ENGR 1540: Physics Workshop (Engineering Special Topics)
Drawing and Analyzing Vectors
Lesson Plan
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Overview: Physics deals with the laws that govern our physical world and links the study of energy, matter, force, and motion. In fact, a large part of physics involves using visual and mathematical models to describe the motion of objects as they move through space over a duration of time. When analyzing the motion of an object, we need a way to describe the magnitude (size) and direction of quantities such as displacement (change in position), velocity (change in position with respect to time), acceleration (change in velocity with respect to time), and force (a natural influence that causes the motion or shape of an object to change). Vectors allow for such a description by indicating size and direction and play an important role in physics problem-solving applications.

Introduction

The analysis of vectors is dependent on trigonometry. Trigonometry, which unifies geometry, algebra, and physics, plays an important role in vector analysis especially as it concerns vector addition. We will use an x-y-z coordinate system as a graphical tool for laying a foundation for the study of vectors and their components. Vector components simplify vector analysis because they are based on existing knowledge of the x-y-z coordinate system. Thus, the purpose of this lesson is to: 1) develop an understanding of vectors based on prerequisite knowledge of the x-y-z coordinate system, 2) recognize how trigonometry is coupled with vector physics, and 3) gain practice on how to use trigonometry to solve vector problems.

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

Basic Objectives (To be mastered prior to class)

1. Explain the difference between scalars and vectors.
2. Add and subtract vectors graphically.
3. Apply knowledge of the x-y coordinate system and properties of right triangles to decompose a vector into its x-and y-components.
4. Recognize that the Pythagorean Theorem is a mathematical model that defines the relationship between the lengths of the sides of a right triangle and can be used to find the length of one side of a right triangle given the lengths of the other two sides.
5. Add two vectors at right angles.
6. Apply knowledge of how to use the lengths of the sides of a right triangle to find its angles via SOH CAH TOA.
7. Apply knowledge of the x-y-z coordinate system to visualize what unit vectors are.

Advanced Objectives (To be mastered during and after class)

Apply knowledge of right triangles and their related trigonometric functions to:

8. Perform vector calculations using components (Adding vectors using their components).
9. Perform a simple vector addition in three dimensions
10. Find the x-and y-components of a vector given its magnitude and angle.
11. Perform vector calculations using unit vectors. (Adding vectors using unit vectors).
12. Calculate the scalar product using components.
13. Calculate the vector products using components.

RESOURCES FOR LEARNING

Vector Worksheet: http://www.wou.edu/~walshk/ph_211_f10/Vector%20Worksheet%202.pdf

Text: Read *University Physics (Young & Freedman)*, Sections 1.7-1.9

EXERCISES:

In preparation for class, read Sections 1.7 – 1.9 to gain a general understanding of vectors and their components. The videos do a good job at summarizing vector analysis and will help reinforce what you read in the textbook but do not rely on them alone. Reading the textbook provides you with more detailed information that may not be included in the videos so it is important to do both. We will focus on physics problem-solving during class so if you familiarize yourself with the information delivered through this guided practice you will be ready to start working!

For each of the below exercises, prepare a neat set of notes based on the assigned reading (Section 1.7-1.9) and each video lecture. Remember to use a ruler to draw straight lines. Work out these exercises in your own notes so you'll have a record of your work. Remember your work is graded on a Pass/Fail based on completeness, effort, and timeliness.

There will be a quick quiz at the beginning of the class to assess your learning. The above document titled "Vector Worksheet" under 'Resources for Learning' contains practice problems starting on page 11. I will randomly choose two to three questions from there so be prepared.

Step	Learning Activity	Estimated Time
1	a) Using graph paper, draw an x-y-z coordinate system. Use the following videos for references: https://www.youtube.com/watch?v=VA5AmjhTA3A https://www.youtube.com/watch?v=vZNtoloJbEI Note that although most of the work in this lesson related to vectors and their components will be drawn using an x-y (rectangular) coordinate system, it is important to build a good understanding of the x-y-z coordinate system for more advanced 3D problem-solving applications especially as it concerns calculating vector (or cross) products.	15 minutes
2	a) Watch the following Introductory videos about vectors (and scalars): https://www.youtube.com/watch?v=ihNZlp7iUHE https://www.youtube.com/watch?v=1G5E_x0MgLc b) Using graph paper, begin with an x-y (rectangular) coordinate system then practice drawing vectors and adding them graphically. Use the following video as a reference: https://www.youtube.com/watch?v=5Zfe4kfcZio	25 minutes

