8: Independent Samples

Data

In the previous chapter we looked at paired samples. In this chapter, we look at independent samples. With independent samples, data points in the two samples represent random samples from distinct populations. To review, we have addressed three different sampling situations this semester:

- Single samples, which reflect the experience of a single group.
- Paired samples, in which each data point in one sample is uniquely matched to a data point in the second sample.
- Independent samples, which represent random samples from two separate populations.

Illustrative data (PETS). It has been suggested that pets provide a social support in buffering adverse responses to psychological stressors. Thirty pet owners volunteer to participate in an experiment. All will be subjected to a psychological stressor. Half (group 1) will have their heart rate monitored in the presence of their pet. The other half will have their heart rate monitored in the presence of one of their human friends (Allen 1991). Data are:

Group 1	Group 2
69.2	99.7
68.9	91.4
70.2	83.4
64.2	100.9
58.7	102.2
79.7	89.8
69.2	80.3
76.0	98.2
86.4	101.1
97.5	76.9
85.0	97.0
69.5	88.0
70.1	81.6
72.3	87.0
65.4	92.5

Descriptive Statistics

Start by describing the groups separately with summary statistics. Use subscripts in the summary statistic symbols to denote group membership. Calculate these statistics with your calculator or whatever computational device your instructor permits. For the current data:

$\bar{x}_1 = 73.49$ $s_1 = 9.959$ $n_1 = 15$
$\bar{x}_2 = 91.32$ $s_2 = 8.329$ $n_2 = 15$

Four significant digits are maintained because these statistics will be required in future calculations.

The group medians are 70 and 91 (calculated below), which are similar to their respective means. Therefore, the distributions are roughly symmetrical.

Before quartiles can be calculated, data are sorted into rank-order.

Data in rank order:

Rank / depth	Group 1	Group 2	
1	58.7	76.9	←Q0
2	64.2	80.3	
3	65.4	81.6	
4	68.9	83.4	← Q1
5	69.2	87.0	
6	69.2	88.0	
7	69.5	89.8	
8	70.1	91.4	← Q2
9	70.2	92.5	
10	72.3	97.0	
11	76.0	98.2	← Q3
12	79.7	99.7	
13	85.0	100.9	
14	86.4	101.1	
15	97.5	102.0	←Q4

Both sample have n = 15. Therefore, the depths of the medians will be at rank (15 + 1) / 2 = 8. The median of group 1 is 70.1. The median of group 2 is 91.4.

The depths of the Q1s will be at (8 + 1) / 2 = 4.5 from the bottom (smallest) number. The Q3s will be at this depth from the top (largest) number.

Thus, for group 1:
Q0 = 58.7
Q1 = avg(68.9, 69.2) = 69.05 = 69.1
Q2 = 70.1
Q3 = avg(79.7, 76.0) = 77.85 = 77.9
Q4 = 97.5

For group 2:
Q0 = 76.9
Q1 = avg(83.4, 87.0) = 85.2
Q2 = 91.4
Q3 = avg(99.7, 98.2) = 77.85 = 98.95 = 99.0
Q4 = 102.0

Reporting and interpretation. When the distributions are roughly symmetrical, report group means, standard deviations, and sample sizes. Otherwise, defer to the 5-point summaries. Round results appropriately *when reporting*. For example: "Group 1 had much lower heart rates on average (means, 73.5 vs. 91.3 bpm). The groups demonstrated comparable variability (standard deviations, 10.0 vs. 8.3)."

Exploratory graphs

Stemplots

Side-by-side stemplots are useful for exploring distributions and group differences. To facilitated comparisons, data can be plotted on a common stem:

```
       8 | 5 |
  999854 | 6 |
   96200 | 7 | 6
      65 | 8 | 013789
       7 | 9 | 12789
         |10 | 012
          x10
```

Interpretation. Group 1 typically had much lower heart rates (location). The groups have comparable variability (spreads). The distributions are mound shape and relatively symmetrical (shape).

