



$\pi = \frac{c}{d}$ $\sigma_{rms} = \sqrt{\frac{\sum_i \delta_i^2}{N}}$ $I_{disk} = \frac{1}{2}mr^2$

Physics 50 & 70 Laboratory Manual

$f_s \leq \mu_s N$ $\frac{1}{2}(m_b + m_c)$ kx

$\vec{\tau} = \vec{r}$ $v = \frac{\Delta x}{\Delta t} = 2\pi r f$ $\frac{e}{t}$

$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

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Introduction

This laboratory portion of Physics 50 has three primary objectives.

1. Reinforce concepts from lecture
2. Introduce the concept of uncertainty in measurements
3. Develop your ability to communicate technical data through written documents

You will be required to write a lab report for each experiment. Each report must properly address the physics and measurement uncertainty involved in the experiment, and must be written in an appropriate form for a scientific document. Your data and calculations for each lab must be saved for inspection by your instructor (though you won't necessarily include all raw data in your lab reports). This manual contains descriptions of the physics involved in each experiment, details of what is expected in a lab report, and an overview of error analysis sufficient for writing complete lab reports. Please read the manual thoroughly, as it will save you time in lab and when writing up your reports.

Much of the data analysis performed in this class will be done with the aid of “Logger Pro”, a software package from Vernier. The software is installed on the lab computers, but you can also download a copy for use on your personal machine from the following URL <http://tinyurl.com/sjsuLP> or by making a copy of the installer which is saved on the desktop of the lab computers. The installers require the password “property”. There are step-by-step instructions in appendix C for most of the common tasks you will be doing with logger pro.

Lab Report Tutorial

Throughout this laboratory course you will write 12 lab reports (one for each of the 12 labs). These reports will be written from scratch by hand or using a computer. This overview will explain the key components of any lab report, and guide you in using the desired format.

Important: Each lab report your turn in must be in original (i.e. in your own words). Copying phrases from your lab partner, any other student *or the lab manual itself* is plagiarism, and will not be tolerated, either at SJSU or in the real world. Infractions will result in action at the university level, and may also lead to an F in your Physics 50 course.

General Lab Report Guidelines

Your imagined audience for these lab reports is a scientist in a different laboratory, using different equipment. This scientist knows all of the physics concepts you are discussing, but does not know the information about the particular experiment that is found in this lab manual. Imagine that after reading your report, this scientist will attempt to reproduce your results (using different equipment).

Be short and concise. Lab reports that are too wordy, or contain information outside the scope of the required framework, will not be accepted (technical writing in the real world must also be short and concise.)

Generic Lab Report Outline

Title Just use the title of the lab that week, include the Lab number.

Background and Objective Describe the motivation for performing this experiment and state the objective of your measurement. The objective is a clear statement of what is being tested, that can be addressed in your conclusion

Measurement Section What system did you measure?

- Draw a labelled diagram (don't copy from lab manual).
- Explain any terminology on your diagram(s).
- Give values for any fixed components on your diagram(s). Which physical quantities of the system did you measure?
- Refer to the corresponding notation on the diagram.
- Do not explain *how* you made the measurement. Under what conditions did you make the measurement(s)?
- Which parameters were varied? (Refer to diagram notation.) Where are the results of your measurements?
- Refer to the data tables in the next section. If multiple systems were measured, make it clear which table goes with which measurements.

Data Design easy-to-read and easy-to-interpret tables in which to enter your data and other relevant parameters. Always use the appropriate number of significant figures for the entries and include your estimate for the uncertainty. If there are other important parameters, list them in this section.

- Measured values: Label the column appropriately (with correct units!).
- Other experimentally-controlled values with label and units.
- Calculated values with label and units. Also give the equation that you are using near the top of the column; further explanation can go in the next section.
- Attach any graphs in this section (with titles, labeled axes, and correct units).

Analysis Give any equations that you are using (you don't have to re-derive anything that you've already been given.) Make sure that all of the parameters in these equations have

been defined in the earlier sections, or define them here. If you have graphed any data, refer to it here. If you used a curve fit on your data, explain how the parameters in the curve fit can be interpreted using your equations. Compare your results to the expected results; when appropriate, give a percent error (but NOT for each individual data point!).

Conclusion Answer any lab-specific questions not addressed in the Analysis section. Give a one-sentence summary of your results (this will usually address the “lab objective” as described in the lab manual

What does not need to be in the lab report

Save your valuable time by not including:

- How to use a particular piece of equipment. (Remember, your “audience” is a scientist who is using different equipment.)
- How a particular measurement was made. (But you still need to say what was measured!)
- Needless repetition. Any time you are writing phrases that you’ve already written elsewhere in the lab report, stop and think of a more concise way to summarize the information.
- Re-derivations of equations derived or given in the lab manual.
- Problems that you encountered before taking your data.

More time-savers:

- Read the lab manual before class, so you know what you’ll be expected to do.
- If you are editing your report in a word processor and don’t know how to enter equations or symbols, leave a blank space and draw them by hand on your hardcopy. These reports do not have to look beautiful to be accepted.
- Don’t let yourself get bogged down on the same issue for more than 5-10 minutes without asking your instructor for assistance.

How NOT to save time:

- Don't speed through your data-taking; there's no surer way to forget to write something down or make a mistake. Think about the results as you take the data; if they don't make sense, stop and try again. You don't want to have to do it twice.
- Don't copy text from the lab manual or your lab partner.
- Don't ask your instructor to explain every step without first reading the manual and trying to figure it out yourself.

Style

Beyond the content of the lab report, a well written lab report should adhere to a set of style guidelines which include the following: The text of the report should not contain incomplete sentences or other basic grammatical errors. The report should not be written in the first person. Passive voice is preferred, as it puts the focus where it should be - on the experiment, not the experimenter. Your description of the experiment should not contain the instructions for completing the experiment or a detailed list of what you did, but only the relevant facts critical to the procedure. Your lab instructor may have further comments on appropriate style.

The two example lab reports in Appendix A show a poorly written report and a well written report for a fictitious lab. Both use the same set of data and both are about the same length. Compare and contrast these two reports. Which of these two reports more clearly communicates the meaning and results of the lab? How many examples of bad style can you find in the poorly written report?

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Experiment 1

Statistics

Objective

The objective of this laboratory is to see if there is a measurable difference between your reaction time measured by catching a falling ruler with your right hand, versus catching it with your left hand. A secondary objective is to give you experience in treating how errors and uncertainties affect the outcome of an experiment.

Theory

All measured quantities have inherent uncertainty. By expressing the size of the uncertainty in a measurement along with the measured value you can express the range of values consistent with your measurement.

Consider the example of a bus driver who is asked how high his bus is. He might look at the bus and state that it is 3 m high, however if you told him the bridge that he has to pass under has a clearance of 3.1 m he might not be confident his bus will pass under. Why? Because he knows that his statement that his truck is 3 m high was only a guess and has a significant amount of uncertainty. If he were to give the keys to another driver at the end of his shift and were to tell her “the bus is 3 m high” she might assume it is safe to drive under the bridge with a clearance of 3.1 m. In common practice we’d use phrases like “the bus is *about* 3 m high” or “the bus is *around* 3 m high”, but in a scientific context, such general

phrases are not useful - instead we express our uncertainty quantitatively (with numbers). When pressed the bus driver might admit that his estimate that the bus is 3 m high is only a guess, but he is confident it is less than 3.3 m high and that it is greater than 2.7 m high, so he could say that the bus is between 2.7 m and 3.3 m high. Even better would be a statement that he believes the bus is 3.0 m high with an uncertainty of ± 0.3 m (i.e. between 3.0-0.3 m and 3.0+0.3 m). This is the proper way to report the result of a measurement in scientific context.

In the previous example the bus driver didn't actually *measure* the height of the bus. If he had parked the bus in front of a tile building with tiles that he knew to be 0.3 m square he might have been able to count the rows of tiles from the ground to the top of the bus to determine the height of the bus. If he sees the bus is between 9 and 10 tiles high he can state the height of the bus is 9.5 ± 0.5 tiles or 2.85 ± 0.15 m.



Figure 1.1: The bus is between 9 and 10 tile high

Note that the uncertainty has units, and should be given in the same units as the measurement. He wouldn't say the bus is $9.5 \text{ tiles} \pm 0.15 \text{ m}$ because that is confusing to mix the units like this. Even worse would be to say the bus is 9.5 ± 0.5 since now the units are ambiguous (is it 9.5 tiles high or 9.5 m high? Is the uncertainty 0.5 tiles or 0.5 m?) If he had used a tape measure, he would have more confidence in his knowledge of the bus height. Perhaps a measurement with a tape measure reveals the bus to be a height of 2.905 m, however, when he measures the bus again the following day he measures 2.901 m. Even though he is able to read the tape measure accurately to the nearest mm, he is measuring a value that is 4 mm different than the day before. The difference may be due to a change in the tire pressure, a difference in the height of the ground where the bus wheels are compared to where the tape measure is placed, or even changes in the length of the tape measure due to temperature

changes. In any event, he can estimate the height of the bus more accurately than when he was guessing, but cannot eliminate the uncertainty. In this case he could report the height to be $2.903 \text{ m} \pm 0.002 \text{ m}$. While he could pursue the source of uncertainty further, it likely is not necessary, since he can now feel confident driving under the bridge that is 3.1 m high.

The uncertainty in a measurement can be thought of as a random error (and is often called the measurement error). Statistically, random errors have an average value of zero, so if we measure a quantity many times and average our results, the effect of the random errors is reduced and the uncertainty associated with the measurement is improved. When a quantity x is measured many times its average value \bar{x} is given by a statistical property of the data called the *mean*, which is simply the average value. Specifically, if there are N measurements for the quantity x , and we label them x_1, x_2, \dots, x_N the mean is:

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}.$$

The spread of the data around this value represents the precision of the measurement and is given by a different statistical property called the *standard deviation*, often denoted by the greek letter σ (sigma):

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

The uncertainty in a single measurement, δx (δ is the lower case greek letter “delta”) and the standard deviation of many measurements σ both represent the same thing – the precision of the measurement (A physicist would be more likely to refer to δx since that is a property of the measurement while a mathematician might refer to σ which is a result of the uncertainty in the measurement that appears in the statistics of the data). In this experiment the distinction between δ and σ is not particularly important. If a measurement is repeated many times, the values can be averaged to give a more precise result, such that the uncertainty of average value is found using Equation B.3 and is $\delta \bar{x} = \sigma / \sqrt{N}$.

To understand these properties it is often useful to construct a special type of graph called a *histogram* that plots the number of times a value was measured to be within a certain range as a function of the value. For example the histogram below shows the distribution of ages for students in a class. For a normal distribution the mean is typically near the location of the peak in the histogram, and the standard deviation describes the width of the peak (such that 68% of the measured values fall within the peak, i.e. within one standard deviation of the mean). Thus if a measurement is repeated many times the uncertainty in a single measurement and the standard deviation of the collection of measured values become

equal. The uncertainty in a measured value is *not* determined by how close the measurement is to some expected value.

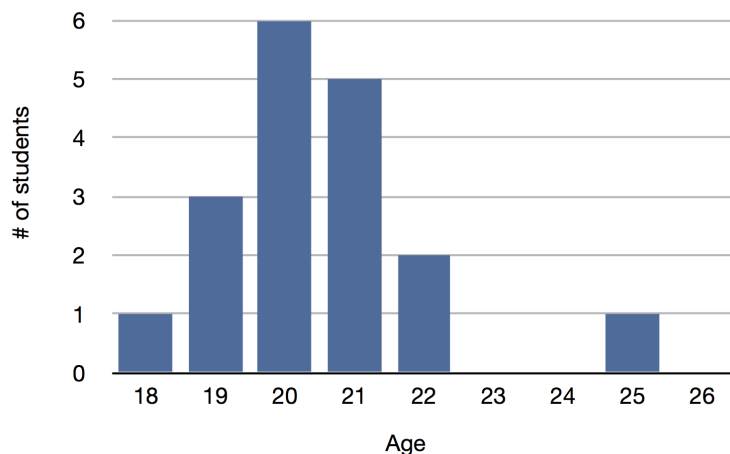


Figure 1.2: Histogram showing the distribution of ages for students in a class

Often we have to calculate a quantity from a measured value (for example calculate the area of a room from measurements of its length and width). When calculating a quantity from a measured value the uncertainty in the measured value causes uncertainty in the calculated quantity. To determine *how* the uncertainty in a measured quantity affects the uncertainty in a calculated quantity we use calculus. If we measure a quantity x and wish to use it to compute a value $f(x)$ we can use the fact that the derivative $\frac{df(x)}{dx}$ gives the relationship between a change δx , and the corresponding change δf , according to $\delta f = \frac{df(x)}{dx} \delta x$, so letting δx be the uncertainty in the measured quantity x , δf is the uncertainty in the calculated quantity $f(x)$. Appendix B described how the uncertainty in a value calculated from *several* measured quantities is determined.

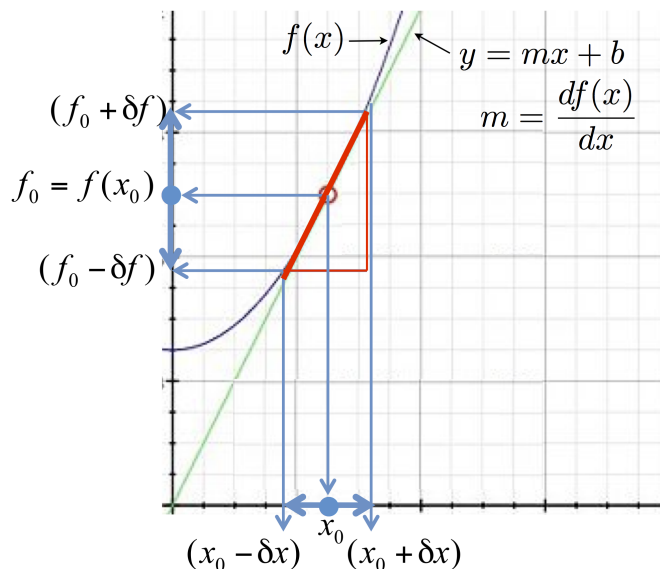


Figure 1.3: Graphical representation of how uncertainty in x affects uncertainty in $f(x)$

Frequently we wish to measure the same quantity in two different ways to validate our measurements, or we wish to compare our measured values to the measurements that others have made. Because measurements have random errors we don't expect multiple measurements of the same quantity to give the exact same results, however, we can use some basic statistical analysis to say how well they match. One common way to do that is to describe how far apart two values are in units of the total measurement uncertainty, for example if you measure the height of Alice to be $h_{\text{alice}} = 154 \pm 2$ cm (that is a measured value of 154 cm with an uncertainty of 2 cm) and you wish to ask if she is the same height as Bob who was measured to be 157 ± 2 cm you could say that their heights differ by 3 cm with a total measurement uncertainty of 4 cm (There is 2 cm of uncertainty in Alice's height and 2 cm uncertainty in Bob's height), which allows us to say that the difference in their heights was measured to be $0.75 \times$ the total measurement uncertainty. Generally if two values differ by less than $1 \times$ the total measurement uncertainty we say they "agree" with each other. In this case the measured height of Alice and Bob agree (i.e. they are the same height), given our measurement uncertainty – Alice could be anywhere from 152 cm to 156 cm, and Bob could be from 155 cm to 159 cm, so the measurement doesn't exclude the possibility that they are the same height (for example they could both be 155.5 cm)

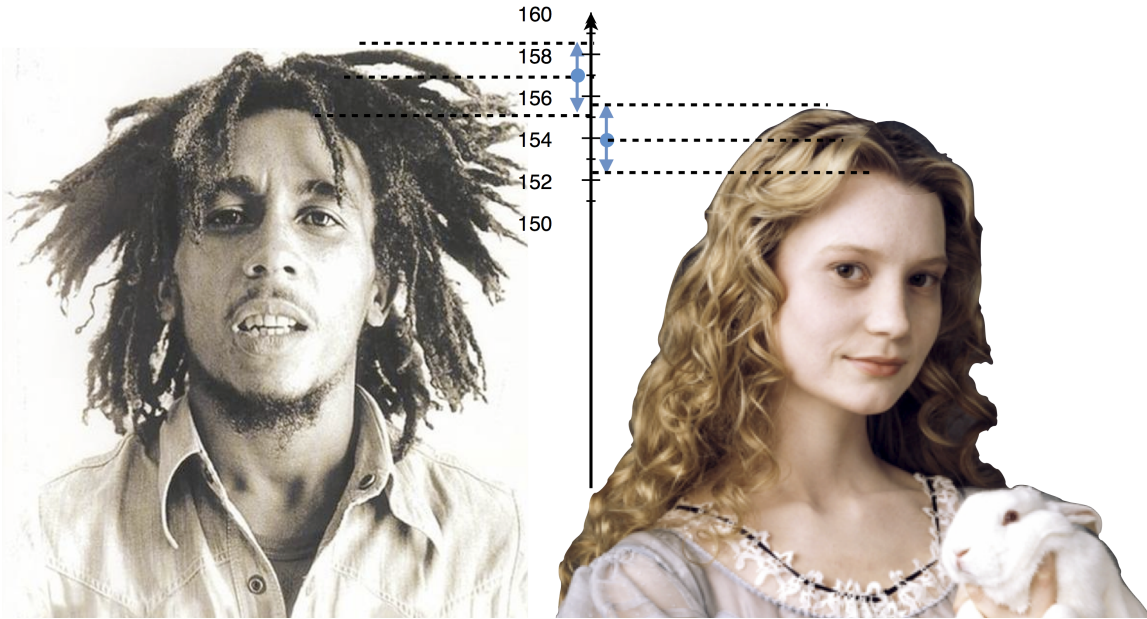
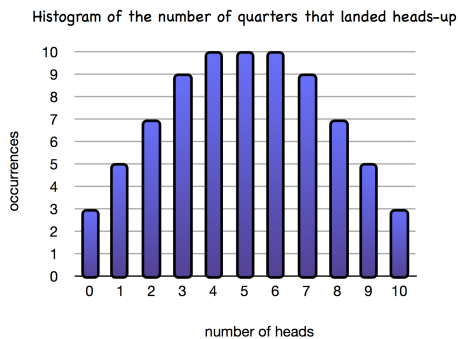


Figure 1.4: The measured heights of Alice and Bob have ranges that overlap

Practice

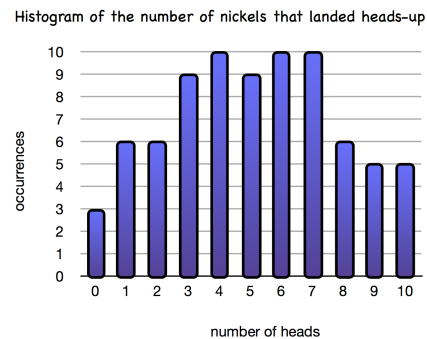
Please complete the following questions before beginning the lab.

1. You drop 10 quarters and 10 nickels onto the floor and count the number of each that land heads up. You repeat this process many times and come up with the following results



$$\bar{x} = 5.0$$

$$\sigma = 2.6$$



$$\bar{x} = 5.1$$

$$\sigma = 2.7$$

- (a) Based on these histograms, how many times did 3 quarters land heads up?
- (b) Based on these measurements do nickels and quarters have the same probability of landing heads up?
2. Consider the expression for the height of an object as a function of time

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$

when moving with constant acceleration, where y_0 is the initial position, v_{0y} is the initial velocity component in the y-direction, and a_y is the acceleration in the y-direction.