3	a) Using graph paper, begin with an x-y (rectangular) coordinate system, then practice graphing the x- (horizontal) component and the y- (vertical) component of a vector. Use the following video as a reference: https://www.youtube.com/watch?v=C7iVmueqkZ0	15 minutes
4	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding the resultant of two vectors at right angles using the Pythagorean Theorem noting that the distance from the starting point to the ending point is equal to the length of the hypotenuse. Use the following video as a reference: https://www.youtube.com/watch?v=szgQQ5l1SNw b) For added practice, do example 1.5 (Young and Friedman) on your own.	15 minutes
5	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding vector components given the magnitude of a vector and its angle. Use the following video as a reference: https://www.youtube.com/watch?v=r382kfkqAF4 b) For added practice, do example 1.6 (Young and Friedman) on your own.	15 minutes

Procedure (In-class activities)

Step	**Learning Activity**	**Estimated Time**
6	a) Take quick quiz (See Worksheet Questions 1-5 b) Using graph paper, begin with an x-y (rectangular) coordinate system, practice adding vectors using their components in accordance with vector addition problem solving strategy 1.3. Use the following video as a reference: https://www.youtube.com/watch?v=g_TnqKX5ybY c) For added practice, do examples 1.7 and 1.8 (Young and Friedman) on your own. Compare your results with textbook solution.	25 minutes
7	a) Using graph paper, begin with an x-y (rectangular) coordinate system, recognize unit vector notation and practice using unit vectors in accordance with vector addition problem solving strategy 1.3. Use the following videos as references: https://www.youtube.com/watch?v=FaF3v-ezbSk https://www.youtube.com/watch?v=595Tiga1gIg b) For added practice, do example 1.9 (Young and Friedman) on your own. Compare your results with textbook solution.	25 minutes

Procedure (During and after class activities)

Step	**Learning Activity**	**Estimated time**
8	a) Calculate the scalar product using components. Use the following videos as references: https://www.youtube.com/watch?v=tObaMIGgiI4	25 minutes

	https://www.youtube.com/watch?v=tObaMIGgiI4 b) For added practice, do examples 1.10 and 1.11 (Young and Friedman) on your own. Compare your results with textbook solution.	
9	a) Calculate the vector products using components. Use yhe following video as a reference. https://www.youtube.com/watch?v=qsgK1d-_8ik b) For added practice, do example 1.12 (Young and Friedman) on your own. Compare your results with textbook solution.	25 minutes

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Lesson: Drawing and Analyzing Vectors

Overview: Students apply basic knowledge about coordinate systems, geometry, and trigonometry (right triangles) to understand how to analyze vectors.

Timeframe: 1.5 weeks

Materials needed:

- Computer with internet access
- Graph paper
- Pencils
- (x3) 12 inch/30 cm rulers

Learning Objectives (LO):

By the end of this lesson, you will be able to:

1. Explain the difference between scalars and vectors.
   Key Concepts
   a. A scalar is described by a single number.
   b. Calculations that combine scalar quantities use the operations of ordinary arithmetic.
   c. A vector quantity has both a magnitude and a direction in space.
   d. Vectors are drawn as a line with an arrowhead at its tip
   e. The combination of vectors requires a different set of operations that require knowledge of the x-y-z coordinate system and trigonometry.
2. Add and subtract vectors graphically.
   Key Concepts
   a. Vectors may be added graphically using the tip-to-tail method
   b. Vectors may be added graphically using the parallelogram method.
3. Apply knowledge of the x-y coordinate system and properties of right triangles to decompose a vector into its x-and y-components.
   Key Concepts
   a. It is important to note that the base and height of a right triangle represents the x- and y-components of a vector and that the hypotenuse of a right triangle represents the resultant vector of these two components.
   b. Components are not vectors- they are quantities/numbers that simplify vector analysis by allowing us to use trigonometry to add vectors.
4. Recognize that the Pythagorean Theorem is a mathematical model that defines the relationship between the lengths of the sides of a right triangle and can be used to find the length of one side of a right triangle given the lengths of the other two sides.
   Key Concepts

   a. Given the side of the triangle parallel to the x- and y-axes, we can use the distance formula to find the length of the hypotenuse which is the distance between two points on an x-y coordinate system.
5. Add two vectors at right angles.
   a. When adding two vectors at right angles, the distance from the starting point of the first vector and the endpoint of the second vector is equal to the length of the hypotenuse which is the same as using the distance formula discussed above (See LO 4)
6. Apply knowledge of how to use the lengths of the sides of a right triangle to find its angles via SOH CAH TOA.

   Key Concepts
   a. It is important to note that knowing how to find the angles of a right triangle can be used to find the resultant direction of a combination of vectors.
7. Apply knowledge of the x-y-z coordinate system to visualize what unit vectors are.
   a. Beginning with a rectangular coordinate system, we draw the tail at the origin and extend it over the x, y, and z axes. As the name implies, the magnitude of a unit vector is equal to one.
   b. A unit vector describes a direction in space. They are used to express the relationship between component vectors and components.

