L = \lambda/\pi$ so almost the full half-wave voltage is necessary to fully deflect the spot.

Guided Wave Optics

The limitations of diffraction can be overcome by embedding the waveguide in the modulator crystal



Tunable Lasers

Femtosecond Laser

WDM Laser Sources

Benchtop
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HeNe Lasers

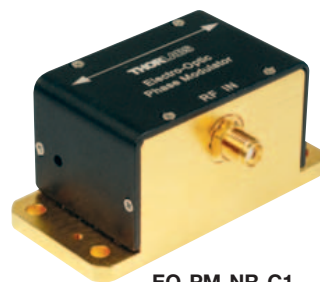
ASE Sources

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**Electro-Optic
Modulators**

Laser & ASE Systems

Electro-Optic Modulators



EO-PM-NR-C1



EO-AM-NR-C2
With EO-GTH5M



EO-HVA

Thorlabs' free-space electro-optic (EO) amplitude and phase lithium niobate modulators combine crystal growth and electro-optic materials. Our standard modulators use undoped lithium niobate. For higher-power operation, we offer MgO-doped lithium niobate. The standard EO modulators are broadband DC-coupled, and a high Q resonant model option is available.

Our standard DC-coupled broadband EO modulators consist of an EO crystal packaged in a housing optimized for maximum RF performance. The RF drive signal is connected directly to the EO crystal via the SMA RF input. An external RF driver supplies the drive voltage for the desired modulation. The crystal may be modulated from DC up to the frequency limits of the external RF driver.

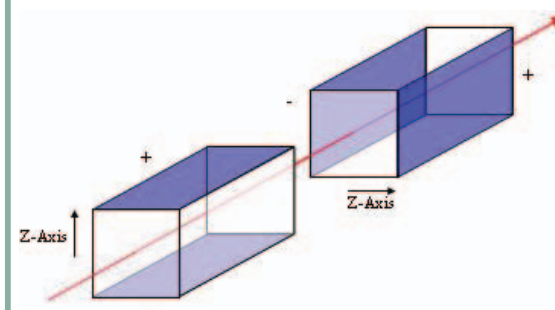
For flexibility, Thorlabs also offers a second modulator option. Resonant frequency modulators simplify the driver requirements for many applications where the modulator is operated at a single frequency. A high Q resonant tank circuit located inside the modulator boosts the low-level RF input voltage from a standard function generator to the high voltage needed to get full depth of modulation. Call our tech support team for details on the specific resonant frequencies available.

EO Amplitude Modulator

The electro-optic amplitude modulator (EO-AM) is a Pockels cell type modulator consisting of two matched lithium niobate crystals packaged in a compact housing with an RF input connector. Applying an electric field to the crystal induces a change in the indices of refraction (both ordinary and extraordinary), giving rise to an electric field-dependent birefringence,

Features

- High Performance in a Compact Package
- Broadband DC Coupled and High Q Resonant Models for Low RF Drive
- Standard Broadband AR and Custom Coatings
- Ø2mm Clear Aperture
- SMA Female Modulation Input Connector
- MgO-Doped Versions for High Power
- DC to 100MHz
- Custom OEM Versions Available



References



- Yariv & Yeh "Optical Waves in Crystals" chapter 8