- (a) Derive an expression for the time it takes an object to fall to a height $-y$ when released from $y_0 = 0$ while at rest with an acceleration of $-g$.

$$t =$$

- (b) From our expression in part a, determine the derivative of $t(y)$ with respect to y

$$\frac{dt}{dy} =$$

- (c) Using your expression from part b, what is the uncertainty in time δt that is present when an object is measured to fall a distance y with an uncertainty of δy

$$\delta t =$$

Procedure

Measure your reaction time by having your lab partner drop a ruler while you try to catch the ruler as soon as it is dropped. The distance the ruler has fallen will be recorded for several attempts and used to calculate your reaction time. Repeat at least 20 times with each hand.

Analysis

1. Think about the uncertainty in your measurement of distance the ruler has fallen - after catching it are you measuring to the top of your fingers, the bottom? How well do you know the original position of the ruler?. Determine the minimum significant distance that your measurement can resolve (i.e. 1 mm? 5 mm? 1 cm? 2 cm? etc...) and round all values to the nearest measurement unit. Explain quantitatively the factors that affect this uncertainty, such as the uncertainty in the initial position of the ruler, the uncertainty in determining the position at which the ruler was caught, the uncertainty in making measurements with the ruler, etc.
2. Enter your data into a spreadsheet (such as *Excel*, *Numbers*, or *Google Docs*). Find the average and standard deviation for the data for each hand (for example by entering into two empty cells “=AVERAGE(B1:B20)” and “=STDEVP(B1:B20)” to get the average and standard deviation of the data in the first 20 rows of column B). Your graphing calculator may also have the capability to calculate such statistics, or you can always calculate the values by hand from the equations in the *Theory* section. If the average falling distance is $\bar{y} = 15.4$ cm, and the standard deviation is $\delta y = 4.6$ cm, then you should state your result as $y = 15.4 \pm 4.6$ cm. Include only significant figures in your result. if you don't know what significant figures mean, ask. Make sure to include the correct units on your result and everywhere your measured value appears in your lab report.
3. Create separate histograms for the data of the distance the ruler fell for each hand.
 - The x-axis for each histogram should be 5 or 6 ranges of distance the ruler fell, for example if the distances you measured range from 7-27 cm, you might choose ranges of 5-10 cm, 10-15 cm, 15-20 cm, 20-25 cm, 25-30 cm, but don't use the same ranges as this example - choose your own to fully cover the values you measured.
 - The y-axis of your histogram should be the number of data points that you recorded that were within each range (or “bin”)

Label the axes of your histogram.

4. Draw a *vertical* line on your histogram representing the mean value of the distance the ruler fell, as well as dashed vertical lines 1 standard deviation above and below this. The range between the dashed lines represents your measured range. If you do this correctly the number of trials falling within this range should be close to 68% of the total number of trials.

5. Calculate your (mean) reaction *time* (for each hand) based on the distance the ruler fell along with the uncertainty in this value (from the standard deviation). You can use the formulas you derived in part 2 of the prelab “Practice” section. Express your reaction time in the form $t = \bar{t} \pm \delta t$. Note the reaction time should be in seconds, and must be *calculated* from the average distance the ruler fell. Does this value make sense? If the value you got seems to big or too small, check you calculations and make sure you didn’t use inconsistent units in the values you plugged into your formula.
6. Which hand had a faster reaction time. Is the difference significant in light of the measurement uncertainty? If not you did not measure a significant difference int he reaction time. Otherwise you did. State this result in the conclusion of your lab report.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include

1. An appropriate title
2. The objective of the lab
3. A description of the experiment being performed with any specific details that influence the quality of your data.
4. The mean and standard deviation for the distance measurements
5. Your calculation of the reaction time (both the mean value and the uncertainty) based on the distance measurements.
6. A summary of your results giving your average value for your reaction time along with an uncertainty
7. A conclusion that addresses your stated objective, stating your measured reaction time and answering the question “is your measured reaction time the same for both hands”.

Experiment 2

Measurement and Precision

Objective

To determine the value of π from the ratio of the circumference to the diameter of a circle while properly accounting for uncertainty.

Theory

It has been known since ancient times that the ratio of the circumference of a circle to its diameter is equal to the transcendental number represented by π . That is,

$$\pi = \frac{c}{d}$$

The instruments you use to measure this have a limit to their precision. You will have to estimate to the nearest mark on the scale you use. As a general rule your uncertainty in a measurement can never be less than $\pm 1/2$ of the smallest ruled increment on your measurement tool, but *can be* (and often is) higher if there are larger sources of uncertainty than those inherent in the measurement tool. For example if you have a watch that lacks a second hand, then any measurements made with the watch will have to be rounded to the nearest minute, and will have an uncertainty of ± 30 s, although some things that you measure with the watch might have a *greater* uncertainty, for example the amount of daylight would be hard to measure to the nearest minute because the time the sun rises and sets is

difficult to determine so precisely. Can you think of another example of a measurement where the uncertainty is greater than the precision of the measurement tool?

One method to estimate the precision of a measurement is to repeat the measurement several times. When you measure a value several times and get slightly different results, you express your measured value as $x_0 \pm \delta x$ where x_0 is the average value from your measurements and δx is called the uncertainty and is half the deviation from the largest to smallest measured values.

Note that the number of significant figures you include in your results is limited by the number of significant figures in the least precise measurement in your data. Suppose you measured the circumference to be $c = 508$ mm (which has 3 significant figures) and you measured the diameter to be $d = 148$ mm (which also has 3 significant figures). According to your calculator, the ratio c/d is equal to 3.432432432. This has 10 significant figures! You must round off this answer to three significant figures and write $c/d=3.43$.

Numbers containing too many significant figures are wrong because they imply a precision that is not justified by your measurements! For the same reason quantities calculated from measured values that do not include the uncertainty are wrong. To reiterate, your final result should never have more significant figures than your least-precise measurement, and should always be written with the associated uncertainty.

While it is important to note the uncertainty in measured quantities, (such as the circumference and diameter of a circle), it is also necessary to use those values to determine how much uncertainty there is in any quantity calculated from those measurements (such as π). There are two basic premises that govern how the uncertainty in measured quantities affects the uncertainty in any calculated quantity, the first is that for a function $f(a, b, c, \dots)$ the uncertainty in the function f caused by the uncertainty in the measured quantity a is given by

$$\delta f_a = \left(\frac{\partial f}{\partial a} \right) \delta a,$$

with similar expressions for the uncertainty due to b , c , etc. The second premise is that if the measured values a , b , c , etc. are *uncorrelated* the uncertainty in f from each adds *in quadrature* as follows

$$(\delta f)^2 = (\delta f_a)^2 + (\delta f_b)^2 + (\delta f_c)^2 + \dots$$

while correlated uncertainties add directly (i.e. $\delta f = \delta f_a + \delta f_b + \delta f_c + \dots$).

Measurements are uncorrelated when they are completely independent of each other. For example consider a “measurement” of the sum of 2 dice that are rolled. If you had 4 dice and

wished to perform this measurement you could roll the 4 dice twice and add up the values for each die to get 4 uncorrelated measurements, or you could roll the 4 dice once and count each value twice to get 4 *correlated* values.

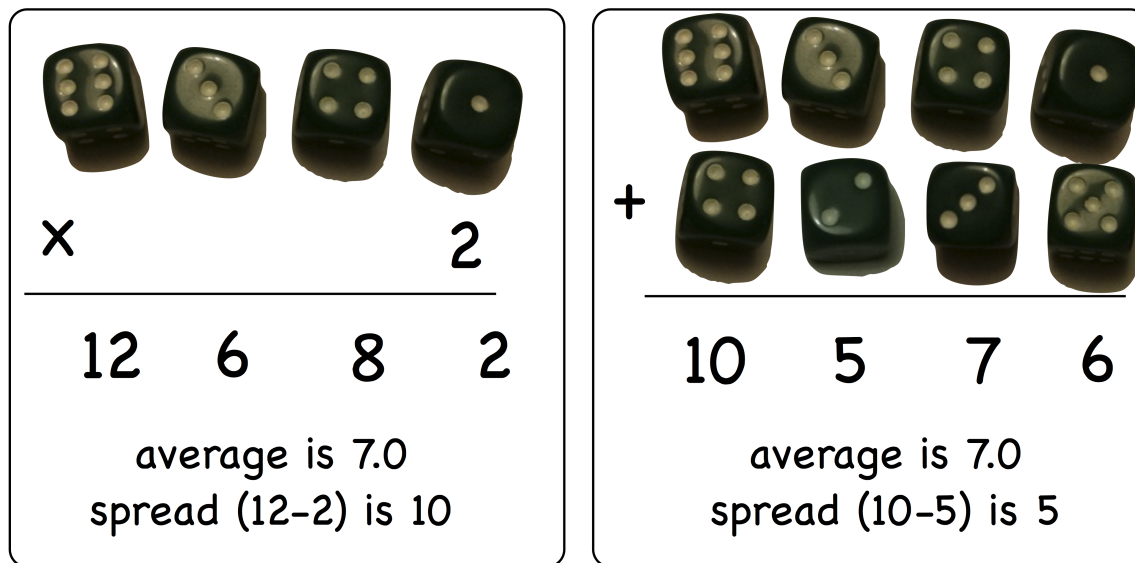


Figure 2.1: Example of correlated and uncorrelated measurements

The randomness of the values of the dice ensure that, on average, the spread in values you get with multiple rolls is less than when you only roll once and count the values multiple times. The same principle applies to measurement uncertainty, as the equations above show that uncorrelated uncertainties produce less spread in a measurement than correlated uncertainties.

When the above two premises are applied to functions that involve the sum or difference of two or more quantities, such as $f(a, b, \dots) = a + b + \dots$ or $f(a, b, \dots) = a - b - \dots$ the resulting uncertainty in f due to each term is found from

$$\begin{aligned}\delta f_a &= \frac{df}{da} \delta a \\ \delta f_b &= \frac{df}{db} \delta b\end{aligned}$$

where for $f(a, b, \dots) = a \pm b + \dots$ and $\frac{df}{da} = \frac{df}{db} = \pm 1$ with the sign depending on if b is being added or subtracted. This gives $\delta f_a = \delta a$ and $\delta f_b = \pm \delta b$. When adding these uncorrelated uncertainties using $(\delta f)^2 = (\delta f_a)^2 + (\delta f_b)^2 + \dots$ we get the general expression

$$(\delta f)^2 = (\delta a)^2 + (\delta b)^2 + \dots \quad (2.1)$$

Sum and difference formula for uncertainty

If the function $g(a, b, \dots) = a \times b \times \dots$ or $g(a, b, \dots) = a/b \dots$ is the product or quotient of two or more quantities the resulting uncertainty in g is found from

$$\begin{aligned} \delta g_a &= \frac{dg}{da} \delta a \\ \delta g_b &= \frac{dg}{db} \delta b \end{aligned}$$

where for $g(a, b) = ab$ and $\frac{dg}{da} = b$, $\frac{dg}{db} = a$. This gives $\delta g_a = b\delta a$ and $\delta g_b = a\delta b$. When adding these uncorrelated uncertainties using $(\delta g)^2 = (\delta g_a)^2 + (\delta g_b)^2$ we get

$$(\delta g)^2 = (b\delta a)^2 + (a\delta b)^2$$

dividing on the left by g^2 and on the right by $(ab)^2$ (which we can do since $g = ab$ and generalizing for more factors gives the general expression

$$\left(\frac{\delta g}{g}\right)^2 = \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \dots \quad (2.2)$$

Product and quotient formula for uncertainty

If formula 2.1 is applied to the N measurements of the same quantity a , each with identical uncertainty δa , the uncertainty in the average of those N results, \bar{a} , is reduced according to

$$\delta \bar{a} = \frac{\delta a}{\sqrt{N}} \quad (2.3)$$

Formula for uncertainty in an average

Note that the above expressions do not give the uncertainty δf directly, but rather are written in a way that is easier to remember but requires that you solve the expressions for δf to determine the uncertainty that you will quote in the calculated quantity. These rules can be applied in turn on more complicated functions. For example the uncertainty in a function that involves both the sum and product of measured quantities $h(a, b, c) = a \times b + c$ can be determined by first finding the uncertainty in the individual components, i.e. find $\delta g(a, b)$ for $g(a, b) = a \times b$ using formula (2) and then rewrite h as $h(g, c) = g + c$ and apply formula (1).

Practice

Consider measurements of the width w and length l of a piece of paper used to calculate the area of the paper using $A = w \times l$.

1. If the length of the paper is measured twice using a ruler marked with 1mm increments, and the measurements give values of $l=298$ mm and $l=294$ mm, what value should be used for l ? for δl ? Express your answer in the form $l \pm \delta l$
2. If the width of the paper is also measured twice using a ruler marked with 1mm increments to give a value of $w=210$ mm both times, what value should be used for w ? for δw ? Express your answer in the form $w \pm \delta w$
3. What is the area of the paper? What is the uncertainty in the area? Express your answer in the form $A \pm \delta A$.
4. What is the value for the perimeter of the paper ($p = 2l + 2w$)? What is the uncertainty in this value? Express your answer in the form $p \pm \delta p$.

Procedure

Work in groups of two. Each student will measure the diameter and the circumference of each provided plexiglas disk several times. The group results will be recorded in a table similar to the one shown on the next page. Draw your own data table. Make sure it has a descriptive title and contains units in the headers of the appropriate columns or after each data value.

Analysis

1. For each measurement of circumference (c) and diameter (d), determine the value of π by taking the ratio c/d . Since both c and d have uncertainty in their measurements, be careful not to quote more than the appropriate number of significant figures in your value of π and to include the uncertainty in your result. To calculate the uncertainty in c/d given your measured uncertainties in c and d you can use the rules given in appendix B in the back of the lab manual
2. Determine your measured value of π by averaging the results from each disk. You will have to use the formulas given in appendix B to determine the uncertainty in this value given the calculated uncertainty in π measured for each disk.
3. Determine the percent difference between the average value of π you have calculated and the accepted value of $\pi=3.14159\dots$

$$\left| \frac{\pi_{\text{measured}} - \pi_{\text{accepted}}}{\pi_{\text{accepted}}} \right| \times 100\%$$

Your accepted value of π should only contain the same number of significant figures as your calculated value.

4. Consider the possibility of a systematic error caused by the width of the string wrapped around the circumference of the circle. Since the finite width causes the (center of the) string to follow a circle of greater diameter than the outer edge of the plexiglas circle, the measured circumference is likely for a larger circle than the measured diameter. Would this systematic error cause the measured value of π to be too big or too small? How can this systematic error be eliminated? (*hint: how much greater is the distance from the center of the circle to the center of the string than to the edge of the circle?*). Explain in your report the cause of this systematic error, and draw a diagram to support your description. Correct the values for the circle diameters to account for this error and recompute π .

Example Data and Results Table

Disk #	c (mm)	d (mm)	c/d
1	393 ± 8	124 ± 2	3.17 ± 0.08
2	625 ± 2	200 ± 1	3.13 ± 0.02
3	943 ± 3	290 ± 1	3.25 ± 0.02
4	1310 ± 10	410 ± 2	3.20 ± 0.03

In the first row in the example data table the calculated value of π is 3.17 ± 0.08 meaning, within the experimental uncertainty a value as low as 3.09 or as high as 3.25 would be consistent with our measurement, therefore this example measurement agrees with the accepted value of π . If the experimental uncertainty were not included however, the values would not agree since $3.17 \neq 3.14$. The calculated values for π in row 2 also agrees with theory, but those in row 3 and 4 do not.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include

1. An appropriate title
2. The objective of the lab
3. The general procedure for the experiment
4. Your groups data in an appropriate format
5. An example calculation of your uncertainty in π for one of the disks
6. A description of any systematic errors present in the experiment and a discussion of how they were dealt with.
7. A summary of your results giving your average value for π along with an uncertainty with and without any systematic errors corrected for.
8. A conclusion that states if you met the objective of the lab (i.e. did your measured value agree with the accepted value for π ?)

Experiment 3

Free Fall Analysis

Objective

The objective is to determine gravitational acceleration by studying the position of a falling object as a function of time.

Theory

An object that is only acted on by gravity is said to be in free-fall. Near the surface of the Earth the magnitude of acceleration due to gravity is given by g , and is nearly constant with a measured value of 9.81 ± 0.02 (where the uncertainty comes from differences in elevation at various points on the Earth's surface as well as variations in the centripetal acceleration for objects at different latitudes.) The acceleration is related to the velocity of an object and the velocity is related to the displacement of an object through the definitions:

$$\vec{a}(t) \equiv \frac{d\vec{v}(t)}{dt}$$

and

$$\vec{v}(t) \equiv \frac{d\vec{x}(t)}{dt}$$

so that an object moving in 1-dimension with constant acceleration has a displacement of the form

$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2$$

Since we have an expectation for the functional form of the measurements of the displacement versus time, we can use a process called *curve fitting* to determine the parameters of the function relating the measured quantities. In this case the initial position y_0 , initial velocity v_0 and the acceleration a are the parameters relating displacement to time. Knowing these parameters would allow one to calculate the displacement $y_{calc}(t)$ at any time and plot a continuous curve. Because of uncertainty in measurements, however, the measured data points would not be expected to lie exactly on this curve (although the range they include - denoted by “error bars” should overlap the plot for 68% of the data).

When given a set of data points one can seek the parameters of the function that provide the best matching of the calculated values to the measured values. Starting with some initial guess as to the parameters of the function, one can (typically using a computer) vary these parameters a small amount and calculate some measurement of how well the function matches the data for each value of the parameters, choosing the parameters that give the best match. This process is called curve fitting, and the method we will use to determine how well a given calculated curve matches the experimental data is called the “least squares” method. With this method, the error, δ_i , from the calculated value to the measured values is computed for each data point according to

$$\delta_i = y_i - y_{calc}(t_i)$$

where y_i is the measured value and $y_{calc}(t_i)$ is the calculated value at t_i . The “root mean square error” is a measure of the average magnitude of the error, and for N data points is given by

$$\sigma_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^N \delta_i^2}$$

The root mean square error can be used as a measure of how well the curve fits the data. A lower value for σ_{rms} means the curve is a better fit, and the measurement more *precise*.

The quality of a measurement can be expressed by its accuracy and its precision. *Accuracy* is how close the measured value is to an accepted value and is often quantified by a percent error

$$\left| \frac{a_{measured} - a_{accepted}}{a_{accepted}} \right| \times 100\%,$$

EXPERIMENT 3. FREE FALL ANALYSIS

while *precision* is how repeatable the measurement is, and is measured by the size of σ_{rms} (which has the same units as the measured quantity.)

In choosing the parameters for a curve fit, we generally seek to create a curve that makes the measured points as precise as possible (i.e. we try to minimize σ_{rms}). There are various algorithms used to determine the parameters of the best fit line, but one of the most general is a simple form of “trial and error”. While it would be possible, but tedious, to choose a value for each parameter, manually calculate σ_{rms} for the resulting curve, and then repeat with different values until you found the values that gave the most precise fit, this process can be done quickly by computers, and many software tools exist that will easily do curve fitting.

Practice

Consider the 4 sets of data shown below for $v(t)$ the velocity of an object in freefall with the corresponding linear curve fits. The slope of the best fit line gives the acceleration, and for comparison a dashed line with a slope of $-g$ is shown.

