Worksheet 11: Eigenvalues and eigenvectors

Example 0.74. Let

\[ A = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix}, \ v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ v_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \]

Compute \( Av_i \) for \( i = 1, 2, 3 \). Are they multiples of \( v_i \)?

Example 0.75. Let

\[ A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}, \ v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}. \]

Determine if \( v \) is an eigenvector of \( A \). If yes, find the corresponding eigenvalue.

Example 0.76. Determine if \(-4\) is an eigenvalue of the matrix \( A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix} \). If yes, find all eigenvectors associated to it.

Example 0.77. It is known that the following matrix

\[ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \]

has two eigenvalues, 1 and 3. Find a basis for each of the two eigenspaces corresponding to them. What are the geometric multiplicities of the eigenvalues?
Example 0.78. The matrix

\[ A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

is not invertible because 0 is an eigenvalue

\[ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

Example 0.79. For each of the following matrices \( A \), first find an expression in \( \lambda \) for \( \det(A - \lambda I) \):

(a) \( A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \)

(b) \( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \)

and then use it to find the eigenvalues of \( A \) and corresponding algebraic multiplicities.

Example 0.80. Find the eigenvalues of

\[ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix} \]

This is an example of a matrix that has complex eigenvalues.

Example 0.81. Determine the eigenvalues of the following matrix

\[ A = \begin{bmatrix} 1 & 2 \\ 3 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & 2 \\ 5 & 6 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 \\ 3 \end{bmatrix}. \]