

Worksheet 1: Sample space, event, and probability

Example 0.1. Write down the sample space for each experiment below:

- Tossing a coin: $S =$
- Rolling a die: $S =$
- Drawing a card from a deck: $S =$

Example 0.2. Find the sample space for each experiment below:

- Throw a coin twice: $S =$
- Throw two dice: $S =$
- Throw a coin repeatedly until a head first appears:
 $S =$

Example 0.3 (Continuous sample spaces).

- Life time of a new light bulb. The sample space is an interval $S = (0, \infty)$.
- Waiting time (in minutes) to talk to a customer service representative:
 $S = (0, \infty)$
- Throw a dart to a unit disk and measure its distance to center: $S = (0, 1)$

Example 0.4 (Roll a single die). The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. The following are events:

- $A = \{1\} = \{\text{The smallest number}\}$
- $B = \{6\} = \{\text{The largest number}\}$
- $C = \{2, 4, 6\} = \{\text{An even number}\}$
- $D = \{1, 3, 5\} = \{\text{An odd number}\}$

If an outcome of 1 was observed when performing the experiment, then which of the above events occurred (and which events did not occur)?

Example 0.5 (Throw two dice). The sample space is $S = \{(i, j) \mid 1 \leq i, j \leq 6\}$. The following are events:

$$A = \{\text{Sum equals 6}\} = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$$
$$B = \{\text{Two identical numbers}\} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$
$$C = \{\text{Two even numbers}\} = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}.$$

Example 0.6. Consider the experiment where you repeatedly toss a coin until you see the first head. The following is an event: $E = \{\text{At most 4 tails occurred}\} = \{H, TH, TTH, TTTH, TTTTH\}$.

Example 0.7 (Throw two dice). Let

- $A = \{\text{Sum equals } 6\}$
- $B = \{\text{Two identical numbers}\}$
- $C = \{\text{Two even numbers}\}$

Compute $|C|, A \cap B, A \cup B, B^c, A - C$

Example 0.8 (Toss two fair dice). Are the following two events disjoint?

- $A = \{\text{Sum equals } 7\}$.
- $B = \{\text{Two identical numbers}\}$.

Example 0.9 (Fair coin model). Let $S = \{H, T\}$ with $P(\{H\}) = P(\{T\}) = \frac{1}{2}$.

Example 0.10 (Biased coin model). Let $S = \{H, T\}$ with $P(\{H\}) = .55, P(\{T\}) = .45$.

Example 0.11 (Fair die model). Let $S = \{1, 2, \dots, 6\}$ with $P(\{1\}) = P(\{2\}) = \dots = P(\{6\}) = \frac{1}{6}$. What is the probability of getting an even number?

Example 0.12 (Throw a fair die). Find the following probabilities:

$$\begin{aligned}P(\{\text{An even number}\}) &= \\P(\{\text{At least } 5\}) &= \\P(\{\text{Not a } 3\}) &= \end{aligned}$$

Example 0.13 (Toss a fair coin 5 times). What is the probability of getting at least one head? (Answer: $\frac{31}{32}$)

Example 0.14. In a large discrete math class, 55% of the students have a major in math, and 35% of the class have a major in CS. Among the two groups of students combined, 5% of them are dual majors (in math and CS). What is the probability that a randomly selected student from the class majors in

- at least one of math and CS,
- one and only one of math and CS,
- neither math nor CS?