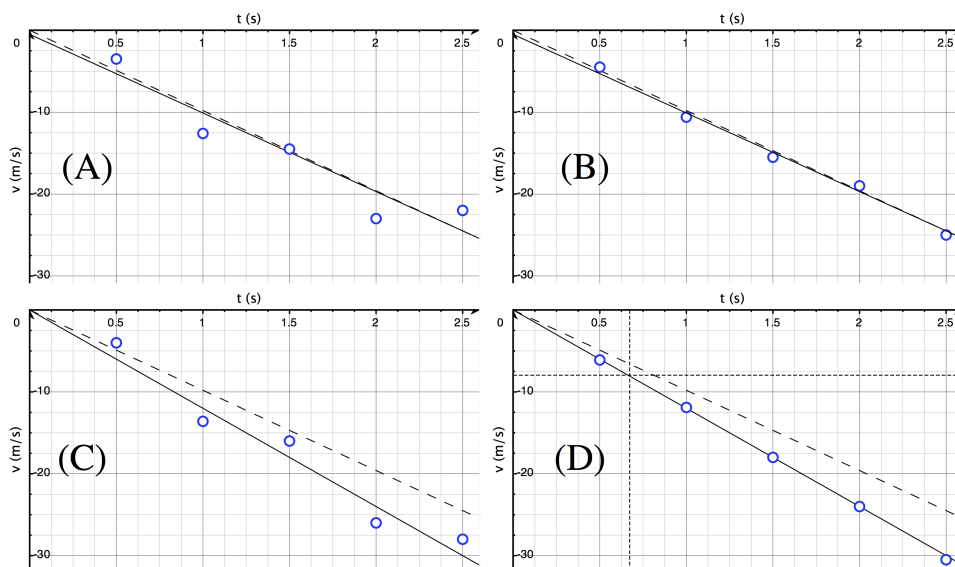


Figure 3.1: Example data

1. Which data set, or sets has the best accuracy?
2. Which data set or sets has the best precision?
3. Consider just two data points such that $v_1 = -12$ m/s at $t_1 = 1$ s, and $v_2 = -19$ m/s at $t_2 = 2$ s. If the data is fit by a function $v(t) = -(10 \text{ m/s}^2)t$ what is σ_{rms} for this data? What are the units?

Procedure

Your lab instructor will demonstrate how to record the motion of a falling object using the tape timer.

1. Cut a 1.5-meter length of recording tape. Make a small loop in one end and seal it with a piece of masking tape. Hang a load of between 500g and 1kg from the loop (make sure to record the amount of mass that you use). The value you choose should be different than that of your lab partner. Trim the tape so that the free end will pass through the timer before the falling mass hits the floor.
2. Feed the other end of the tape up through the guides in the timer. Make sure the tape runs between the carbon paper and the strike plate. Pull up the free end of the tape until the falling mass is suspended just below the timer. The tape should be vertical and aligned with the guide slots to minimize drag as it is pulled through the timer.
3. With the tape vertical and with the weight stationary, switch the timer on at its 40-Hz pulse rate and release the tape. The timer will stamp spots on the tape giving the position of the falling mass at $\frac{1}{40}$ second intervals.

Analysis

Look at your raw data; you will observe a track of spots on the paper tape. Since the separation between the points gets larger, you can see that the speed of the falling mass is increasing. If you don't see an increasing separation between the first 20 or so dots, your data is defective and you should repeat the experiment. To fit a quadratic curve to the data we will use the software "Logger Pro" that is available on the lab machines. If you prefer to work on your own machine, you may download a copy from <http://tinyurl.com/sjsuLP> using the password "property" to install it.

1. Data: Pick a spot near the start of your data track and label it "point 1". Start where the separation between spots is about 1cm; if less, the distance increments cannot be measured accurately. Label the spots consecutively: 1, 2, 3 Record the uncertainty in the spot positions. Also record the value of the falling mass.
2. In Logger Pro enter your data for time and position into the cells in columns x and y respectively.
3. Logger pro will automatically plot the data. If your plot does not look like figure 3.2 you may need to set the axes to "Autoscale from 0" by double clicking on the axis and adjusting the "scaling" settings in the "Axes Options" tab of the pop-up window.

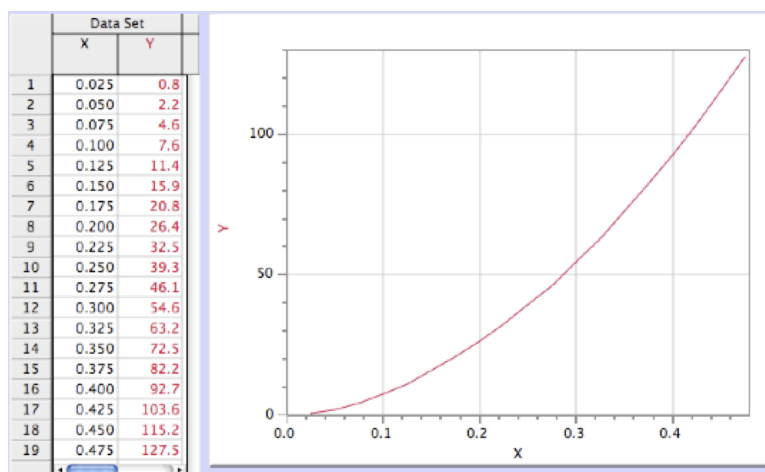


Figure 3.2: Logger Pro data and plot

4. Tell Logger Pro that you want to do a curve fit. From the "Analyze" menu select "Curve Fit..."

5. Select the functional form that you wish to fit. Initially we will assume the mass was in freefall so that the displacement versus time is parabolic. Select “ Ax^2+Bx+C (Quadratic)” as the function that describes this model.
6. Estimate the free parameters. If the curve fit has the form $y = Ax^2 + Bx + C$, and your model for the trajectory of the object has the form $y(t) = y_0 + v_0 t + \frac{1}{2}gt^2$, determine what the values of A , B , C and x are in terms of the quantities y_0 , v_0 , g and t .
7. Try out your expected parameters. Select “Manual” from the options for “Fit Type” in the curve fitting window and type in values for A , B , and C based on the values that you expect y_0 , v_0 and g should be. Record the value of the root mean square error (RMSE) determined by Logger Pro and shown at the bottom of the coefficients window. This describes the distance (in the vertical direction) of a typical data point from the curve with the parameters you entered. Observe the plot of this curve superimposed on your data. Try modifying the parameters by hand to see if you can make the curve fit better. You will be able to see if the curve fits better both qualitatively by looking at the plot with the curve on it, and quantitatively by the size of the RMS error (which will be smaller for a better fit). Give an example of how you reduced the RMS error by changing the estimates for A and/or B . What does this tell you about your initial estimates for the initial position and initial velocity?
8. Let Logger Pro find a better fit. Select “Automatic” from the options for “Fit Type” in the curve fitting window and click the “Try Fit” button. Record the fit parameters A , B , and C as well as their uncertainties and the root mean square error σ_{rse} for this fit.
9. Print the graph of your data with the best fit curve superimposed
10. Record the values and uncertainty of y_0 , v_0 , g corresponding to best fit parameters A , B , and C . Write these on the graph and include them in your lab report.
11. Comparison to measured values: How far is your measured value for the acceleration from the accepted value of g ? Does your measured value agree with the accepted value?
12. How does the uncertainty you determined in step 3a compare to the observed standard deviation σ_{rse} ? If you have estimated the uncertainty properly they should be similar in magnitude. If they are not it suggest an error in your original estimation of the uncertainty in the position measurements.
13. If (and only if) your result (taking uncertainty into consideration) differs from the accepted value of g , discuss (as quantitatively as possible) whether friction or air resistance might account for the difference. You might discuss any difference between

your results and your partners results which were based on a different amount of falling mass as evidence for or against friction or air resistance accounting for any inaccuracy of your result.

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. An appropriate title.
2. The objective (One or two sentences).
3. Graph of displacement versus time including the best fit line.
4. A conclusion that addresses the questions asked in step 3k and 3l of the analysis section.

Experiment 4

Newton's Laws

Objective

To demonstrate and confirm Newton's 3 laws of motion and to test the equivalence principle relating inertial mass to gravitational mass.

Theory

The field of Classical Mechanics is based on Newton's 3 laws of motion, that can be summarized as follows:

1. An object at rest will remain at rest unless acted on by a net force, and an object in motion will continue in motion with constant velocity unless acted on by a net force.
2. The net force on an object is equal to the time rate of change of its momentum. (i.e. $\vec{F} = m\vec{a}$ for an object with constant mass).
3. For every action (Force) there is an equal and opposite reaction.

Newton's first law is really just a specific case of his second law. Newton's second law states that the net force on an object equals its mass times its acceleration. With this relationship

we can say that the mass of an object is a constant of proportionality between the force applied on the object and its acceleration - we will call the mass found this way the “inertial mass”, as opposed to the “gravitational mass” that is found from the objects weight. The *equivalence principle* tells us that these two definitions of mass are equivalent, and results in objects having the same acceleration due to gravity, regardless of their weight. In this lab we will determine the mass of an object by measuring the net force on it and its acceleration. This measured mass will then be compared to the mass found from its weight.

In this experiment, the test object will be suspended by a string from a force sensor and the object will be accelerated in the vertical direction. That means there will be two forces on the object, its weight W and the tension in the string T which is measured by the force sensor. The acceleration of the object will be determined by measuring the angular acceleration of a pulley that the string passes over using a digital rotation sensor. The linear acceleration of the string a as well as the test object that it connects to will be related to the angular acceleration α of the pulley of radius r by $a = \alpha r$. Newton's 2nd law applied to the hanging mass is

$$T - W = ma$$

which can be written in the form

$$T = W + ma$$

Allowing us to fit a line to our data and relate the slope of the line to m and the y-intercept to W .

Newton's third law will be tested by connecting two force sensors together and measuring the force on each independently as various forces are applied by one sensor on the other

Practice

Consider a test object with a weight of 2.00 N, attached to a string (as described in the “Theory” section) with a tension of 2.10 N that passes over an ideal pulley of radius 2.50 cm.

1. What is the magnitude of the net force on the object?

2. If the pulley has an angular acceleration of 19.7 rad/s^2 what is the magnitude of the linear acceleration of the test object?
3. What is the measured inertial mass of the test object?
4. What is the gravitational mass of the test object?



Procedure

First we will test the statement from Newton's first law that an object at rest will stay at rest unless acted on by a net force.

1. Choose a mass between 100g and 500 g and place it on the table so that it is stationary with no net force acting on it.
2. Observe the motion (or lack thereof) of the object.
3. Repeat several times

Now we will investigate Newton's 2nd law




1. Measure the radius of the arc the string will take as it passes over the pulley on the rotation sensor. As in the lab where you calculated π you should add the radius of the string to the radius of the pulley to avoid a systematic error due to the finite string width. Make sure to record an estimate of your uncertainty in this value.
2. Make sure the LabQuest data logger is turned on, the digital rotation sensor and the force sensor are plugged in and the unit is reading an angle and force.

3. If the data logger is not reading a force it may need to be configured to use the force sensor: From the tab, select “Sensor setup...” choose “Force” for whichever channel the sensor is plugged into and then from the submenu select “Dual Range Force 10N” and click “OK”.
4. Determine the weight of the hook on the end of the force sensor. Hold the sensor vertically with the hook pointed down and record the reported force. Flip the sensor upside down so the hook points upwards and again record the measured force (it should be negative). The difference in these measurements is twice the weight of the hook W_{hook} .
5. With nothing connected to the hook of the force sensor, hold it horizontally (with the hook on the side) and observe the measured force on the handheld data logger. If the force is not zero it needs to be zeroed using the following step.
6. Zero your force sensor. While holding the sensor horizontally (with the hook on the side), go to the  tab, select **Sensor**→**Zero**→**Dual Range Force ± 10 N**.
7. Estimate the uncertainty in the force readings from the force sensor by observing the fluctuations in the reading while the sensor is at rest.
8. Place between 200g and 500g of mass onto the end of a string. Connect the other end of the string to the hook on the force sensor and mount the force sensor on a cart on a spring. Pass the string 90° around the pulley so that the side connected to the force sensor is horizontal and the side connected to the mass is vertical.
9. Pull the cart all the way to the end of the track and make sure the hanging mass is stationary and not swinging back-and-forth. Click  on the data logger device to start recording data and release the cart allowing it to move in and out to raise and lower the mass for 10 seconds while the LabQuest data logger collects force and acceleration data.
10. Transfer the collected data onto one of the lab computers for analysis by connecting it via a USB cable and opening “Data Logger Pro” on the computer. The device will be auto-detected and you will be prompted to import the data. If you are successful you can skip step 3k, otherwise see the instructions for manually importing data in the next step.
11. If your data did not import automatically, save your data on the handheld device by selecting “save” from the “file” menu. Then when it is connected to the computer,

EXPERIMENT 4. NEWTON'S LAWS

select “LabQuest Browser” from Logger Pro’s “File” menu and select the file that you just saved.

Finally we will investigate Newton’s 3rd law of motion. This will require borrowing a force sensor from another lab group, but the measurement is quick so you can likely find a way to share resources without introducing delays

1. Plug two force sensors into the back of your LabQuest data logger (and disconnect the rotation sensor if it is still connected) and select . You should see readings for both force sensors. If you do not, power off the data logger and power it back on to get it to recognize both force sensors.
2. Zero your force sensors. With the sensors placed horizontally on the table (with the hook on the side), go to the  tab, select **Sensor**→**Zero**→**Dual Range Force ± 10 N**. Do this procedure for both sensors.
3. Connect the hooks on the end of the sensors together. Work with a lab partner so that you are each holding one of the force sensors.
4. Click  on the data logger device to start recording data and begin to pull *gently* on the force sensor. Try to coordinate with your lab partner so that one of you pulls harder than the other. If your sensor saturated (exceeds the maximum value of 10N) you will see a “plateau” on your graph of force versus time. In this region of the graph accurate data was not recorded. If you have this in your data repeat the experiment using less force.
5. Again, transfer the data to Logger Pro on the computer. If you haven’t yet analyzed the data from the experiment on the Second law, you might save that or complete the analysis on it first so that you do not have problems if you overwrite the data with the newly imported data.

Analysis

Import your data from the experiment on Newton’s second law, or open the file containing this data. The data table in Logger Pro will have columns for (among other things) *total* Force and *angular* acceleration. We will need to create a “calculated column” for the *linear* acceleration. We will then plot force versus acceleration and perform a linear curve fit to determine the weight and the inertial mass of our object

1. Create a column for the linear acceleration by clicking “New calculated column...” from the “Data” menu. Give the new column an appropriate name (in both the “Name:” field and the “Short Name:” field). In the equation panel create an expression for this column of the form $r * \alpha$ where r is the measured radius of the pulley that the string passed over, and α is the column for angular acceleration (which by default is incorrectly called “Acceleration”)
2. When Logger Pro imported your data it created some graphs of the measured functions versus time. Delete all but one of these. The remaining one we will change to plot what we are interested in: force versus linear acceleration.
 - (a) Click on the label of the Y-axis on the graph you wish to modify. From the pop-up menu select the “Force” column.
 - (b) Click on the label of the X-axis on the graph you wish to modify. From the pop-up menu select the “linear acceleration” column.
 - (c) Right click on each of the axes and select “Autoscale” to have Logger Pro set the scale so that your data fills the graph.
 - (d) Perform a linear fit to the data by choosing “Linear Fit” from the “Analyze” menu. The parameters of the fit will be shown on the graph.
 - (e) If your graph has a negative slope it simply means the rotation sensor and force sensor were oriented with their “positive” directions opposing. If this is the case you can fix this by explicitly including a negative sign in your equation for the linear acceleration column (double click the column header to edit the equation).
3. The y-intercept of the fit gives the force applied when the object is not accelerating, which is the weight of the hanging object. Record this value as the hanging weight $W_{hanging}$, and calculate the total gravitational mass from this weight and the weight of the hook calculated in step 3d as $m_g = (W_{hanging} + W_{hook})/g$.
4. Note the slope of the linear fit, this is the inertial mass of the hanging mass and the hook (both of which were being accelerated). Record this value as the inertial mass m_i .
5. Uncertainty: If the box showing the parameters of the fit does not show the uncertainty in the parameters, double click it and select the “show uncertainty” checkbox. Record the uncertainty in the y-intercept parameters as $\delta W_{hanging}$.
6. Find the uncertainty in the gravitational mass of the object. First use the sum/difference rule for uncertainty to find the uncertainty in the total weight $W = W_{hanging} + W_{hook}$

EXPERIMENT 4. NEWTON'S LAWS

from the uncertainty in the hanging weight $\delta W_{\text{hanging}}$ and the uncertainty in the weight of the hook δW_{hook} . From your uncertainty in total weight, δW and the relationship $m_g = W/g$ use the product/quotient rule to get the uncertainty in the gravitational mass δm_g assuming the uncertainty in g is zero.

7. Find the uncertainty in the inertial mass of the object. The uncertainty in the slope δm_i (and therefor in the inertial mass) will be dominated by the uncertainty in r with a magnitude of $\delta m_i = m_i \frac{\delta r}{r}$.
8. Did your inertial mass and gravitational mass agree within their uncertainty?

You will now analyze the data from the experiment on Newton's third law. Open the file containing the data for this experiment in Logger Pro, or import the data from the LabQuest data logger if you haven't already done so.

9. Because the force sensors were oriented anti-parallel to each other, they each have a different direction for what was recorded as a positive force. To fix this we will invert the sign of the data from force sensor 2 and call that value F'_2 . Create a new calculated column with an appropriate name and enter an expression of the form $-F_2$
10. Create a column for the force difference by choosing "New calculated column..." from the "Data" menu. Give the new column an appropriate name (in both the "Name:" field and the "Short Name:" field) and enter an equation of the form $\Delta F = F_1 - F_2$ to get the difference in forces measured by each force sensor.
11. Create a graph of force versus time showing the force from each sensor F_1 , F'_2 and the difference ΔF .
12. Find the mean and standard deviation of the force difference by selecting "Statistics" from the "Analyze" menu.
13. Is the force difference ΔF equal to zero within the uncertainty?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. Your plot of applied force versus acceleration, showing the best fit line.
2. For Newton's third law experiment, your plot of the two forces and their difference versus time

Experiment 5

Vector Addition

Objective

To add vectors graphically, analytically, and experimentally (using the force table).

Theory

Vectors are quantities with magnitude and direction, such as force, displacement and velocity. They can be represented graphically by arrows that have a length proportional to the magnitude of the physical quantity they represent and the same direction. When vectors quantities are added care must be taken not to simply add the magnitudes of the quantities, but to add the vectors taking into account their direction. For example if you drive 4 km north-east and then 3 km north-west you do not end up 7 miles from where you started, but rather 5 miles in a direction mostly north. In this example *displacement* vectors were being added. There are two general methods for adding vectors: adding their components, and adding graphically.

