Worksheet 9: Coordinate system

Example 0.61. Find the coordinate vector of \( \mathbf{x} = [2, 5]^T \in \mathbb{R}^2 \) relative to the basis given by the columns of \( \mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \).

Example 0.62. We have previously showed that the columns of the matrix form a basis for \( \mathbb{R}^3 \):
\[
\mathbf{A} = \begin{bmatrix} 3 & -4 & -2 \\ 0 & 1 & 1 \\ -6 & 7 & 5 \end{bmatrix}
\]
and for \( \mathbf{b} = [1 \ 0 \ 2]^T \in \mathbb{R}^3 \), we obtained that
\[
\mathbf{b} = (-1)\mathbf{a}_1 + (-2)\mathbf{a}_2 + 2\mathbf{a}_3.
\]
Therefore, the coordinates of \( \mathbf{b} \) relative to the basis (columns of \( \mathbf{A} \)) are \( [-1, -2, 2]^T \).

Example 0.63. Let \( V = \{ \text{all polynomials of degree at most 2} \} \). Then \( V \) is a vector space with basis \( \mathcal{B} = \{1, t, t^2\} \). The coordinate mapping from \( V \) to \( \mathbb{R}^3 \) is
\[
\mathbf{v} = c_0 + c_1 t + c_2 t^2 \mapsto [\mathbf{v}]_\mathcal{B} = [c_0, c_1, c_2]^T \in \mathbb{R}^3.
\]
It can be shown that this is a 1-to-1 linear transformation (isomorphism) from \( V \) to \( \mathbb{R}^3 \) and the two vector spaces are identical algebraically.

Example 0.64. Let \( \mathbf{v}_1 = [1, 1]^T \). Then \( \mathcal{B} = \{\mathbf{v}_1\} \) is a basis for \( H = \text{Span}\{\mathbf{v}_1\} \subset \mathbb{R}^2 \). Determine if \( \mathbf{x} = [5, 5]^T \) is in \( H \), and if yes, find its coordinate vector relative to \( \mathcal{B} \).

Example 0.65. Let \( \mathbf{v}_1 = [1, 1, 0]^T, \mathbf{v}_2 = [1, 0, 1]^T \). Then \( \mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\} \) is a basis for \( H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\} \subset \mathbb{R}^3 \). Determine if \( \mathbf{x} = [3, 2, 1]^T \) is in \( H \), and if yes, find its coordinate vector relative to \( \mathcal{B} \).