Advanced Objectives (To be mastered during and after class)

Apply knowledge of right triangles and their related trigonometric functions to:

8. Perform vector calculations using components (Adding vectors using their components).
9. Perform a simple vector addition in three dimensions
10. Find the x-and y-components of a vector given its magnitude and angle.
11. Perform vector calculations using unit vectors. (Adding vectors using unit vectors).
12. Calculate the scalar product using components.
13. Calculate the vector products using components.

Background

Students pursuing engineering degrees must pass a series of physics courses which serve as a foundation for engineering design and problem-solving applications. Oftentimes, students struggle with physics because they lack understanding of basic concepts and principles. If mastered early on, the difficulties with physics learning will be greatly reduced. A key concept in physics is vectors. They are used in a variety of physics problem solving applications and require the following prerequisite knowledge:

1. How to draw, label, and interpret an x-y-z coordinate system.
   a. Basics of number lines
   b. Plotting points in a two-dimensional coordinate system
   c. Plotting points in a three-dimensional coordinate system.
2. How to calculate the lengths of the sides of a right triangle using the Pythagorean Theorem.
3. How to calculate the angles of a right triangle using SOH CAH TOA.

Introduction to Lesson

Physics Workshop Lesson Plan Drawing and Analyzing Vectors

During the week prior to class, students will be required to read their textbook and develop a set of notes (a.k.a. learning toolkit) based on assigned readings and videos to gain a basic understanding of vectors and their components. Students must complete pre-class activities prior to meeting in class. Students will receive a grade for completing each pre-class activity. To receive full credit, 100% of the activities must be completed.

Procedure (Prior to class activities)

In preparation for class, follow Steps 1-5 to gain a basic understanding of vectors. During class, we will apply what you learn to solve problems related to HW and Exam Questions.

Step	Activity For each of the below activities, prepare a neat set of notes based on each video. Use a ruler to draw straight lines.	Learning Objectives	Estimated Time
1	a) Using graph paper, draw an x-y-z coordinate system. Use the following videos for references: https://www.youtube.com/watch?v=VA5AmjhTA3A https://www.youtube.com/watch?v=vZNtoloJbEI Note that although most of the work in this lesson related to vectors and their components will be drawn using an x-y (rectangular) coordinate system, it is important to build a good understanding of the x-y-z coordinate system for more advanced 3D problem-solving applications especially as it concerns calculating vector (or cross) products.	Prerequi-site	15
2	a) Using graph paper, begin with an x-y (rectangular) coordinate system then practice drawing vectors and adding them graphically. Use the following video as a reference: https://www.youtube.com/watch?v=5Zfe4kfcZio	2	15
3	a) Using graph paper, begin with an x-y (rectangular) coordinate system, then practice graphing the x- (horizontal) component and the y- (vertical) component of a vector. Use the following video as a reference: https://www.youtube.com/watch?v=C7iVmueqkZ0	2,3	15
4	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding the resultant of two vectors at right angles using the Pythagorean Theorem noting that the distance from the starting point to the ending point is equal to the length of the hypotenuse. Use the following video as a reference: https://www.youtube.com/watch?v=szgQQ5l1SNw b) For added practice, do example 1.5 (Young and Friedman) on your own.	4,5	15
5	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding vector components given the magnitude of a vector and its angle. Use the following video as a reference: https://www.youtube.com/watch?v=r382kfkqAF4 b) For added practice, do example 1.6 (Young and Friedman) on your own.	6	15

Procedure (In-class activities)

Step	Activity	Learning Objectives	Estimated Time
6	a) Take quick quiz (See Worksheet Questions 1-5 b) Using graph paper, begin with an x-y (rectangular) coordinate system, practice adding vectors using their components in accordance with vector addition problem solving strategy 1.3. Use the following video as a reference: https://www.youtube.com/watch?v=g_TnqKX5ybY c) For added practice, do examples 1.7 and 1.8 (Young and Friedman) on your own. Compare your results with textbook solution.	7	25
7	a) Using graph paper, begin with an x-y (rectangular) coordinate system, recognize unit vector notation and practice using unit vectors in accordance with vector addition problem solving strategy 1.3 (Young and Friedman). Use the following videos as references: https://www.youtube.com/watch?v=FaF3v-ezbSk https://www.youtube.com/watch?v=595Tiga1gIg b) For added practice, do example 1.9 (Young and Friedman) on your own. Compare your results with textbook solution.	8-11	25

Procedure (During and after class activities)