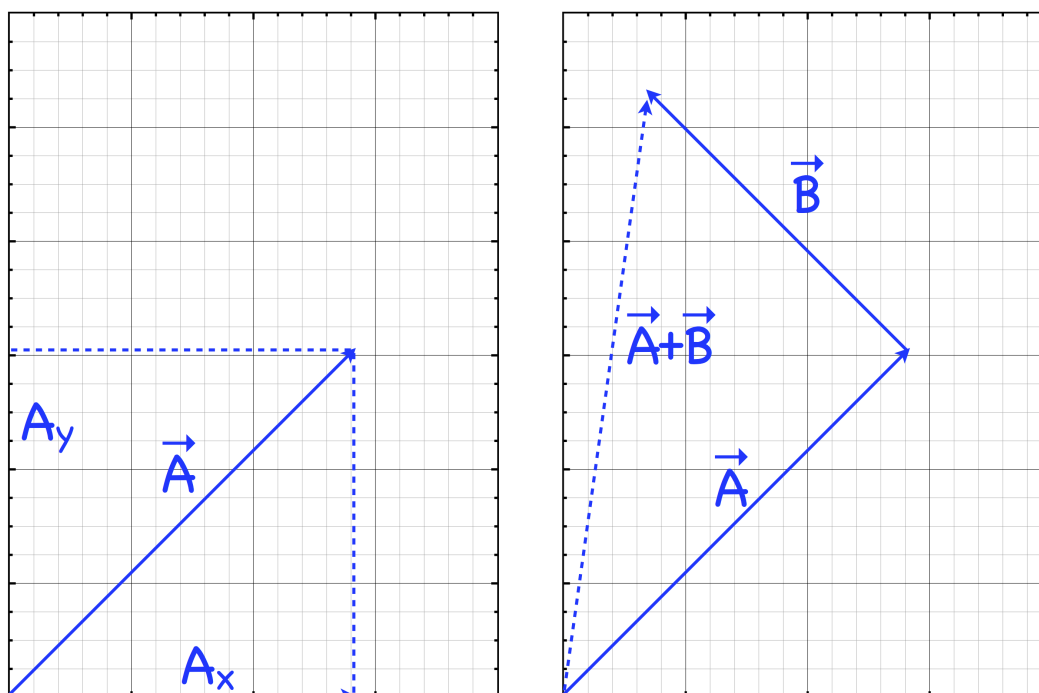


Figure 5.1: (left) Vector components and (right) graphical addition of vectors

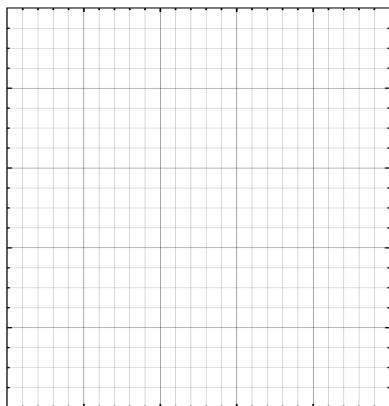
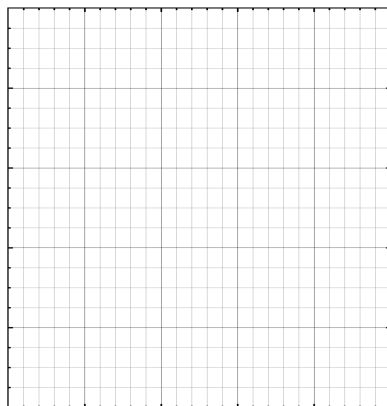
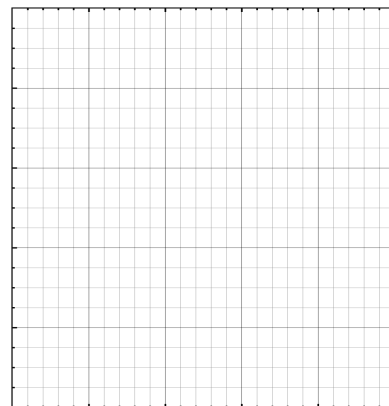
Adding vector components requires that you know the magnitude of the vectors' components. In the preceding example the first vectors being added has components of 2.8 km in both the north and east direction (corresponding to A_x and A_y in the left hand figure), while the second had components of 2.1 km north and -2.1 km east. Thus the sum of these displacements has components of 2.8 km+2.1 km=4.9 km north and 2.8 km-2.1 km=0.7 km east.

To find the sum of two (or more) vectors graphically the vectors being added should be lined up tip-to-tail. The sum is then the vector from the tail of the first to the tip of the last (the dashed vector in the right hand figure).

Practice

Consider two vectors, vector A has a magnitude of 3 cm and at an angle of 35° , while vector B has a magnitude of 2 cm and points at 150° .

1. Use a protractor and ruler to draw each vector, first separately, then tip-to-tail.

 \vec{A}  \vec{B}  $\vec{A} + \vec{B}$ (tip-to-tail)

2. From your scale drawings, estimate the following values:

$$A_x = \underline{\hspace{2cm}} \quad A_y = \underline{\hspace{2cm}} \quad B_x = \underline{\hspace{2cm}} \quad B_y = \underline{\hspace{2cm}}$$

$$\text{For } \vec{A} + \vec{B}, \text{ Magnitude} = \underline{\hspace{2cm}} \quad \text{Angle} = \underline{\hspace{2cm}}$$

Procedure

The force table will be demonstrated by your instructor. Set up 2 forces of unequal magnitude using masses between 0.20 kg and 0.50 kg separated by an angle between 40° and 70° . Find the balancing force, \vec{F}_b , on the force table and the amount you can deviate the angle $\delta\theta$ and the weight δF_b of the balancing force and maintain equilibrium. Record your values in a data table

1. Choose an appropriate scale for your diagram which will allow your diagram to be large yet fit on a single page. Then add your two original vectors using the tip-to-tail method as illustrated below. Since the balancing force \vec{F}_b has an uncertainty in magnitude δF_b and angle $\delta\theta$ you should draw an ellipse around the tip of vector \vec{F}_b indicating the region of uncertainty. Within your experimental uncertainty, is the length of the vector sum of the forces $\vec{F}_1 + \vec{F}_2$ equal to the length of the balancing force vector, \vec{F}_b , and oppositely directed to it? What is the percent error in the magnitude of the \vec{F}_b compared to the magnitude of $\vec{F}_1 + \vec{F}_2$
2. Draw a new scale diagram of the vectors with the same scale factor as in step 3a, but this time draw all of the vectors separately (i.e. don't draw them tip-to-tail) and draw the x- and y-components of each vector. *Measure* the length of the vector components. Since vector \vec{F}_b has uncertainty the components must have a corresponding uncertainty as well. Record the component values in your data table and add up the vector components numerically. Is $F_{1x} + F_{2x} = -F_{bx}$ within the uncertainty δF_{bx} ? Is $F_{1y} + F_{2y} = -F_{by}$ within the uncertainty δF_{by} ?

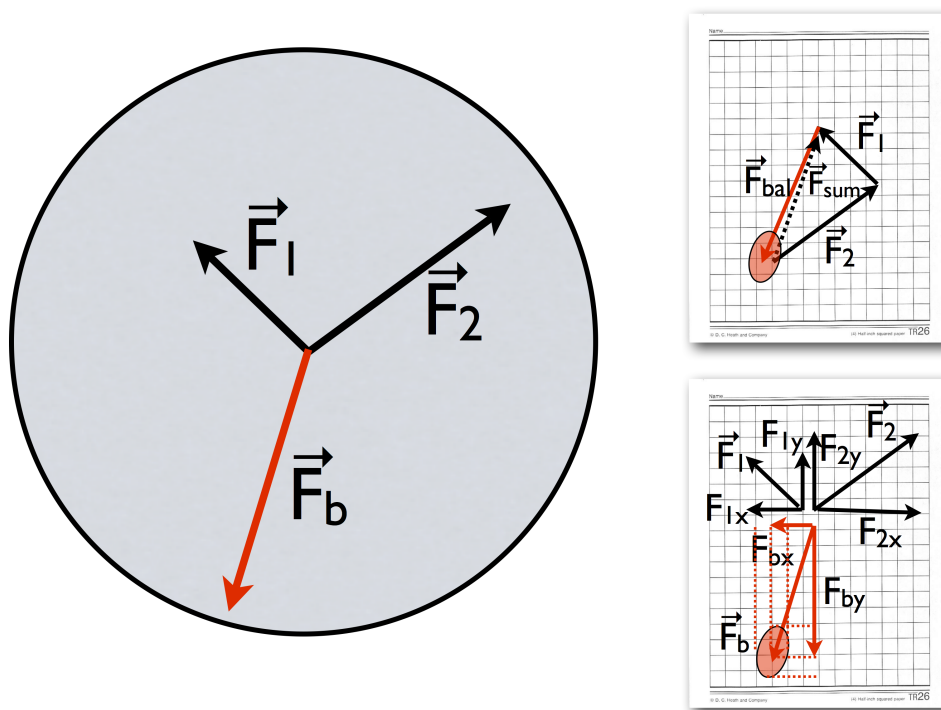


Figure 5.2: Force table setup and corresponding vector diagrams

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. Your two full page vector diagrams
2. The completed data table
3. The percent error you determine in step 3a
4. Were your sum vectors and balancing vectors equal and opposite, within the experimental uncertainty?

Data Table

Name	Angle θ	Mass m	Magnitude of Force mg	Vector Length	x-component F_x	y-component F_y
\vec{F}_1						
\vec{F}_2						
$\vec{F}_1 + \vec{F}_2$		XX	XX			
\vec{F}_b						

Scale factor _____

Experiment 6

Centripetal Force

Objective

The objective of this lab is to verify that centripetal force is equal to $m\frac{v^2}{r}$

Theory

The magnitude of centripetal force on an object moving with speed v in circular motion with radius r is given by

$$F_c = m\frac{v^2}{r}$$

This is frequently written in terms of the *angular velocity*, ω of the object, rather than its linear velocity. From the definition of an angle θ measured in radians,

$$\theta = \frac{s}{r}$$

in terms of the arc length s that it traces out at a distance of r we can take the time derivative of both sides to get the magnitude of the angular velocity $\omega \equiv d\theta/dt$ so that

$$\omega = \frac{d\theta}{dt} = \frac{ds/dt}{r}$$



recognizing ds/dt as the magnitude of linear velocity, we get the relation $\omega = v/r$ which we can use to express the centripetal force as

$$F_c = m\omega^2 r$$

The validity of this relationship will be tested by simultaneously measuring the angular velocity ω and centripetal force F_c on a mass rotating in a circular trajectory with a radius of r .

Procedure

A force sensor is set up to measure the force pulling on a mass that is moving in circular motion. The force sensor is mounted on a rotation sensor so that the force and angular velocity can be simultaneously recorded by the LabQuest Data Logger as the whole system is rotated around the axis of the rotation sensor.

1. With the apparatus completely at rest and with the LabQuest Data Logger in the  tab, tap the Sensors menu, select Zero and then select Dual Range Force Sensor to calibrate the force sensor.
2. Press the  button on the LabQuest data logger to begin recording data, then twist the apparatus a half turn and release it so that it spins back and forth several times while the data logger collects data. If the peak magnitude of your data is less than 0.5 N repeat the measurement using a larger amplitude of rotation (i.e. pull it back further), a tighter (or additional) rubber band or using a larger mass.
3. Rotate the apparatus so that the mass hangs vertically and is stationary. Record the weight of the mass from the force readout on the data logger W and the uncertainty δW .
4. Determine the radial position of the center of mass r_{cm} . The mass on the force sensor can be treated as two pieces. One piece is internal to the force sensor (mostly the attachment rod that the external mass screws into) with a mass of $m_{\text{int}} = 8 \pm 2$ g, while the remainder is the external mass that has been screwed on. *Do not attempt to separate the external mass to weight it or the internal mass - as this can result in small parts to the force sensor falling off and getting lost.* The center of mass of the total is given by

$$r_{\text{cm}} = \frac{r_{\text{int}}m_{\text{int}} + r_{\text{ext}}m_{\text{ext}}}{m_{\text{total}}}$$

You can determine m_{total} from the weight you recorded in step 3c, the position r_{ext} can be measured from the axis of rotation to the geometric center of the disk, but r_{int} and its associated uncertainty must be estimated (the uncertainty is bounded by the size of the force sensor box!).

Analysis

We will plot the measured centripetal force F_c versus the measured values of $m\omega^2 r$, while keeping in mind that there may be two types of errors present in the data: Systematic error which affects all of our calculated values the same (for example due to an error in measuring r_{cm}) and random error due to the intrinsic noise of the sensors we are using.

1. Transfer the collected data onto one of the lab computers for analysis by connecting it via a USB cable and opening “Data Logger Pro” on the computer. The device will be auto-detected and you will be prompted to import the most recent data set. Delete all of the graphs that were automatically generated when importing your data - we won’t need them.
2. Create parameters for the measured quantities r_{cm} , W and for the acceleration due to gravity $g = 9.80 \text{ m/s}^2$ that we will use later. Select “User parameters...” from the “Data” menu, and add a parameter for each quantity giving each an appropriate name and recording the numerical value (in SI units).
3. Insert a column to hold the calculated value of the centripetal force on the mass. From the “Data” menu select “New Calculated Column...” and give the column an appropriate name (and short name) and then enter a formula of the form $(W/g)\omega^2 r_{\text{cm}}$ where ω is accessed from the “variables” drop-down list (the column incorrectly labeled “Velocity”). The other quantities are accessed from the “parameters” drop down list.
4. Uncertainty in m . The dual range force sensor has a sensitivity of 0.01 N (when set to a range of 10 N). Taking this as the uncertainty in the weight of the mass, determine the uncertainty in its mass.
5. Uncertainty in ω : The rotary motion sensor has a sensitivity of 1 degree. Using the time interval Δt between successive data points (which you can determine from the data column) calculate the uncertainty in ω , $\delta\omega = \delta(\Delta\theta)/\Delta t$. Make sure to convert to rad/s.

6. Uncertainty in r_{cm} . Use the general expression for the center of mass of two objects $r_{\text{cm}} = \frac{r_{\text{int}}m_{\text{int}} + r_{\text{ext}}m_{\text{ext}}}{m_{\text{total}}}$ and the product/quotient rule and sum/difference rule for error propagation to estimate the uncertainty in r_{cm} .
7. Create parameters for the measured and calculated uncertainties δm , $\delta \omega$ and δr .
8. Create an uncertainty column. Using the product/quotient rule for uncertainty propagation, determine the formula for the uncertainty in the calculated centripetal force $m\omega^2 r$ in terms of the uncertainties δm , $\delta \omega$ and δr . Enter this into a new calculated column, the uncertainty in the calculated centripetal force.
9. Create a graph of the measured centripetal force F_c versus the calculated value $m\omega^2 r$. From the “Insert” menu select “Graph” and set the x-axis to be the measured centripetal force F_c and the y-axis to be the calculated centripetal force $m\omega^2 r$.
10. Error bars. We can express the uncertainty in our data points by including *error bars* on the plot. These vertical and/or horizontal bars around the data points represent the range of y and/or x values respectively that are within the experimental uncertainty associated with the measured data point.
 - (a) Click on the heading of the column for the measured centripetal force. In the “Options” tab, check the “error bar calculations” box and enter a fixed value error or $\pm 0.01N$ which represents the sensitivity limit of the force sensor
 - (b) Repeat the previous step for the column for the calculated centripetal force, but this time instead of entering a fixed value for the uncertainty, tell it to use the column created in step 3h
 - (c) Double click on the graph to bring up the graph options window and turn on the option to show the x and y error bars and turn off the option to connect the data points with lines.
11. Fit a curve to your data by selecting “Curve Fit...” from the “Analyze” menu. Choose “Linear” as the general equation, enter “1” as the value for the parameter A , the slope of the curve, and click OK. This line represents the expression $F_c = m\omega^2 r$
12. What percentage of your data points agree with the line (i.e. what percentage of the error bars does the line pass through? If all of the uncertainties were estimated properly *and there are no systematic errors unaccounted for*, 68% of the data points should fall within one error bar of the curve.

13. Fit another curve to your data by selecting “Curve Fit...” from the “Analyze” menu. Again choose “Linear” as the general equation, but this time select “try fit” to have Logger Pro find the slope of the line that best fits the data. Record the slope of this line.
14. What is the percent difference between the measured slope of your line and the theoretical value of 1? If more than 68% of your data points lie within one error bar of the slope=1 line, then the difference is not significant. Was the deviation from the expected value that you observed significant?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A discussion of the magnitude and likely origin of any systematic errors in your measurement.
2. The graph of F_c versus $m\omega^2 r$.

Experiment 7

Friction

Objective

To verify that the force of static friction obeys $f_s \leq \mu_s N$ and the force of kinetic friction obeys $f_k = \mu_k N$ where $\mu_s \geq \mu_k$ and are independent of the area of contact.

Theory

Friction is a contact force that opposes the motion of objects. For objects that are stationary relative to each other the force of static friction between them has a magnitude that obeys $f_s \leq \mu_s N$, where N is the normal force between the objects and μ_s is called the *coefficient of static friction*, a constant that depends on the materials in contact. If you were to gradually push harder-and-harder on an object sitting on a surface with friction, the force of friction would increase to oppose your applied force, rising to the maximum value $f_{s,max} = \mu_s N$ before being overwhelmed by the applied force, at which point the block would begin to slide and *kinetic* friction with a magnitude of $f_k = \mu_k N$ would oppose the motion. Since the kinetic friction is not strong enough to prevent the motion it cannot be greater than the maximum force of static friction. According to these relationship the force of friction does not depend on the area in contact, only the materials. These relationships will be tested in this lab.

Practice

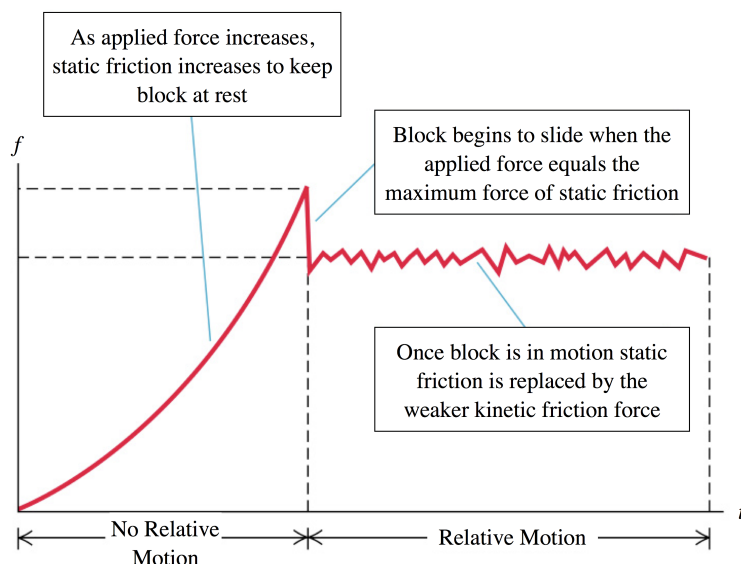


Figure 7.1: Magnitude of friction force as a function of time as a force is applied




Identify and label on the above plot the maximum value of static friction ($f_{s,max}$) and the force of kinetic friction (f_k).

Procedure

A block, loaded with various masses will be set on a piece of paper which is pulled out from under it, while the block is restrained by a “force sensor” with the data logged for later analysis to determine μ_s and μ_k between the block and the paper. In these experiments the block will sit on three of its four surface, a large wooden surface, a small wooden surface, and a large felt surface.

