Worksheet 1: Sample space, event, and probability

Example 0.1. Write down the sample space for each experiment below:

- Tossing a coin: \( S = \)  
- Rolling a die: \( S = \)  
- Drawing a card from a deck: \( S = \)

Example 0.2. Find the sample space for each experiment below:

- Throw a coin twice: \( S = \)  
- Throw two dice: \( S = \)  
- Throw a coin repeatedly until a head first appears: \( S = \)

Example 0.3 (Continuous sample spaces).

- Life time of a new light bulb. The sample space is an interval \( S = (0, \infty) \).
- Waiting time (in minutes) to talk to a customer service representative: \( S = (0, \infty) \).
- Throw a dart to a unit disk and measure its distance to center: \( S = (0, 1) \).

Example 0.4 (Roll a single die). The sample space is \( S = \{1, 2, 3, 4, 5, 6\} \). The following are events:

- \( A = \{1\} = \{\text{The smallest number}\} \)
- \( B = \{6\} = \{\text{The largest number}\} \)
- \( C = \{2, 4, 6\} = \{\text{An even number}\} \)
- \( D = \{1, 3, 5\} = \{\text{An odd number}\} \)

If an outcome of 1 was observed when performing the experiment, then which of the above events occurred (and which events did not occur)?

Example 0.5 (Throw two dice). The sample space is \( S = \{(i, j) \mid 1 \leq i, j \leq 6\} \). The following are events:

- \( A = \{\text{Sum equals 6}\} = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} \)
- \( B = \{\text{Two identical numbers}\} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} \)
- \( C = \{\text{Two even numbers}\} = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\} \).

Example 0.6. Consider the experiment where you repeatedly toss a coin until you see the first head. The following is an event: \( E = \{\text{At most 4 tails occurred}\} = \{H, TH, TTH, TTTH, TTTTH\} \).
Example 0.7 (Throw two dice). Let
- \( A = \{ \text{Sum equals 6} \} \)
- \( B = \{ \text{Two identical numbers} \} \)
- \( C = \{ \text{Two even numbers} \} \)

Compute \( |C|, A \cap B, A \cup B, B^c, A - C \)

Example 0.8 (Toss two fair dice). Are the following two events disjoint?
- \( A = \{ \text{Sum equals 7} \} \)
- \( B = \{ \text{Two identical numbers} \} \)

Example 0.9 (Fair coin model). Let \( S = \{ H, T \} \) with \( P(\{ H \}) = P(\{ T \}) = \frac{1}{2} \).

Example 0.10 (Biased coin model). Let \( S = \{ H, T \} \) with \( P(\{ H \}) = .55, P(\{ T \}) = .45 \).

Example 0.11 (Fair die model). Let \( S = \{1, 2, \ldots, 6\} \) with \( P(\{1\}) = P(\{2\}) = \cdots = P(\{6\}) = \frac{1}{6} \). What is the probability of getting an even number?

Example 0.12 (Throw a fair die). Find the following probabilities:

\[
P(\{ \text{An even number} \}) = \\
P(\{ \text{At least 5} \}) = \\
P(\{ \text{Not a 3} \}) =
\]

Example 0.13 (Toss a fair coin 5 times). What is the probability of getting at least one head? (Answer: \( \frac{31}{32} \))

Example 0.14. In a large discrete math class, 55% of the students have a major in math, and 35% of the class have a major in CS. Among the two groups of students combined, 5% of them are dual majors (in math and CS). What is the probability that a randomly selected student from the class majors in
(a) at least one of math and CS,
(b) one and only one of math and CS,
(c) neither math nor CS?