Step	Activity	Learning Objectives	Estimated Time
8	a) Calculate the scalar product using components. Use the following videos as references: https://www.youtube.com/watch?v=tObaMIGgiI4 https://www.youtube.com/watch?v=tObaMIGgiI4 b) For added practice, do examples 1.10 and 1.11 (Young and Friedman) on your own. Compare your results with textbook solution.	12	25
9	a) Calculate the vector products using components. Use the following video as a reference: https://www.youtube.com/watch?v=qsgK1d-_8ik b) For added practice, do example 1.12 (Young and Friedman) on your own. Compare your results with textbook solution.	13	25

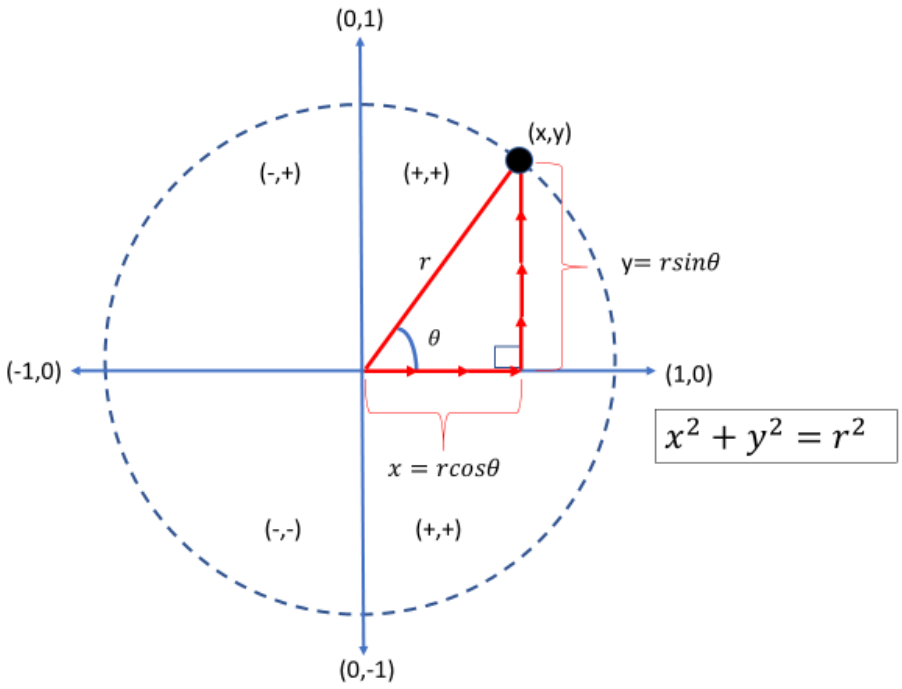
Additional Notes

Vector analysis represents the intersection between geometry, trigonometry, algebra, and physics. Vectors are merely extensions of previously learned concepts and principles involving coordinate geometry wherein the location of points can be determined using a pair of numbers and the distance between two numbers can be found using the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Upon careful inspection it is not difficult to see that the above formula is merely a variation of $c^2 = a^2 + b^2$. Moreover, finding the vertical and horizontal components of a vector (or a combination of vectors) involves using prior knowledge of SOH CAH TOA. It is important to note that all the knowledge

needed to perform vector analysis is encapsulated in the diagram of a unit circle as shown below where a=x, b=y, and c=r:



EVALUATION

Analysis

Resistance to Change

The students in this class are primarily freshman and sophomore engineering and computer science majors who have completed their pre-calculus requirements (geometry, trigonometry, algebra) and are either taking Calculus I or have completed Calculus I. They typically rely on lecture to acquire knowledge of physics concepts and principles with little or no course preparation in terms of reading and taking notes based on readings. There will be some resistance to reading and developing their notes for problem solving applications (a.k.a learning toolkits) prior to class but with time they will realize the benefits of developing these good study habits and skills.

Prior Knowledge

Also, at first, they may find vectors to be a bit complicated but with time and practice they will realize that vectors are merely extensions of geometry, trigonometry, and algebra that they already have prior knowledge of.

Pre-class Individual Activity

The pre-class activity will give students their first exposure of the content alone so that students are familiar with independent work. The in-class activity will help students deepen their understanding by working out more advanced problem-solving applications. Lastly, the post-class activity will give students the opportunity to practice these concepts individually to prepare them for exams.

Post-Class Individual Activity

Many freshman-level students have a difficult time transferring what they have learned as a group and performing individually on a class exam. Students will apply their newfound knowledge by completing sample test question for each concept that they have learned. They will then be required to post ONE unique test question with the answer on the course's Moodle "Students' Test Questions" discussion board where the instructor will choose student questions for the in-class exam. Students could use the Student Test Questions as a study guide.

Connections to Future Lesson Plans:

The topics covered in this activity will serve as a foundation for understanding future topics involving 2D motion (linear and circular), force, momentum, and torque. It also reinforces how to organize work involving physics problem solving by drawing neat diagrams which are used to visualize interactions between objects in the physical world.