1. Tape a long piece of paper on top of a sheet of plexiglass with the wood block on top of the paper near one end. Position the plexiglass so that its edge hangs off the edge of the lab table.
2. With the force sensor range set to ± 10 N, connect it to the wooden block by a string tied to the force sensor on one end and the wooden block on the other. Plug it into

EXPERIMENT 7. FRICTION

- the data logger unit. The sensor should be immediately recognized and the LabQuest display updated with a reading of the current value from the sensor. If so you may skip to step 3d.
3. Configure LabQuest to use the force sensor. From the  tab, select “Sensor setup...” choose “Force” for whichever channel the sensor is plugged into and then from the submenu select “Dual Range Force 10N” and click “OK”.
 4. Calibrate your force sensor. With no tension in the string between the force sensor and the block, go to the  tab, select **Sensor**→**Zero**→**Dual Range Force ± 10 N**.
 5. Practice data collection by having your lab partner press down on the force sensor to hold it in position while you gently pull the paper and plexiglass together horizontally out from under the block, slowly increasing the applied force until they slide out.
 6. Once you are comfortable with the previous step, click  on the data logger device to begin recording data and repeat step 3e for a 10 second interval. Your data should resemble figure 7.1.
 7. Record the relevant values for the frictional force from the force vs. time curve on the data logger:
 - (a) Using the stylus, tap the peak of the curve (Note that unlike today’s smart phones, this touch sensitive display requires a moderate amount of pressure), use the arrow keys to fine tune the point you have selected until the readout displays the maximum force. Record this value as the maximum force of static friction, $f_{s,max}$.
 - (b) Using the stylus, tap and drag over the “flat” region of the graph (where the block was moving with constant velocity) to highlight it. Select **Analysis**→**Statistics** from the drop down menus to measure the mean value and standard deviation of the force of kinetic friction, f_k . Record these values.
 - (c) Record any other parameters relevant for this trial, such as the value of any masses that were placed on top of the sliding block.
 8. Repeat the measurement (steps 3f and 3g) one more time to get an average value and a spread (uncertainty) in $f_{s,max}$. Note that unlike the maximum value of static friction, the value of kinetic friction is represented by many data points in each experiment, so average and spread in values of f_k can be obtained from a single experiment.
 9. Place a 500 g mass on top of the cart and repeat the measurement 2 more times. Finally with two 500 g weights on the block repeat the measurement 2 *more* times.

10. Repeat the above procedure testing different surfaces of the block: the large and small wooden surfaces and the large felt surface in contact with the paper for one set of measurements.

Analysis

Enter your data into logger pro (you could also use Excel if you prefer). Using a separate column for each block surface and each type of frictional force (i.e. “Static friction of small wood surface”, “Kinetic friction of small wood surface”, “Static friction of large wood surface”, ...), labeled appropriately, complete the following analysis

1. Plot the average force of static friction and the average force of kinetic friction versus the normal force on the cart (both should be on the same plot). Give your plot an appropriate title, axes labels indicating units and any other formatting necessary for it to be easily understood.
2. Fit a line to the data using the “linear fit” option in the “Analyze” menu (in Excel you would use the “add trendline” option). Record the slope of the line - this is the value of the coefficient of friction for the particular type of friction (static or kinetic) being plotted.
3. From your uncertainty in the force of friction with the cart fully loaded, estimate the uncertainty in the coefficient of friction from $\delta\mu \approx \frac{\delta f}{mg}$ (note this neglects the uncertainty in the mass because the total uncertainty will be dominated by δf)
4. Print out your graphs and record on each the measured coefficients of kinetic and static friction, along with their associated uncertainty.
5. Do your measured values for μ_s between the large and small wood surfaces on the table agree with each other within the experimental uncertainty? Do they differ from the coefficient of static friction observed in the experiments with the felt surfaces? Which surface had the higher value for the coefficient of static friction?
6. Was the measured value of the coefficient of kinetic friction less than that of static friction for each surface being considered? If not postulate a reason why it wasn't.

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Experiment 8

Work and Energy

Objective

To validate the work-energy theorem and to study the conservation of energy principle.

Theory


The work-energy theorem states that the net (total) work done on a system is equal to its increase in kinetic energy. You will determine the work done on a (nearly) frictionless cart and show that the work done is equal to the increase in kinetic energy of the cart. Furthermore, you will show that the increase in energy of the cart is equal to the decrease in potential energy of the falling weight that supplies the force on the cart.

The apparatus consists of a cart that is accelerated along a linear track by the constant force due to the tension in a cord attached to a falling mass. The cord passes over a pulley mounted on a rotation sensor that will be used to determine the velocity of the cart as a function of time.

Practice

1. Consider a pulley of radius 15 mm rotating at an angular velocity of 67 rad/s. What is the linear velocity of a cord moving over the pulley rim without slipping.
2. Given a mass $m_1 \pm \delta m_1$ and $m_2 \pm \delta m_2$ moving at speed $v \pm \delta v$ calculate the kinetic energy and its associated uncertainty for the system (i.e. $KE \pm \delta KE$).
3. Given a mass $m_1 \pm \delta m_1$ at a height $h \pm \delta h$ calculate the potential energy and associated uncertainty, i.e. $h \pm \delta h$.

Procedure

1. Make sure the handheld data logger is turned on, the digital rotation sensor is plugged in and the unit is reading an angle.
2. Load the cart with about 1kg of mass and record its loaded weight as m_c
3. To simulate a frictionless system, tip the track until the cart rolls toward the pulley at a constant speed. (No forces should be acting on the cart!)
4. Use vernier calipers to measure r , the radius of the pulley on the rotation sensor which the cord will pass over.
5. Select a falling mass, m_f between 0.05 kg and 0.20 kg, that is different than that your lab partner used. Weigh it together with any mass hanger that you might use to connect it to the end of the string.
6. Place the cart at the starting position, record the height of the falling mass as h_0 begin recording data on the data logger by pressing  and release the cart.

7. Transfer the collected data onto one of the lab computers for analysis by connecting it via a USB cable and opening “Data Logger Pro” on the computer. The device will be auto-detected and you will be prompted to import the data. If you are successful you can skip step ??, otherwise see the instructions for manually importing data in the next step.
8. If your data did not import automatically, save your data on the handheld device by selecting “save” from the “file” menu. Then when it is connected to the computer, select “LabQuest Browser” from Logger Pro’s “File” menu and select the file that you just saved.
9. Correct the column names that are automatically created. Since you were using a *rotation* sensor it was the *angular* displacement $\Delta\theta$, *angular* velocity ω and *angular* acceleration α that were recorded. Double click on the column headers and correct the titles for these columns.

Analysis

1. On your spreadsheet remove any data points that were before the cart was released or after it had reached the end of the track. Highlight the rows associated with these data points and select “Strike Through Data” from the “Edit” menu.
2. Create calculated columns for the linear displacement $\Delta x = r\Delta\theta$ and linear velocity $v = r\omega$ using your measured values for r and the columns for $\Delta\theta$ and ω
3. Create a column for the kinetic energy of the falling mass $KE_f = \frac{1}{2}m_f v^2$ and one for the kinetic energy of the rolling cart $KE_c = \frac{1}{2}m_c v^2$, and finally a column for the total kinetic energy $KE_{tot} = KE_f + KE_c$
4. Create a column for the potential energy of the falling mass, relative to the ground. Using h_0 as the starting distance for the falling mass above the ground, the potential energy is $PE = m_f g(h_0 - \Delta x)$
5. Create a column for the work done on the system by gravity $W_g = m_f g \Delta x$
6. Uncertainty in ω : The rotary motion sensor has a sensitivity of 1 degree. Using the time interval Δt between successive data points (which you can determine from the data column) calculate the uncertainty in ω , $\delta\omega = \delta(\Delta\theta)/\Delta t$. Make sure to convert to rad/s.

- Using your estimated uncertainty in the mass of the cart (δm_c), falling mass (δm_f), pulley radius (δr) and angular velocity ($\delta \omega$), determine the equation for the uncertainty in the kinetic energy. Create a new calculated column using this equation. From the expression for kinetic energy $KE = \frac{1}{2}M_{tot}(r\omega)^2$ the uncertainty can be found using equation B.2, the product rule for calculating uncertainties:

$$\delta KE = KE \sqrt{\left(\frac{\delta M_{tot}}{M_{tot}}\right)^2 + 4\left(\frac{\delta r}{r}\right)^2 + 4\left(\frac{\delta \omega}{\omega}\right)^2}$$

where $M_{tot} = m_c + m_f$ and the uncertainty in this quantity (δM_{tot}) can be found from the equation B.1, the sum rule for adding uncertainties. Note the factor of 4 in front of the last two term arises because the quantities r and ω are squared in the expression for KE so its uncertainty appears twice and is correlated so the two uncertainties add directly (not in quadrature).

- Determine an expression for the uncertainty in *potential* energy $PE = m_fgh = m_fg(h_0 - r\Delta\theta)$ again neglecting the uncertainty in $\Delta\theta$. (*hint* first find the uncertainty in the first term m_fgh_0 due to the uncertainty in m_f , then find the uncertainty in the second term $-m_fgr\Delta\theta$ due to the uncertainties in m_f and r then add these uncertainties using the sum rule for adding uncertainties.) Enter this expression into another new calculated column.
- From your calculated uncertainty in kinetic energy and potential energy, calculate the uncertainty in the total energy $E_{tot} = KE + PE$ and enter the expression into a new calculated column.
- Use the product rule for combining uncertainties to determine an expression for the uncertainty in the total work done on the system $W_g = m_fg\Delta x$. Enter this expression in a new calculated column.
- Plot the total kinetic energy, the potential energy of the falling mass and the total energy as a function of Δx along with error bars for each quantity. Is the total energy constant within the experimental uncertainty? (i.e. if it is possible to draw a horizontal line on the graph that passes through all of the error bars in total energy then you have shown that energy was conserved)
- Plot the work done by gravity on your graph, along with its associated uncertainty. Does the work agree with the change in total kinetic energy within the experimental uncertainty?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. Expressions for the energy uncertainties.
2. A formula showing how each quantity you report on (Work, Kinetic Energy, Potential Energy, Total Energy) was calculated from the measured parameters (there is no need to include the full spreadsheet though)
3. A single graph of Work, Kinetic Energy, Potential Energy and Total Energy versus displacement with error bars for each quantity that summarize your analysis.
4. Your statement of observations and conclusions. If the total energy of the system was not constant, is its variance reasonable in light of the experimental uncertainties?

Experiment 9

Conservation of Linear Momentum

Objective

To compare the sum of the momentum vectors before and after a two-dimensional collision of steel balls. To demonstrate the principle of conservation of linear momentum.

Theory

A consequence of Newton's laws of motion is that during a collision of objects, the *total* momentum of the objects remains constant. Consider that Newton's second law states

$$\vec{F} = \frac{d\vec{p}}{dt},$$

with $\vec{p} = m\vec{v}$ this reduces for objects with a constant mass to $\vec{F} = m\frac{d\vec{v}}{dt}$ or in the more familiar form $\vec{F} = m\vec{a}$. The original form can be rewritten as $\Delta\vec{p} = \vec{F}\Delta t$. Newton's third law states that the force exerted by an object on another is equal in magnitude and opposite in direction to the force that the other object exerts on the original object, which can be expressed by

$$\vec{F}_{12} = -\vec{F}_{21}$$

so we can say that the momentum change for object 1 is $\Delta\vec{p}_1 = \vec{F}_{21}\Delta t$ when it collides with object 2 and is in contact for a time Δt , while for object 2 the momentum change is

$\Delta \vec{p}_2 = \vec{F}_{12} \Delta t$ and thus the total momentum change is $\Delta \vec{p}_{tot} = \Delta \vec{p}_1 + \Delta \vec{p}_2 = (\vec{F}_{12} + \vec{F}_{21}) \Delta t = 0$, i.e. the total momentum for the system is conserved. We will test this relationship by observing the momentum of two objects before and after they collide.

Practice

Consider an object of mass 2 kg moving with a velocity of $\vec{v} = 0.3 \text{ m/s } \hat{x} + 0.4 \text{ m/s } \hat{y}$. What is the momentum for this object?

Procedure

Each student will record the collision of a pair of steel balls. The resulting video will be analyzed with the help of “Logger Pro” data analysis software to determine the velocity of each ball before and after the collision.

1. Record the mass of each steel ball.
2. Mount the webcam using the supplied mounting equipment so that it is looking straight down at the floor.
3. Open “Logger Pro” and select “Insert → Video Capture ...” to get a video preview. You can change the resolution of the video by selecting the “Set up camera” button. The 800x600 setting provides a good balance of high resolution with reasonably fast frame rate.
4. Place a meter stick on the floor *completely* in the frame of the video (this will be used to calibration distances in the video)
5. Plan out the path you want the pair of balls to follow: Place the starting ramps fully outside the field of view of the camera so that when released one ball approaches the other and they collide directly underneath the camera and then exit the field of view in a straight line before being stopped - if the balls acquire spin during the collision they may travel in a curved path making the data analysis problematic. If you observe this

repeat the experiment until you achieve a collision where the spin is negligible - it may help to decrease the speed of the balls. It will require careful timing, practice and a bit of luck to get the balls to collide properly. Try several times to get the timing right for a collision. If you are unable to get a collision with both balls rolling towards the collision point, you can instead start with one ball at rest directly under the camera.

Analysis

To analyze the video you will use the program “Logger Pro” on the lab computers. Refer to the figure below for location of various buttons. Use the following procedure to analyze the motion of your recorded collision.

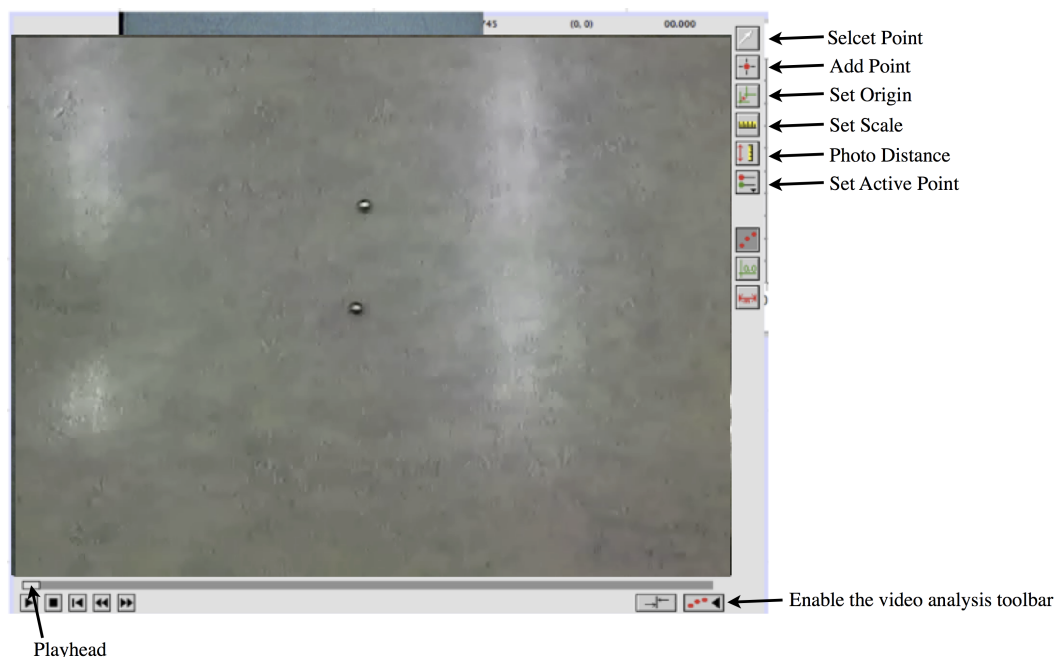


Figure 9.1: *Logger Pro* video window with video analysis toolbar enabled

Set up video for motion analysis

1. Open the video analysis toolbar by clicking on the button with three red dots in the bottom corner of the movie window.

2. Set an origin for your coordinate system using the “Set Origin” button of the video analysis toolbar.
3. Calibrate the scale of the video using the image of the meter stick by clicking the “Set Scale” button and drawing a line the length of the meter stick and entering 1 m for the line length.
4. Position the playhead at the first frame where both balls are in the video frame and are moving freely (i.e. are not still on the ramp). This will be your first data frame.

Capture the motion

1. Click the “Add Point” button, then click on the center of the first ball. The video will advance by one frame immediately after you click.
2. Continue to click on the center of the same ball in all additional frames until the first of the two balls has moved off screen - this will be your last data frame.
3. Click the “Set Active Point” button and select “Add point series” from the popup menu.
4. Return the playhead to the first data frame. Click on the center of the second ball to identify its location, and continue to click on the center of the ball in each successive frame until the last data frame.
5. **Save your data.** If your computer crashes or Logger Pro scrambles your data you don’t want to have to go through that time consuming step again!

Determine the momentum

As you were recording the position of each ball in each frame, the properly scaled x and y coordinates were being added to data columns in the logger pro spreadsheet. Additionally columns for the x and y components of velocity were generated from the position columns. We will complete the spreadsheet by creating columns for the components of the momentum vectors adjusting the origin of the time axis to be at the moment of collision, and adjusting for the effect of external forces on the system. You can choose to do the analysis directly in logger pro (using the instructions below) or you can choose at this point to export your data in “csv” format to import into Excel or another spreadsheet for analysis.

1. Looking at the graph of position versus time that was generated when you captured the motion, determine the time t_0 at which the collision occurred. Since the collision may have occurred *between* two frames of the video, it may be necessary to estimate the value between the two points closest to the collision. This process is called *interpolation*.
2. Record the time of collision as a parameter. Select “User parameters...” from the “Data” menu, and add a parameter t_0 for this quantity, giving it an appropriate name and recording the numerical value. While you are in the edit parameters menu add a parameter for the mass of each ball m_1 and m_2 and enter the numerical values, and create two parameters that we will use to remove the effects of external forces that you can call F_x and F_y . You can leave the values of these set to zero for the moment.
3. Create a column for time relative to the collision. Click “New calculated column...” from the “Data” menu and give the new column an appropriate name (in both the “Name:” field and the “Short Name:” field). In the equation panel create a formula for this column of the form $t_r = t - t_0$ by clicking in the equation panel to get a cursor there, selecting “Time” from the list of variables, typing in the minus symbol, then selecting the collision time parameter t_0 from the parameters list. You will use this same method to enter formulas in the other calculated columns that you create in the next few steps.
4. Create a calculated columns for the momentum of ball 1 in the x-direction of the form $p_{1x} = m_1 v_{1x}$. Repeat three more times for p_{1y} , p_{2x} and p_{2y} .
5. Add another calculated column for the corrected total momentum in the x-direction using the formula $p_x = p_{1x} + p_{2x} - F_x * t_r$ where t_r is the relative time column you created earlier and F_x is a parameter that corrects for any external forces. We will use the term $F_x * t_r$ to subtract the momentum gained (or lost) from external forces such as friction or due to the floor not being level, however at the moment $F_x = 0$ so this term has no effect. Repeat for the total momentum in the y-direction.
6. Determine the appropriate correction factor for external forces. Plot the x-component of total momentum versus relative time (p_x vs t_r) by clicking on the y-axis label of the graph and selecting the appropriate column for p_x and clicking on the x-axis label and selecting the appropriate column for t_r . If the data has an overall upwards or downwards trend to it then an external force was pushing (or pulling) it in the x-direction. We’ll check the magnitude of this effect and then compensate for it. Perform a linear fit to the data by choosing “Linear Fit” from the “Analyze” menu. The parameters of the fit will be shown on the graph. Note the slope of the fit, and record this as the value of your parameter F_x by accessing the “User Parameters...” option

from the “Data” menu again. Once you have done this the momentum data should adjust to remove this trend. Repeat for the y-component of total momentum.