Boxplots

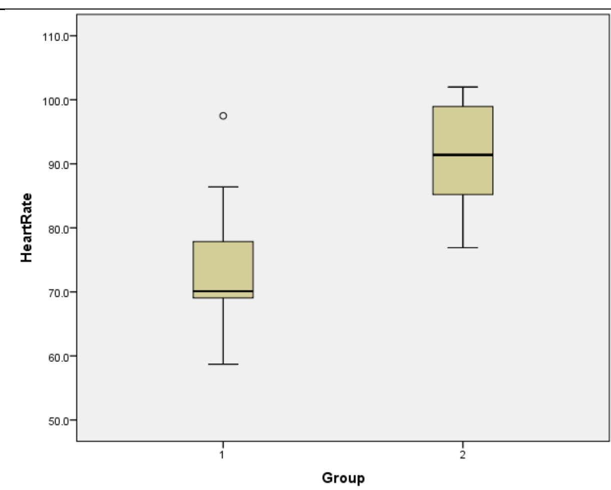
Side-by-side boxplots make nice displays but can be labor intensive to do by hand because we must first check for outside and inside values. (If you do not remember how to properly create or interpret a boxplot, review Chapter 3.)

Recall the ordered array and 5-point summaries on the prior page:

Group 1:	58.7	69.1	70.1	77.9	97.5
Group 2:	76.9	85.2	91.4	99.0	102.0

The IQR for group 1 $IQR_1 = 77.9 – 69.1 = 8.8$. The lower fence $F_L = 69.1 – (1.5 \cdot 8.8) = 69.1 – 13.2 = 55.9$. Q0 was 58.7, so there are no outside values on the bottom and 55.9 is the lower inside value. The upper fence $F_U = 77.9 + (1.5 \cdot 8.8) = 91.1$. Hence the value of 97.5 is an outside value and the value 86.4 is the upper inside value.

The IQR for group 2 $IQR_2 = 99.0 – 85.2 = 13.8$. The lower fence $F_L = 85.2 – (1.5 \cdot 13.8) =85.2 – 20.7 = 64.5$. Q0 was 58.7, so there are no outside values and 76.9 is inside. The upper fence $F_U = 99.0 + (1.5 \cdot 13.8) = 119.7$. Hence, there are none outside on top and 102.0 is the upper inside value.



"A picture is worth 1,000 words, but to be so, it may have to include 100 words" (Tukey, 1986, p. 74). How would you describe the above plot?
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Inferential statistics

Parameter and point estimate of effect

The effect of the intervention or exposure is quantified by the **independent mean difference**. This **parameter** is denoted $\mu_1 - \mu_2$. The **point estimator** is $\bar{x}_1 - \bar{x}_2$. In our example, the point estimate of effect = 73.49 – 91.32 = −17.83 beats per minute (i.e., lower heart rate in pet-exposed group).

NHST

We may ask if the observed mean difference is noteworthy or if it is "random"? The **null hypothesis** is $\boldsymbol{H_0: \mu_1 - \mu_2 = 0}$.

The test (called the **independent *t* test**) is performed with this *t* statistic:

$$t_{\text{stat}} = \frac{\text{observed mean difference} - \text{expected mean difference when } H_0 \text{ true}}{SE_{\bar{x}_1 - \bar{x}_2}}$$

where the observed mean difference is $\bar{x}_1 - \bar{x}_2$, the expected mean difference under H_0 is nearly always set to 0, and $SE_{\bar{x}_1 - \bar{x}_2}$ is the standard error of the independent mean difference.

The **standard error of the independent mean difference** can be calculated several different ways. We will use this conservative approach.[1]

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

For the illustrative data, $SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{9.959^2}{15} + \frac{8.329^2}{15}}$ = 3.352 and $t_{\text{stat}} = \frac{-17.83 - 0}{3.352}$ = –5.32.

Again, there are multiple ways to calculate the **degrees of freedom** for this test statistic. And again we will use *the most conservative approach*.

df = the smaller of $(n_1 - 1)$ or $(n_2 - 1)$

For the current situation, it doesn't matter which *n* we use. Either way, df = 15 – 1 = 14.

The ***P* value** is determined with the *t* app as the areas in the tails of t_{14} beyond the $\pm|t_{\text{stat}}|$. Thus, P = .0001.

Summarize the results in plain language. Include **(a)** the observed mean difference, **(b)** the direction of the difference, and **(c)** the *P* value in your concluding statement. For example, "The group that was stressed in the presence of their pets had an average heart rate that was 17.8 bps *lower* than the control group (P = .0001)."

[1] The approach is conservative because it makes the fewest assumptions and avoids overstating the significance of observed findings.