

HW0 (Algebra and Calculus Review)

This assignment is meant to test your level of preparation for this course (it is assumed that you have already learned these material).

1. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 4, 5\}$. Find $A \cap B$ and $A \cup B$.
2. In how many different ways can you perform each of the following tasks?
 - (a) Arrange 5 people in a row.
 - (b) Select 4 people from a group of 10 to work on a project.

3. Find the domain and range of the following function

$$f(x) = \frac{1}{\sqrt{4-x^2}}.$$

For which value(s) of x , is $f(x) = 1$?

4. Solve the following inequalities:

$$-1 < \frac{3-x}{2} < 2, \quad |x-3| > 5$$

5. Expand the following products:

$$(x+y)^2, \quad (x+y)(x-y)$$

6. For which values of p is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

7. Write down the result directly for each series below:

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + \dots = ? \quad (\text{assuming } |r| < 1)$$

$$\sum_{n=0}^{\infty} \frac{A^n}{n!} = \frac{1}{0!} + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \dots = ? \quad (A \text{ is any fixed real number})$$

8. Evaluate the following series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots = ?$$

Hint: Write each term as a difference of two fractions.

9. Evaluate the following integrals

$$\int_1^{\infty} \frac{2}{x^3} dx, \quad \int_0^1 x(1-x)^3 dx, \quad \int_0^{\infty} xe^{-2x} dx, \quad \int_0^{\infty} xe^{-x^2} dx$$