7. Add a calculated column for the magnitude of total momentum

$$p_{tot} = \sqrt{p_x^2 + p_y^2}$$

where you use the appropriate columns for p_x and p_y .

8. Add a calculated column for the angle of total momentum

$$\theta_{tot} = \tan^{-1} \left[\frac{p_y}{p_x} \right]$$

where you use the appropriate columns for p_x and p_y .

9. Add a calculated column for the total kinetic energy

$$KE_{tot} = \frac{1}{2}m_1(v_{1x}^2 + v_{1y}^2) + \frac{1}{2}m_2(v_{2x}^2 + v_{2y}^2)$$

where you use the appropriate columns for v_{1x} , v_{2x} , v_{1y} and v_{2y} .

Statistical Analysis

You will use the columns for the kinetic energy, magnitude and angle of the total momentum to determine the average and standard deviation of these values before and after the point of the collision.

1. Double click on the “y-axis” of your graph and in the “Axes Options” tab select kinetic energy, magnitude and angle of the total momentum as the (only) three columns to plot.
2. Select “Statistics” from the “Analyze” menu. Select your columns for total magnitude of momentum and angle of momentum vector for analysis.
3. Adjust the brackets so that only the data before the collision point (i.e. $t_r < 0$) is within the brackets. Record the mean values as your measurement and the standard deviations as your uncertainties in the initial magnitude and angle of the total momentum and the kinetic energy.

EXPERIMENT 9. CONSERVATION OF LINEAR MOMENTUM

4. Adjust the brackets to enclose only the data *after* the collision (i.e. $t_r > 0$) and record the mean values as your measurement and the standard deviations as your uncertainties.
5. Compare your initial and final momentum (magnitude *and* angle) for both collisions. Was momentum conserved?
6. Compare your initial and final energy. What percent of the initial energy was lost in the collision? Would you classify this collision as a fully elastic, partially inelastic or fully inelastic collision?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A printout of the graphs of the magnitude and angle of momentum, as well as kinetic energy for your collision. You will need to plot these quantities separately so that each graph can have the y-axis scaled and labeled appropriately for each quantity. Make sure the y-axis scale starts at zero so that the relative magnitude of any fluctuations can be seen on the graph.
2. A conclusion, addressing the two questions asked in the statistical analysis section under steps 3e and 3f.

Experiment 10

Ballistic Pendulum

Objective

To use a ballistic pendulum to determine the velocity of a projectile. To verify this velocity by measuring the range of the projectile.

Theory

We use measurements obtained from a ballistic pendulum to review energy conservation, momentum conservation, and projectile motion. From the measured recoil speed of the pendulum we determine the speed of a ball fired by a spring-loaded cannon. We check this speed by measuring the range of a ball fired horizontally from the cannon from the bench top to the floor.

The spring-loaded cannon fires a steel ball with sufficient speed to shatter eyeglasses. Use appropriate caution. Don't look down the barrel; you can see from the side whether the spring is cocked and if the ball is in place. Don't leave a cocked and loaded cannon on the bench.

To determine the initial velocity of the steel ball fired from the "cannon", we apply two conservation laws, the conservation of momentum (during the collision) and the conservation of energy (after the collision). Conservation of momentum relates the velocity of the ball

before the collision with the velocity of ball plus pendulum after the collision, according to

$$m_b v_0 = (m_b + m_p) v_r$$

Conservation of energy relates the initial (kinetic) energy of the pendulum to its final (potential) energy, according to

$$\frac{1}{2}(m_b + m_p)v_r^2 = (m_b + m_p)gh$$

where h is the maximum height reached by the center of mass of the pendulum. Using these two relations, we can determine the velocity v_0 of the ball when it is shot from the cannon.

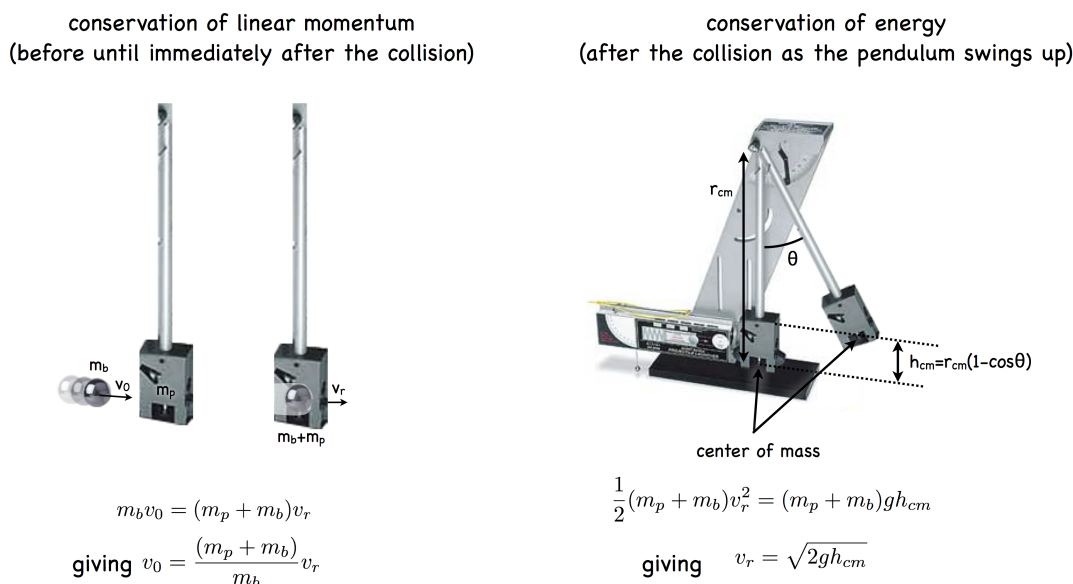


Figure 10.1: The operation of the ballistic pendulum

Practice

1. Consider a ball of mass m_b moving with speed v_0 colliding *inelastically* with an object of mass m_p that is initially at rest. What is the speed v_r of the two objects immediately after the collision in terms of v_0 , m_b , and m_p ?

2. Consider the bob of a pendulum of mass m moving at speed v_r at the bottom of the arc of the pendulum's motion. Using conservation of energy, find the maximum height h_{max} that the pendulum bob will swing to.

3. For a pendulum of length L that swings an angle θ so that the pendulum bob rises to a height of h draw a diagram and use trigonometry to find an expression for the maximum height of the pendulum h_{max} as a function of the maximum angular displacement of the pendulum θ_{max}

4. Using your results from the previous problems, find an expression for the initial speed v_0 of a ball in a ballistic pendulum that produces a maximum angular displacement of θ_{max}

Procedure

1. Remove the pendulum from the stand by unscrewing the pivot axle. Weigh and record the masses of the pendulum m_p and the steel ball m_b .
2. With the ball latched in the catcher, balance the pendulum on a ruler or pencil to locate the center of mass. Measure and record as r_{cm} , the distance from the pivot axis to the center of mass.
3. Reattach the pendulum to the stand. Check that it swings freely. With the pendulum hanging vertically, move the angle pointer to just contact the pendulum arm. Record this angle as θ_0 .
4. With the pendulum latched at 90° , insert the steel ball and use the ramrod to cock the spring to its middle or maximum range position. Let the pendulum hang vertically and

move the angle pointer to θ_0 . While holding the base firmly against the table fire the cannon by pulling up on the trigger. Observe carefully the angle indicator to ensure that it stays at the maximum angle (if it slides back a few degrees from its maximum displacement due to insufficient friction you will have to use a different apparatus). Write down the maximum angle recorded by the angle indicator.

5. Repeat this process five times to check the reproducibility of the apparatus. Be sure to cock the cannon to the same setting each time. To minimize drag on the pendulum set the angle indicator initially to about 5° below the maximum recoil angle. Record the maximum recoil angle, θ_{max} , for each shot.

Analysis

1. Compute the average value of the recoil angle $\theta = \theta_{max} - \theta_0$
2. Find the change in height of the center of mass $h_{cm} = r_{cm}(1 - \cos \theta)$.
3. Calculate the recoil velocity $v_r = \sqrt{2gh_{cm}}$ being sure to use three significant figures for g ; $g = 981 \text{ cm/s}^2$.
4. Calculate the initial velocity of the steel ball $v_0 = \left(\frac{m_p + m_b}{m_b}\right) v_r$. Express this quantity as a value plus or minus its uncertainty, thus: $v_0 \pm \delta v_0$ where

$$\delta v_0 = v_0 \sqrt{\left(\frac{\delta m_p^2 + \delta m_b^2}{(m_p + m_b)^2}\right) + \left(\frac{\delta m_b}{m_b}\right)^2 + \left(\frac{1}{2} \frac{\delta r_{cm}}{r_{cm}}\right)^2 + \left(\frac{1}{2} \frac{\sin \theta}{(1 - \cos \theta)} \frac{\pi \text{ rad}}{180^\circ} \delta \theta\right)^2}$$

and the uncertainty in angle $\delta \theta$ is measured in degrees.

5. Now calculate where the ball would land if it were shot horizontally with this initial velocity from on top of the lab bench. The range is given by $x = v_0 t$ where v_0 is the initial velocity of the ball and t is the time of flight. You've found the uncertainty in step 3d and the uncertainty in time for an object that fell a distance $y \pm \delta y$ was something you calculated in experiment 1, so you can find the uncertainty δx using these quantities and the product/quotient rule for uncertainty. the velocity in the The time of flight can be determined from the fact that the distance the ball falls vertically is given by $y = \frac{1}{2}gt^2$. Place a piece of graph paper this distance away and tape it to the floor. Mark the predicted landing spot with an x and draw a line a distance $t\delta v$ beyond the x, and another line the same distance before the x, this is your uncertainty in the range.

6. Now remove the pendulum and fire the ball horizontally. (make sure it does not hit anyone.) Observe where the ball hits the floor – it will make a mark on the carbon paper. Repeat 10 times. If you have properly calculated your uncertainty 7 out of the 10 trials should hit between the lines.

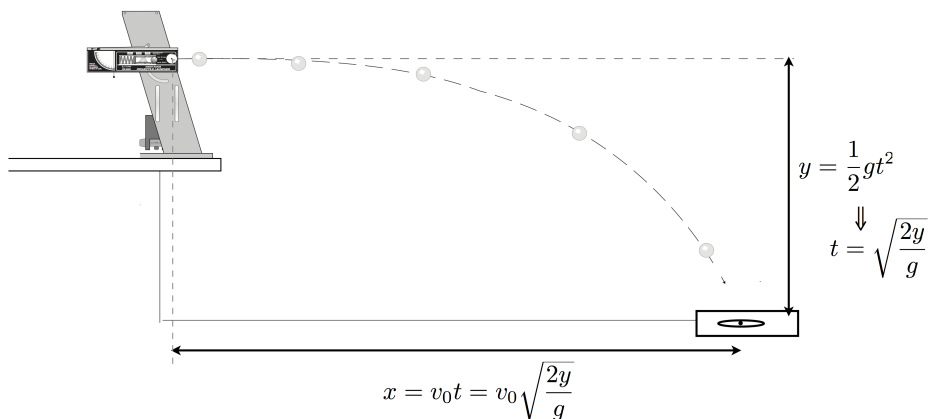


Figure 10.2: Projectile motion from the cannon

7. Does the value of the range calculated using your measured value for v_0 agree with the observed range?
8. Finally, compute the percentage of the balls initial kinetic energy that was lost when the ball collided with the pendulum arm.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A neat and organized presentation of all measured and calculated values.
2. A calculation of the percent uncertainty in the calculated range and the measured range of the projectile. How many of your shots landed within the uncertainty range? What can you conclude from this?
3. A final comment on whether or not mechanical energy was conserved. If not, is that what you would expect in this type of a collision?

Experiment 11

Rotational Dynamics

Objective

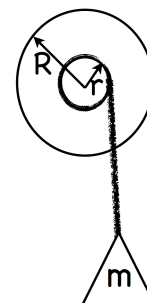
To investigate the relationship between torque and angular acceleration and to verify the work-energy theorem for rotational motion.

Theory

The rotational analogue to $\vec{F} = m\vec{a}$ is $\vec{\tau} = I\vec{\alpha}$ where $\vec{\tau}$ is torque, I is the moment of inertia and $\vec{\alpha}$ is the angular acceleration. If we apply a torque to a body that can rotate about a fixed axis, it will undergo an angular acceleration and a change in its angular velocity, therefore, its rotational kinetic energy will change. The rotational kinetic energy is given by $KE = \frac{1}{2}I\omega^2$, where ω is the angular velocity (radians per second).

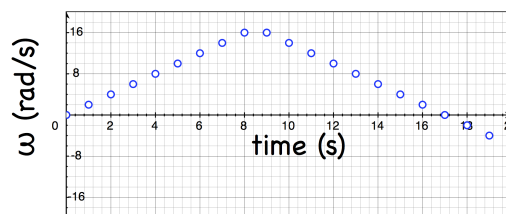
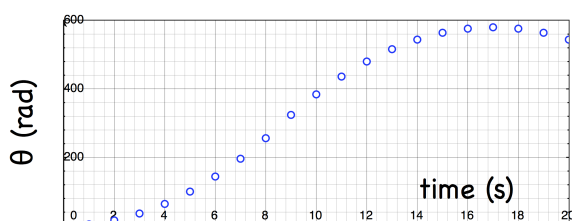
Practice

Consider an object of mass m at rest hanging from a string that is coiled around a pulley with an inner radius of r and an outer radius of R as shown to the right.



1. What is the torque exerted by the string on the pulley about its axis?


If the mass begins to fall it will produce a constant angular acceleration on the pulley as the string is unwinding, and a constant angular acceleration of opposite sign as it winds back up.

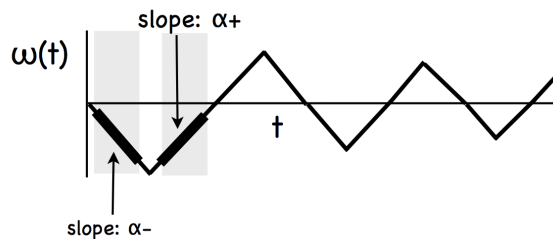


2. If the instantaneous angular displacement and angular velocity are as plotted on the above graphs, what is the magnitude of this constant angular acceleration (be sure to include the correct units)?
3. What was the maximum angular velocity of the pulley?
4. What was the angular displacement of the pulley as it was spinning up?

Procedure

Your instructor will demonstrate proper operation of the apparatus. An aluminum disk (or bar) is attached to a “quadrature encoder” based rotation sensor that spins about a horizon axis and relays information about its angular displacement to a hand-held data logger. Angular acceleration is produced by the tension in a cord attached to a small load mass. Although you will work in groups of two, each student should record his or her own data using a different value for the small load mass.

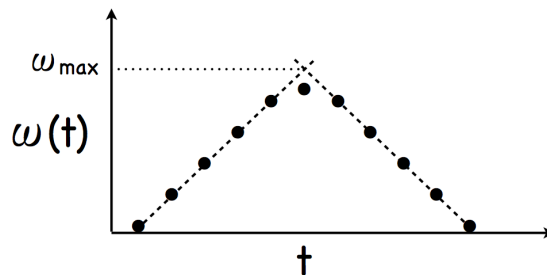
1. Remove the aluminum disk (or bar), and spool and measure their mass, and radius (or length and width as appropriate). Record these values in a data table.
2. Reassemble the system with the thread anchor washer and a small spool attached to the disk with the screw.
3. Ensure that the rotation sensor is plugged into the side of the data logger and that the unit is reading a value for the rotation angle, and that it is set to record for 20s (as displayed in the box labeled “Length”). If the recording length needs to be adjusted, tap on the box labeled “Length” then enter the new value.
4. Wind the string around the spool, lifting the small load mass until it is just below the bottom of the disk (or bar). The first group member should start with 25 g of mass on the hanger and each group member should add an additional 5 g before taking their data.
5. Start logging data on the hand-held data collector by pressing . Release the system from rest and allow the data logger to collect angular displacement and angular velocity data over a 20 second interval as the load mass falls and then returns up towards its starting point. The resulting graph of ω vs t shown on the data logger should look like a “saw tooth” function as shown below.



6. Determine the angular acceleration of the disk from the plot of angular velocity vs. time by selecting the region of the graph (tap the stylus and drag to select a region) just after the mass was released from rest, but before it had reached the bottom (i.e. the first line segment of the saw tooth graph). Choose “curve fit” from the “analysis” menu and select “velocity” and then choose a “linear” curve fit from the “Fit Equation” pull down menu. Record the value for m the best fit slope, with appropriate units taken from the graph. Call the *magnitude* of this value α_- , the magnitude of angular acceleration when the applied torque opposed the frictional torque. Repeat for the region just after the mass had reached the bottom but before it had wound all the way up calling the magnitude of the slope α_+ the magnitude of angular acceleration when the applied torque is in the same direction as the frictional torque.

7. When the spool is unwinding the applied torque is opposed by friction, while when it is winding up the applied torque adds to the frictional torque to decelerate the disk, thus the magnitude of the acceleration due to only the applied torque is the average magnitude of the acceleration $\alpha_a = (\alpha_+ + \alpha_-)/2$. Calculate this value. Also compute the difference in the magnitude of angular accelerations $\Delta\alpha = |\alpha_+ - \alpha_-|$, this is a measure of the effect of only the frictional torque and will be used to determine the energy dissipated by friction.

8. Estimate and record the maximum value of the magnitude of the angular velocity (ω_{max}). Logger Pro determines the angular velocity at a given time by comparing the angular displacement at an earlier and latter time and assuming the velocity in that time interval is constant. At the kinks in the curve (where the angular velocity is a maximum) this is not a valid assumption and results in the sharp corners of the triangular curve being rounded and the reported angular velocity being underestimated. To find the maximum angular velocity extrapolate the linear regions of the graph and find the angular velocity from the point of intersection on the graph as shown below. Also record $\Delta\theta$, the magnitude of the angular displacement between the time when the disk began rotating (i.e. the starting point), until when it was moving at its maximum angular speed. These values will be used for energy calculations.



9. Repeat the experiment 2 more times to obtain three values for α_a . Find the average and standard deviation of these three values to use as $\bar{\alpha}$ and $\delta\alpha$ respectively in your calculations.

