Worksheet 12: Similar matrices and diagonalization

Example 0.82. Verify that

\[
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix} = \begin{pmatrix}
3 & 1 \\
1 & 1
\end{pmatrix}^{-1} \begin{pmatrix}
2 & -3 \\
1 & -2
\end{pmatrix} \begin{pmatrix}
3 & 1 \\
1 & 1
\end{pmatrix}
\]

This shows that \(A\), \(B\) are similar to each other.

Example 0.83. Let

\(A = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}, \quad B = \begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix}\)

Show that they have the same characteristic polynomial and thus the same eigenvalues, but they are not similar.

Example 0.84. The matrix

\(A = \begin{pmatrix}
0 & 1 \\
3 & 2
\end{pmatrix}\)

is diagonalizable because

\[
\begin{pmatrix}
0 & 1 \\
3 & 2
\end{pmatrix} = \begin{pmatrix}
1 & 1 \\
3 & -1
\end{pmatrix} \begin{pmatrix}
3 & 1 \\
1 & -1
\end{pmatrix}^{-1}
\]

but the matrix

\(B = \begin{pmatrix}
0 & 1 \\
-1 & 2
\end{pmatrix}\)

is not (we will see why later).

Example 0.85. For the diagonalizable matrix \(A\) in the preceding example, find its 10th power, i.e., \(A^{10}\).
Example 0.86. The matrix \( B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} \) is not diagonalizable because it has one distinct eigenvalue \( \lambda_1 = 1 \) with \( a_1 = 2 \) and \( g_1 = 1 \) (only one linearly independent eigenvector).

Example 0.87. Is the following matrix diagonalizable? If yes, find the eigendecomposition.

\[
A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & -1 \\ -2 & 2 & 4 \end{bmatrix}.
\]

Example 0.88. Is the following matrix diagonalizable? If yes, find the eigendecomposition.

\[
A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ -2 & 2 & 4 \end{bmatrix}.
\]