Analysis

1. Tension. Calculate the tension in the cord T and its associated uncertainty δT . The linear acceleration of the falling mass, a , is so small it can be ignored: $T = m_f(g - a) \approx m_f g$ with $\delta T = \delta m g$.
2. Applied Torque. The applied torque on the pulley is $\vec{\tau}_a = \vec{T} \times \vec{r}$, but in our case the tension in the string is perpendicular to the radius of the spool so the magnitude of the torque can simply be expressed as $\tau_a = T r$ with uncertainty $\delta\tau_a = \tau_a \sqrt{\left(\frac{\delta T}{T}\right)^2 + \left(\frac{\delta r}{r}\right)^2}$.
3. Moment of inertia. The moment of inertia can be calculated from $I = \frac{\tau}{\alpha}$ with uncertainty $\delta I = I \sqrt{\left(\frac{\delta\tau_a}{\tau_a}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$. Compute your experimental value for the inertia of the disk (or bar) from the applied torque found in step 3b and the acceleration found in step 3i of the procedure and compare it to the calculated value found from the mass and dimensions of the disk (or bar): $I_{\text{disk}} = \frac{1}{2} m_d r_d^2$ and $I_{\text{bar}} = \frac{1}{12} m_b (L^2 + w^2)$. Do these values agree within the experimental uncertainty?
4. Finally, check the work-energy relation, which states the total work done on a system equals the change in kinetic energy. There are two forces (gravity and friction) that do work on the system as the disk is spinning up: $W_g = mgh$ where $h = \Delta\theta r$ is the height the load mass fell while the spool was unwinding, and $W_f = -\tau_f \Delta\theta$, where $\tau_f = |I\Delta\alpha/2|$. The change in kinetic energy also has two terms (one for the rotation of the disk and one for the falling mass) the rotational kinetic energy of the disk is

$KE_{rot} = \frac{1}{2}I\omega^2$ while the linear kinetic energy of the falling mass is $KE_{lin} = \frac{1}{2}mv^2$ where $v = \omega r$. Relate the total work done on the system as the disk spins up to the change in kinetic energy *for one of your trials* (i.e. don't use the average values for ω_{max} or $\Delta\theta$ since averaging the square of the values is not the same as squaring the average!). What is the percent difference in the total work done and the change in kinetic energy? How does this compare to the experimental uncertainty (the experimental uncertainty will be dominated by the uncertainty in your measured value of I , so you can use $(\delta I/I) \times 100\%$ for the percent experimental uncertainty.) Are they equal within the experimental uncertainty?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A data table.
2. A comparison of the known and the experimental values of the moment of inertia. These values should agree to within the experimental uncertainty. If they do not, then state the sources of uncertainty in this experiment that could have caused this discrepancy.
3. A comparison of the total work done on the system and the maximum kinetic energy of the system. Do these values agree? Recall the Work-Energy Theorem.

Example Data and Results Table

Disk Radius $r_d =$ _____

Spool Radius $r =$ _____

Disk Mass $m_d =$ _____

Falling mass $m_f =$ _____

Trial	α_- (rad/s ²)	α_+ (rad/s ²)	$\alpha_a = \frac{ \alpha_+ + \alpha_- }{2}$ (rad/s ²)	$\Delta\alpha = (\alpha_+ - \alpha_-)$ (rad/s ²)	ω_{max} (rad/s)	$\Delta\theta$ (rad)
1						
2						
3						
avg						
stdev						

Experiment 12

Angular Momentum

Objective

The object of this experiment is to observe and analyze one-dimensional rotational collisions between a disk and a bar rotating about a common axis. You will test the law of conservation of angular momentum and determine whether or not rotational kinetic energy is conserved during the collision.

Theory

Angular momentum $\vec{L} = I\vec{\omega}$ is a useful quantity for analyzing the motion of objects over time because it is a “conserved” quantity. To understand this, consider the time derivative of angular momentum

$$\frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt}.$$

Recognizing that the term $\frac{d\vec{\omega}}{dt}$ on the right side of this expression is the angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$, and using Newton’s 3rd law for rotational motion $\vec{\tau} = I\vec{\alpha}$ we can say that

$$\frac{d\vec{L}}{dt} = \vec{\tau}.$$

This implies that if no outside torques act on a system, the angular momentum of the system will not change in time.

Experiment

In this experiment you will devise your own procedure for comparing the angular momentum of a set of objects before and after a collision involving one stationary object being dropped onto a rotating object. You have a rotation sensor, calipers and rulers, a scale, and a pair of objects that can be placed on the rotation sensor (two disks or a disk and a bar depending on your apparatus). As you develop a procedure consider the following important points:

1. The moment of inertia for a uniform disk of mass m and radius r rotating about its center of mass is $I_{disk} = \frac{1}{2}mR^2$
2. The moment of inertia for a uniform bar of mass m length L and width w rotating about its center of mass is $I_{bar} = \frac{1}{12}m(L^2 + w^2)$
3. Angular momentum is only conserved if no external torques act on the system - how can you avoid introducing unwanted external torques when you drop one object onto the other?
4. When one stationary object is dropped onto another the “collision” is not instantaneous - it takes some time for the stationary object to spin up and the rotating object to slow down until they are moving at a common angular velocity.
5. Friction in the rotation sensor is an unavoidable external torque and may cause the system to gradually lose angular momentum. If you are comparing angular momentum before the collision and after the collision you’d expect there to be a difference due to any frictional losses. How can you take this into account?

Analysis

What are the sources of uncertainty in your measured values for the angular momentum? Are you able to show that momentum is conserved within this uncertainty range?

Because there is friction between the objects as they collide we expect energy to not be conserved. In addition to comparing the angular momentum before and after the collision, determine the total rotational kinetic energy ($KE_{rot} = \frac{1}{2}I\omega^2$) of the system before and after the collision. What percentage of the kinetic energy was lost in the collision?

Save your work onto your flash drive or your own laptop to keep until the end of the semester. You may need it later and files left on the lab computers will be deleted.

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A detailed description of your experimental procedure
2. A detailed description of your analysis
3. Your momentum graph and energy graph
4. A conclusion in which you address the following:
 - (a) Whether or not angular momentum was conserved.
 - (b) Whether or not the energy of the system was conserved.

Experiment 13

Numerical Analysis

Objective

The objective of this experiment is to examine the accuracy of numerical solutions to analytical expressions and to determine the effect of a velocity-dependent drag force on a moving object by using computer-assisted numerical methods to solve Newton's equation of motion.

Concepts

Projectile kinematics homework problems instruct you to neglect air resistance because the velocity-dependent drag force invalidates the simple constant-acceleration solution. Fortunately nature ignores this hint; otherwise raindrops would hit with lethal impact. This week we learn to compute numerical solutions to Newton's equation of motion that include the effect of air resistance. Many real-world engineering and science calculations are set up this way.

For a moving spherical object, such as a baseball, the forces acting on the object are gravity and a drag force due to air resistance. The drag force has two main characteristics; it opposes the direction of motion and it increases in magnitude with increasing velocity. We shall assume the drag force is proportional to the velocity squared.



Figure 13.1: Free body diagram for a falling baseball

Newtons 2nd law in the vertical direction is written as:

$$F_d - mg = ma$$

with $F_d = 0.22d^2v^2$. As the ball falls, the velocity increases until the drag force balances the gravitational force so that the net force is zero.

To predict the motion of the ball, we need to calculate the resultant force and velocity produced by the drag using simple step functions over a small time interval in which the acceleration is assumed constant. New values of position and velocity are calculated from previous values of velocity and acceleration. The repetitive arithmetic in these step equations is just what computers were developed for. These equations use the simplest estimate for the velocity and acceleration in each interval. The larger the interval the less accurate the estimate over this interval is. Our text book develops slightly more detailed expressions, but the core concept for this and all the more sophisticated treatments is the same: if we know the forces, Newtons second law lets us predict the motion.

Calculations

1. **Air Resistance on a Baseball.** A thrown ball will encounter an air resistance force proportional to the square of its diameter d and its speed v . This drag force is given by

$$F_d(v) = \left(0.22 \frac{\text{kg}}{\text{m}^3}\right) d^2 v^2$$

for air at STP. The constant (0.22) has units of kg/m^3 so that when MKS units are used for diameter and velocity the resulting force is in Newtons. When the sphere reaches its terminal velocity v_T , the net force on the sphere is zero, so from Newtons second law we have

$$\left(0.22 \frac{\text{kg}}{\text{m}^3}\right) d^2 v^2 - mg = 0.$$

If a baseball is at hand, measure the diameter and mass of the baseball and compute its terminal velocity. If a baseball is not available, assume its mass is 0.14 kg and its diameter is 0.072 m .

2. **Free Fall from Rest.** A baseball is dropped from rest. Use the spreadsheet program Excel to evaluate the motion over a large number of small time steps. Enter the $n=0$ row of initial values. Define the $n=1$ elements as equations using previous values and constants. Then use the fill down command (in the Edit menu) to repeat the calculations for as long and as far as you want the baseball to fall. Program each of the following equations into the following columns, using references to cells containing the value for Δt , v_T , g , and m . You can use a time step of $\Delta t = 0.1 v_T / g$ which is one tenth of a characteristic time v_T / g describing how long before the effects of drag become significant. In step 3c we will investigate the significance of this value. The first couple of rows in your spreadsheet should look similar to this

index	time	No Drag				Drag			
		velocity	position	net force	acceleration	velocity	position	net force	acceleration
0	0	0	0	-1.372	-9.8	0	0	-1.372	-9.8

Figure 13.2: Example of the first row of the spreadsheet

where the values mg and g reference the cells where these constants have been defined. The value of successive cells in the n^{th} row of data should be calculated from the following formulas:

- (a) Index: n
- (b) Time: $t_n = t_{n-1} + \Delta t$
- (c) Exact solution to velocity without a drag force acting: $v_n = -gt_n$
- (d) Exact solution to distance without a drag force acting: $y_n = -\frac{1}{2}gt_n^2$

- (e) Net force for numerical solution without drag force acting: $F_n = -mg$
- (f) Acceleration for numerical solution without drag force acting $a_n = F_n/m$
- (g) Numerical solution to velocity with drag force acting: $v_n = v_{n-1} + a_{n-1}\Delta t$
- (h) Numerical solution to distance with drag force acting: $y_n = y_{n-1} + v_{n-1}\Delta t$ (the $\frac{1}{2}a\Delta t^2$ term can be neglected for sufficiently small timesteps)
- (i) Net force for numerical solution with drag force acting: $F_n = 0.22d^2v_n^2 - mg$
- (j) Acceleration for numerical solution with drag force acting $a_n = F_n/m$

Plot velocity and position as functions of time for at least 32 points. Highlight the column of time values. Holding the ctrl key to skip columns, also highlight the two columns of velocity values. Then select Chart from the Insert menu and Excel will guide you through the plotting options. Show on this graph the terminal velocity you calculated. Also plot distance as a function of time, again comparing the drag-free case to the realistic solution.

3. **Effect of Timestep.** Set the diameter d to zero in your spreadsheet. This has the effect of eliminating air resistance from the numerical solutions (if this causes division by zero errors in your cells, don't worry, the issue will be resolved once you manually set the timestep Δt). We will observe how changing the time step affects the accuracy of these solutions. Set the time step to 2 seconds. Record the exact position of the ball at 2 seconds and the numerical solution to the position at 2 seconds. Reduce your time step by a factor of 2 (i.e. set it to 1 second) and again record the exact and numerical solution to the position at 2 seconds (because the time step has changed, these values will be in a different row than before). Continue decreasing the time step by factors of 2 down to 1/16th of a second, recording the position of the ball at 2 seconds for the exact and numerical solutions for each time step. Determine the percent error in the numerical solution at each time step by comparing it to the exact value. Below what value for the time step does the error decrease to less than 10%? How does this compare to the value of Δt used in step 3b? Explain why smaller time steps give less error and why there is diminishing value to having time steps less than $\delta t = 0.1v_T/g$

Data Table

Timestep	Exact value for $y(2\text{ s})$	Numerical solution to $y(2\text{ s})$	% difference
2 s			
1 s			
$\frac{1}{2}$ s			
$\frac{1}{4}$ s			
$\frac{1}{8}$ s			
$\frac{1}{16}$ s			

Report

In addition to the standard elements of a well written lab report described in the introduction to this manual, your report must include:

1. A combined graph including curve of exact velocity without drag vs. time, and the curve of numerical solution to the velocity with drag vs. time. Indicate on the graph what the terminal velocity is and what the mass and diameter of the ball are for the results appearing in the graphs.
2. A combined graph including the distance fallen as a function of time for the exact solution without drag and the numerical solution with drag.
3. Printout of the spreadsheet data table.
4. A conclusion discussing what you have learned regarding how the time scale used in numerical calculations affect the accuracy of the results.

Appendix A

Example Lab Reports

The following two example lab reports show a poorly written report and a well written report for a fictitious lab. Both use the same set of data and both are about the same length. Compare and contrast these two reports. Which of these two reports more clearly communicates the meaning and results of the lab? How many examples of bad style can you find in the poorly written report?

Example of a well written lab report

The Universal Gravitational Constant

Background:

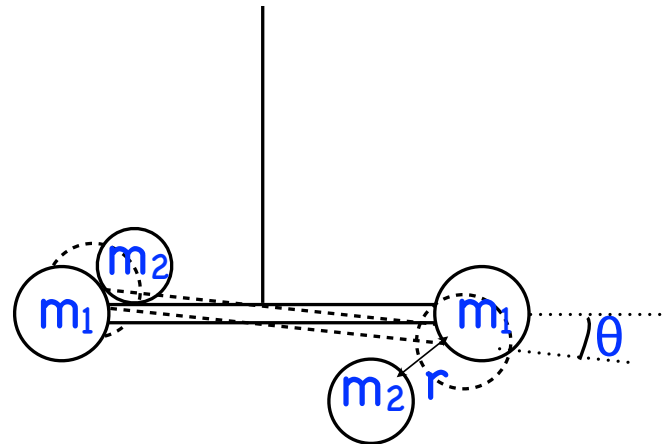
Newton's law of gravitation describes the gravitational force between massive objects. The evolution of the universe, the motion of the planets, and even the behavior of everyday object can be explained using Newton's equation for the gravitational force,

$$\vec{F} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}$$

which says the gravitational force between two objects is directly proportional to the mass of the objects and inversely proportional to their separation squared. Knowledge of the value of the constant of proportionality, G (called the universal gravitational constant), is fundamental to being able to apply this equation, and was experimentally measured in this lab.

Experimental Apparatus

A "torsional pendulum" was used as a tool for measuring the force of attraction between two pairs of massive objects. The torsional pendulum consists of barbell like structure with 1kg masses (m_1 in the figure) on either end of a 30cm rod of negligible mass. The structure is suspended from its center of mass by a thin wire of length 1m, and the angle of the structure in the horizontal plane is measured using capacitive sensors.



The angular displacement of the torsional pendulum caused by a given torque is calibrated by attaching a string to each end of the barbell and running it over a pulley on the edge of the table so that a mass hanging off the string can supply a known tension to the string, while the change in angular displacement is recorded.

Once the torsional pendulum is calibrated, a pair of large test-masses (m_2 in the figure) is placed a distance r from the ends of the barbell so that the gravitational force between each of the

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masses and each end of the barbell causes a rotation of the torsional pendulum that can be measured to determine the gravitational force. For various mass test-masses and test-mass–barbell separations the gravitational force is measured and the value. The value of G is determined from the slope of a plot of force versus m_1m_2/r^2_{12} .

Data

displacement vs. force calibration

Trial	1	2	3	4
Hanging mass (± 0.01 g)	1.00 g	2.00 g	3.00 g	4.00 g
Tension ($\pm 0.1 \times 10^{-3}$ N)	9.80×10^{-3} N	19.6×10^{-3} N	29.4×10^{-3} N	39.2×10^{-3} N
Angular Displacement (± 0.01 mrad)	0.99 mrad	2.01 mrad	2.87 mrad	4.02 mrad

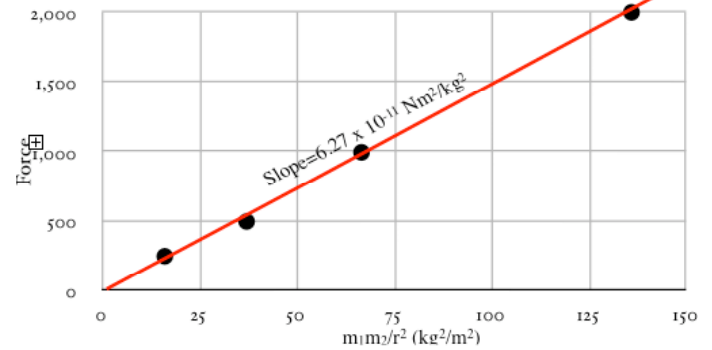
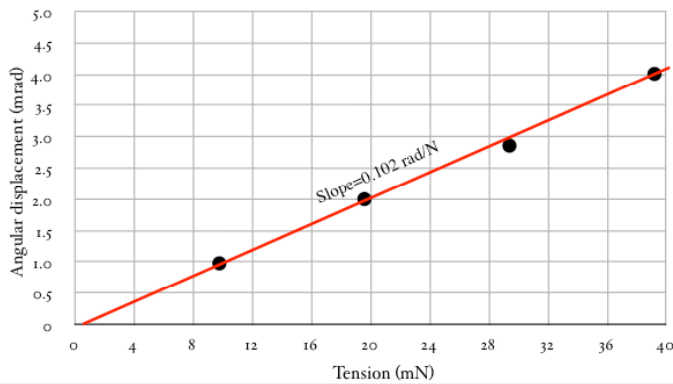
gravitational force measurements

Trial	1	2	3	4
Test mass	10.0 kg	10.0 kg	20.0 kg	20.0 kg
Separation	10 ± 1 cm	20 ± 1 cm	10 ± 1 cm	20 ± 1 cm
Angular Displacement (± 0.01 nrad)	6.81 nrad	1.65 nrad	13.9 nrad	3.80 nrad
Force ($\pm 0.1 \times 10^{-3}$ N)	66.7 nN	16.2 nN	136 nN	37.2 nN

Calibration curve

Force vs displacement measurements

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Analysis

The torsional pendulum was found to have a rotation of 0.102 rad per Newton of force acting on each of the barbell masses. The calibrated response of the torsional pendulum was used to determine the force producing the measured angular displacement of the pendulum for 4 different combinations of test masses and test mass positions. When m_1m_2/r^2 is plotted versus the measured force for each case the slope of the best-fit line is G . G can be expressed as

$$G = \frac{F}{(m_1m_2/r^2)}$$

with uncertainty

$$\delta G = G \sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta m_2}{m_2}\right)^2 + 2\left(\frac{\delta r}{r}\right)^2}$$

and evaluates to $0.9 \times 10^{11} \text{ Nm}^2/\text{kg}^2$. When the results of the 4 measurements are averaged the resulting uncertainty is

$$\delta G_{avg} = \frac{\sqrt{(\delta G_1)^2 + (\delta G_2)^2 + (\delta G_3)^2 + (\delta G_4)^2}}{4}$$

giving an uncertainty in the average of $0.45 \times 10^{11} \text{ Nm}^2/\text{kg}^2$, or 7% of the measured value of G . Taking this uncertainty into account, the measured value of G is $6.27 \pm 0.45 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.

Comparing the measured value for G to the accepted value $G_{acc} = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ gives a percent difference of

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$$\left| \frac{6.67 - 6.27}{6.67} \right| \times 100\% = 6\%$$

which is less than the 7% uncertainty in the measured value, therefore the measured value of G, the universal gravitational constant, agrees with the accepted value.

Example of a poorly written lab report

Objective

In this lab we will use weights hanging off a string to calibrate the response of a torsion pendulum. Then we will put a 10kg and a 20kg weight next to the torsion pendulum and measure its deflection when the weights are 10cm and 20cm apart. We will plot the value for m_1m_2/r^2 versus force and use the graph to determine the value for G.

Data

displacement vs. force calibration

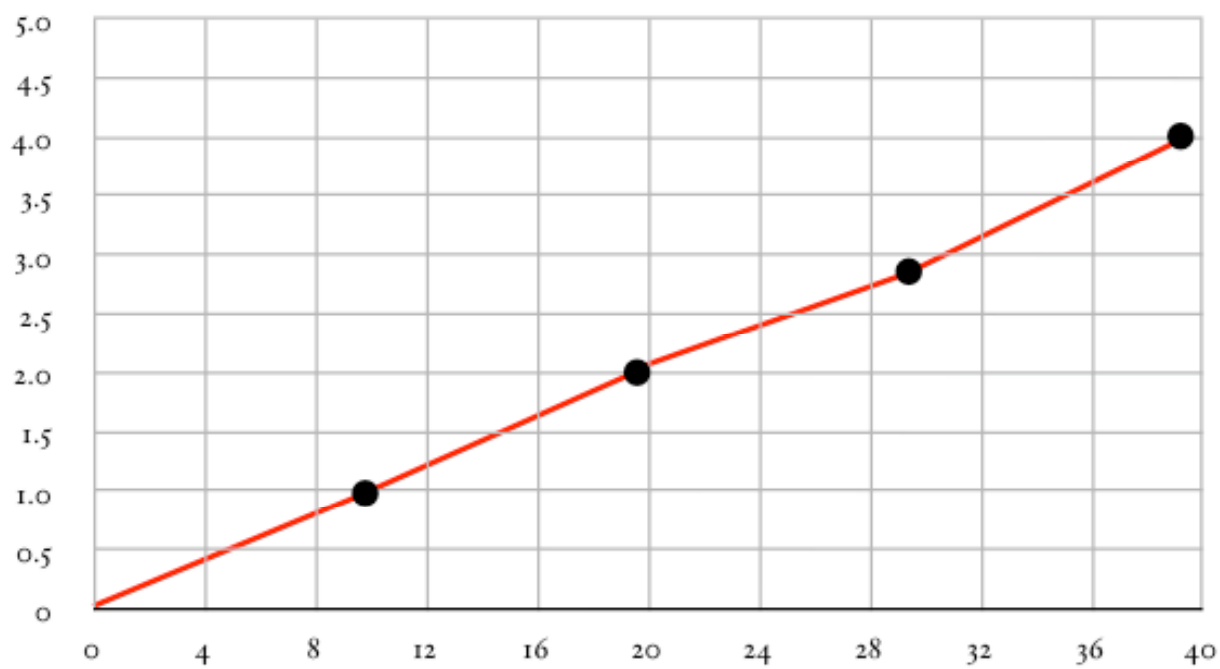
Trial	1	2	3	4
Hanging mass	1	2	3	4
Tension	9.80×10^{-3}	19.6×10^{-3}	29.4×10^{-3}	39.2×10^{-3}
Angular Displacement	0.99	2.01	2.87	4.02

gravitational force measurements

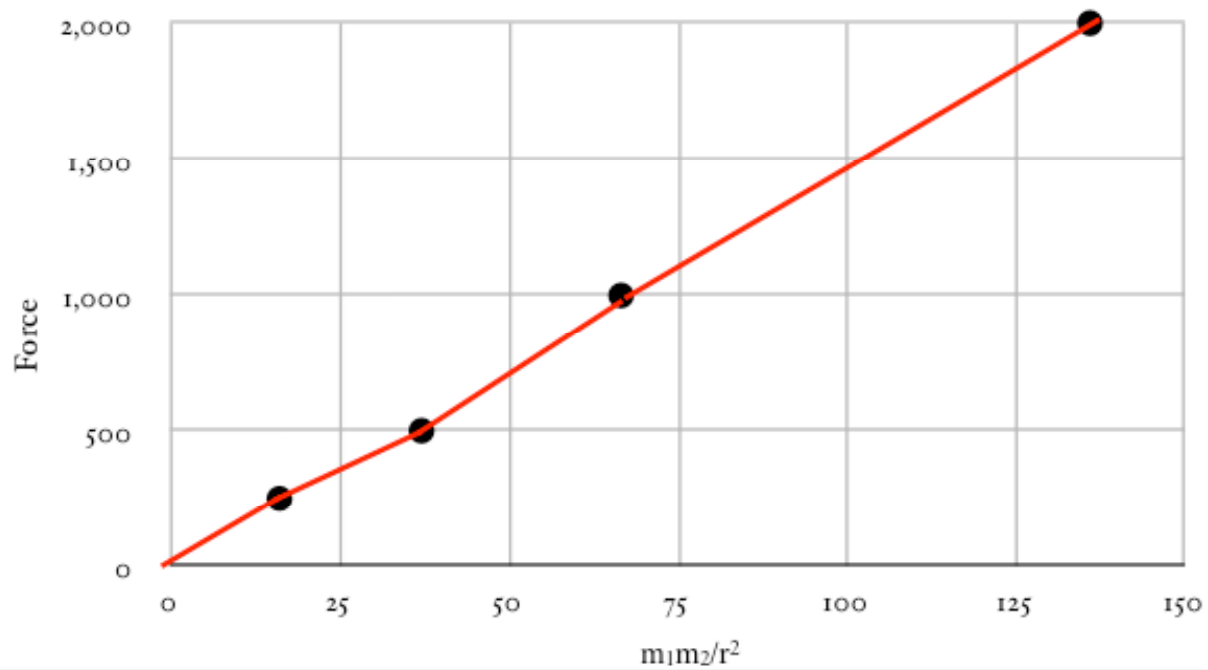
Trial	1	2	3	4
Test mass	10 kg	10 kg	20 kg	20 kg
Separation	10 cm	20 cm	10 cm	20 cm
Angular Displacement	6.81 nrad	1.65 nrad	13.9 nrad	3.8 nrad
Force	66.7102 nN	16.1958 nN	136.1004 nN	37.1697 nN

Example of Poorly Written Lab Report

Graph 1



Graph 2



Analysis

We measured a value of G that was 6.27×10^{-11} , which is pretty close to the accepted value of 6.67×10^{-11} . The percent error is

$$\left| \frac{6.67 - 6.27}{6.67} \right| = 0.06$$

The reason for the difference was probably due to a mistake that we made measuring the distance between the masses. If we had been more careful we probably could have measured it better. Also another source of error was friction in the string and our equipment was kind of old.

Appendix B

An Introduction to Error Analysis

An important part of presenting experimental data is the treatment of the uncertainty in the data. This document provides a brief introduction to the topic and has a few examples that will be useful references when you are preparing your own lab reports.

Uncertainty in *measured* quantities

Whenever you make a measurement there is uncertainty. Any error in the measurement can be minimized by using care and proper experimental techniques, but it cannot be eliminated. For example, if you are determining the mass of an object, you should be careful to “zero” the scale before putting the object on it, and you should ensure the measurement you make is repeatable, this will minimize the uncertainty in your measurement of the mass. When you measure length using a meter stick, you should not measure from the end of the meter stick, which is often worn down, but rather from a line that is more clearly defined, such as the 1 cm line (you’d then have to subtract 1 cm from the value you read on the meter stick). Virtually all other types of measurements that you will make will require similar care and thought to minimize the error in the measurements. In practice when we make a measurement what we are actually determining is a range of values that we believe contains the actual value.

When the error in a measurement is minimized, what is left is the inherent uncertainty. It is necessary to estimate the size of this uncertainty and record it with the value of your measurement in your data table. If we measure a quantity a with an uncertainty δa we

express the result as $a \pm \delta a$. Both the measured value a and its uncertainty δa will have the same units, so for example if a spring is measured to exert 10.2 N of force with an uncertainty of 0.3 N the measured results would be expressed as $F = 10.2 \pm 0.3$ N. The size of the uncertainty (δa) is usually chosen to meet the following criteria: If 100 people were to carefully repeat your measurement using the same equipment, 68 of them should measure a value that is within the uncertainty of your measured value (i.e. between $a - \delta a$ and $a + \delta a$.) A more useful, although not as precise, method for estimating the uncertainty in a measurement is as follows: you make a measurement of some quantity and then have your lab partner measure the same quantity. The difference between your measurements is an estimate of the uncertainty. Be careful though - if both of your measure exactly the same value, this doesn't mean there is no uncertainty - if nothing else the uncertainty will always be as large as half of the least significant digit in your measured value. For example, if you and your lab partner both measure the mass of an object to be 123 g on a scale, then because you are only reading a value to the nearest gram if the actual mass were as low as 122.5 g or as high as 123.5 g your scale would still read it as 123 g so your uncertainty is ± 0.5 g even though you both measured the same value.

When using any measurement device the measurement uncertainty cannot be less than half of the smallest scale increment (i.e. on a meter stick with tick marks every mm, the uncertainty cannot be less than 0.5 mm) regardless of how carefully the measurement is taken, that is because all measurement devices have this inherent uncertainty. That being said it is possible to make a careful measurement with an uncertainty larger than the minimum increment on the scale of the measuring device. For example, if you were to measure the circumference of a disk by wrapping a string around the disk, marking the length of the string and then straightening it out to measure with a meter stick, you might find that the string stretches when it is around the disk, making it difficult to ensure the same length when it is straightened out. In this case you would have to estimate how much the string stretches, and use that as your uncertainty. Again this can be checked by observing the variation in repeated measurements.

Uncertainty in *calculated* quantities

While it is important to note the uncertainty in measured quantities, it is also necessary to use those values to determine how much uncertainty there is in any quantity calculated from those measurements. There are two basic premises that govern how the uncertainty in measured quantities affects the uncertainty in any calculated quantity, the first is that for a function $f(a, b, c, \dots)$ the uncertainty in the function f caused by the uncertainty in the

measured quantity a is given by

$$\delta f_a = \left(\frac{\partial f}{\partial a} \right) \delta a,$$

with similar expressions for the uncertainty due to b , c , etc. The second premise is that if the measured values a , b , c , etc. are *uncorrelated* the uncertainty in f from each adds *in quadrature* as follows

$$(\delta f)^2 = (\delta f_a)^2 + (\delta f_b)^2 + (\delta f_c)^2 + \dots$$

Uncertainties are uncorrelated when they are completely independent of each other. For example if you measure the mass and acceleration of an object to determine the force on it, your measurement of mass should have an uncertainty that has nothing to do with the uncertainty in your measurement on acceleration. An example of correlated uncertainties would be if you measured the radius of a circle and then doubled that to get a value of diameter that you consider a “measured” value - any error in the measurement of the radius would then produce the same relative error in the recorded value for the diameter, so these uncertainties are *correlated*.

When the above two premises are applied to functions that involve the sum or difference of two or more quantities, such as $f(a, b, \dots) = a + b + \dots$ or $f(a, b, \dots) = a - b - \dots$ the resulting uncertainty in f due to each term is found from

$$\begin{aligned} \delta f_a &= \frac{df}{da} \delta a \\ \delta f_b &= \frac{df}{db} \delta b \end{aligned}$$

where for $f(a, b, \dots) = a \pm b + \dots$ and $\frac{df}{da} = \frac{df}{db} = \pm 1$ with the sign depending on if b is being added or subtracted. This gives $\delta f_a = \delta a$ and $\delta f_b = \pm \delta b$. When adding these uncorrelated uncertainties using $(\delta f)^2 = (\delta f_a)^2 + (\delta f_b)^2 + \dots$ we get the general expression

$$(\delta f)^2 = (\delta a)^2 + (\delta b)^2 + \dots \tag{B.1}$$

Sum and difference formula for uncertainty

If the function $g(a, b, \dots) = a \times b \times \dots$ or $g(a, b, \dots) = a/b \dots$ is the product or quotient of two or more quantities the resulting uncertainty in g is found from

$$\begin{aligned} \delta g_a &= \frac{dg}{da} \delta a \\ \delta g_b &= \frac{dg}{db} \delta b \end{aligned}$$

where for $g(a, b) = ab$ and $\frac{dg}{da}b, \frac{dg}{db} = a$. This gives $\delta g_a = b\delta a$ and $\delta g_b = a\delta b$. When adding these uncorrelated uncertainties using $(\delta g)^2 = (\delta g_a)^2 + (\delta g_b)^2$ we get

$$(\delta g)^2 = (b\delta a)^2 + (a\delta b)^2$$

dividing on the left by g^2 and on the right by $(ab)^2$ (which we can do since $g = ab$ and generalizing for more factors gives the general expression

$$\left(\frac{\delta g}{g}\right)^2 = \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \dots \quad (\text{B.2})$$

Product and quotient formula for uncertainty

If formula 2.1 is applied to the N measurements of the same quantity a , each with identical uncertainty δa , the uncertainty in the average of those N results, \bar{a} , is reduced according to

$$\delta \bar{a} = \frac{\delta a}{\sqrt{N}} \quad (\text{B.3})$$

Formula for uncertainty in an average

Note that the above expressions do not give the uncertainty δf directly, but rather are written in a way that is easier to remember but requires that you solve the expressions for δf to determine the uncertainty that you will quote in the calculated quantity. These rules can be applied in turn on more complicated functions. For example the uncertainty in a function that involves both the sum and product of measured quantities $h(a, b, c) = a \times b + c$ can be determined by first finding the uncertainty in the individual components, i.e. find $\delta g(a, b)$ for $g(a, b) = a \times b$ using formula (2) and then rewrite h as $h(g, c) = g + c$ and apply formula (1).

Interpretation of results with uncertainty

Often times the purpose of lab experiments is to experimentally confirm the value of a “known” quantity (such as the acceleration due to gravity) or to compare a quantity that is measured in two different ways (for example the speed a bullet leaves a gun based on a measurement of its trajectory as well as a measurement of its ability to displace a large mass). When comparing two quantities that each have uncertainty (if one of the quantities

is “known” that means the uncertainty is negligible) we can say the measured values “agree” if the range of values for the two measurements overlap, that is if there is one possible value that falls within the uncertainty of both measurements. Thus if we compare a quantity $a \pm \delta a$ to another quantity $A \pm \delta A$ we say they agree if $|a - A| < \delta a + \delta A$ (i.e. the quantities differ by less than the total uncertainty, so they *could* be a measure of the same value.

When the above criteria is not met, i.e. $|a - A| > \delta a + \delta A$ the measurements “do not agree” within the experimental uncertainty. It is useful to describe how much disagreement there is, one meaningful way of doing this is to state how far they were apart relative to the uncertainty

$$\text{relative error} = \frac{|a - A|}{\delta a + \delta A} \sigma,$$

where σ is the greek letter sigma and stands for “standard deviation”. We can think of it as just our unit of relative uncertainty just as the percent sign is our unit of percent error. For example if two measurements have a difference that is twice the total uncertainty we would say the measurements differed by 2σ . Another method is to describe the percent error between the values

$$\text{percent error} = \frac{|a - A|}{(a + A)/2} \times 100\%$$

and also express the total uncertainty as a percentage for comparison

$$\text{percent uncertainty} = \frac{\delta a + \delta A}{(a + A)/2} \times 100\%$$


These types of *objective* statements about the measurement and uncertainty are an important part of a report. *Subjective* statements such as “the measured value was 0.2% off from the expected value so the measurement was successful” are not useful and often lead to incorrect conclusions - without knowledge of the uncertainty it is impossible to tell if an error is significant, even if it *seems* small. Scientific reports have no room for subjective interpretation of what is significant and what is not. With the tools for error analysis presented here there is no reason for such subjective statements.

Appendix C

Logger Pro Tutorial

Many of the experiments in this manual rely on the LabQuest Data Logger to collect data and on Logger Pro to analyze the data. You can find a quick reference manual online at <http://www2.vernier.com/manuals/LP3QuickRefManual.pdf>, and you can download Logger Pro for use on your own computer at <http://tinyurl.com/sjsuLP> (you will need to authenticate via your SJSUone account - you need to sign out of gmail before you can do this.). The instructions below represent just the most common steps you will be asked to do.

Importing data

1. Click  on the data logger device to start recording data and begin to move the force sensor in and out to raise and lower the mass for 10 seconds while the LabQuest data logger collects force and acceleration data. Do not let the mass touch the floor at any point during the data collection.
2. Transfer the collected data onto one of the lab computers for analysis by connecting it via a USB cable and opening “Data Logger Pro” on the computer. The device will be auto-detected and you will be prompted to import the data. If you are successful you can skip step 3k, otherwise see the instructions for manually importing data in the next step.
3. If your data did not import automatically, save your data on the handheld device by selecting “save” from the “file” menu. Then when it is connected to the computer,

select “LabQuest Browser” from Logger Pro’s “File” menu and select the file that you just saved.

Entering Data

Data can be classified in two forms: constants and variables, constant values (such as the mass of an object) are called “User Parameters” in Logger Pro, while variables are called “Columns” because the values the variable takes are organized as cells in a column of the data table.

1. To enter a constant into Logger Pro select “User parameters...” from the “Data” menu, and add a parameter giving it an appropriate name and recording the numerical value.
2. To manually enter the values for a variable click ‘New manual column...’ from the “Data” menu. Enter an appropriate name (and short name). You can have Logger Pro populate each cell with a preset numerical value by clicking the “Generate Values” checkbox and entering the appropriate parameters, or you can simply type the numerical values into the cells in the column after the column is created.
3. To calculate a quantity based on a variable you will create a new column for the quantity. Select “New calculated column...” from the “Data” menu. Give the new column an appropriate name (in both the “Name:” field and the “Short Name:” field). In the equation panel create an expression for this column of the form that you wish to calculate. For instance if you wanted to calculate the value $x = r \cos \theta$ where r is a constant and θ is a variable recorded in another column, you would select “r” from the “parameters” drop down menu, then type “*” to represent multiplication, then you would select “cos” from the “functions” drop down menu (it is in the “trigonometric” submenu - alternatively you could type “cos()” directly). Finally you would place your cursor inside the parenthesis of the cosine function and select the appropriate column name that represents θ from the “variables (columns)” drop down menu.

Graphing Data

When you begin using logger pro a graph is automatically generated for each of the initial columns. If you need to change the quantities plotted it may be easiest to change the default

graphs rather than to create new graphs from scratch, if this is the case you can skip step 3a below

1. To create a graph from scratch select “Graph” from the “Insert” menu and then customize it as described below.
2. To change the quantities plotted on a graph click on the label for the x or y axis and select the new quantity you wish to plot. If you want to plot more than one quantity, click on the y-axis and select “More”, then click the checkbox for every quantity you want to include on the plot.
3. To change the appearance of the graph double click on the graph to bring up the graph/axes options window. You might choose to turn off connecting lines or show error bars from the “graph options” tab, or you might choose to change the scale of the axes from the “axes options” tab.

Curve Fitting

While we often want to fit a particular function to the data on our graph and can do so through the “curve fit...” option of the “Analyze” menu, when we want to fit a straight line to the data we can use the short cut of selecting “linear fit” instead.

1. When fitting a general curve you will first have to choose the form of the function from the “General Equation” pane. You can then manually enter parameters that you wish to use for the fit or you can click “Try Fit” to have logger pro select the parameters that gives the best fit.
2. Once the best fit curve is drawn on the graph a box appears showing the fit parameters. You can double click on the box to bring up a menu that lets you select how the fit parameters are presented, including whether to show the uncertainty of the fit parameters.
3. By clicking and dragging on the brackets that appear around the data you can select a limited portion of the data to use in the curve fit. This could be useful for example if you had some outlying data points at the beginning or end of your data set that you did not want to influence the fitting routine.

Expressing Uncertainty

We can express the uncertainty in our data points by including *error bars* on the plot. These vertical and/or horizontal bars around the data points represent the range of y and/or x values respectively that are within the experimental uncertainty associated with the measured data point. To include these we need to tell Logger Pro how big our errors are and then make sure it is set to show error bars on the plot

1. Click on the heading of the column that you wish to enter the uncertainty for. In the “Options” tab, check the “error bar calculations” box. You can enter a fixed error value, a percentage, or you can point it to a column that holds the error values for each point.
2. Double click on the graph to bring up the graph options window and turn on the option to show the x and y error bars.