# Lesson Plan

**Instructor:** Steven Vukazich

**Lesson: Evaluating the Determinacy of Planar Structures**

**Timeframe: 50 minutes**

**Materials needed:**

Paper
Pencil
Eraser

**Objectives:**

***Basic:***

1. Remember the number of independent equations of equilibrium available for a general planar (two-dimensional) structure (prerequisite course CE 95)
2. Remember the number of unknown reactive forces associated with common structural supports (e.g. pin support, roller support from prerequisite course CE 95)
3. Isolate and draw a Free Body Diagram (FBD) of a structural system (prerequisite course CE 95)

***Advanced:***

1. Identify individual "pieces" of structure required to be isolated, draw FBDs of each "piece", and evaluate determinacy
2. Idealize and evaluate determinacy of an actual planar structure presented
3. Modify the planar process to evaluate determinacy of a general three-dimensional structure

**Background:**

Read chapter material that reviews basic objectives 1, 2, and 3;

Read chapter material and notes (and possible video lecture) provided by the instructor to introduce advanced learning objective 1.

**Procedure [Time needed, include additional steps if needed]:**

***Pre-Class Individual Space Activities and Resources:***

| Steps | Purpose | Estimated Time | Learning Objective |
|---|---|---|---|
| **Step 1:**<br><br>**Read section in Leet text** | **Review basic statics (pre-requisite material) and to introduce the new material** | **20 minutes** | **Basic Objectives 1,2,3** |
| **Step 2:**<br><br>**Read instructor notes (see notes below – these could be turned into a Camtasia video)** | **In depth review and discussion of new material** | **30 minutes** | **Basic Objectives 1,2,3** |
| **Step 3:**<br><br>**Complete example problem in instructor notes.** | **Reinforce the review and extension to new material** | **10 minutes** | **Basic Objective 3** |

# CE 160 Notes – Determinacy for Planar (two-dimensional) Structures

Recall from statics that for a planar structure there are 3 independent equilibrium equations available to solve for unknown forces per free body diagram (FBD).

Recall also that a structure is statically determinate is one that is stable and all unknown reactive forces can be determined from the equations of equilibrium alone. A statically indeterminate structure is one that is stable but contains more unknown forces than available equations of equilibrium. We will see later in the course that statically indeterminate structures can be solved but require information on the deformation of the structure to solve.

The concept of statical determinacy or indeterminacy is meaningful only for stable structures. We will assume, for the moment, that all of the structures we consider are stable and we will study the assessment of stability later. In order to accurately assess if a stable structure is statically determinate or indeterminate we must be able to count all of the unknown forces associated with the structure. The counting of unknowns requires that we cut the structure into the minimum number of "pieces" and draw a FBD of each "piece".

In order to accurately count the total number of unknowns in a structure we must, at the minimum, cut the structure at the following locations:

1. The structure must be cut at all supports (e.g. pins, rollers, fixed supports);
2. The structure must be cut at all locations of known internal force (e.g. internal hinges);
3. The structure must be cut in such a way to "open" all closed rigid loops (we will define a closed rigid loop later).

Once the structure has been cut at the locations above, the number of "pieces" of the structure will be defined. The total number of unknowns can then be determined once a FBD of each "piece" is drawn.

Determinacy can be determined as follows, if:

n = total number of "pieces" associated with the entire structure;
X = total number of unknowns associated with the entire structure;

then

3n = total number of independent equations of equilibrium available to solve for unknowns since there are 3 equations of equilibrium available for the FBD of each "piece"

For stable, planar structures, if:

$X = 3n$

then the structure is **Statically Determinate**

$X \geq 3n$

then the structure is **Statically Indeterminate** and the degree of indeterminacy is:

$X - 3n$

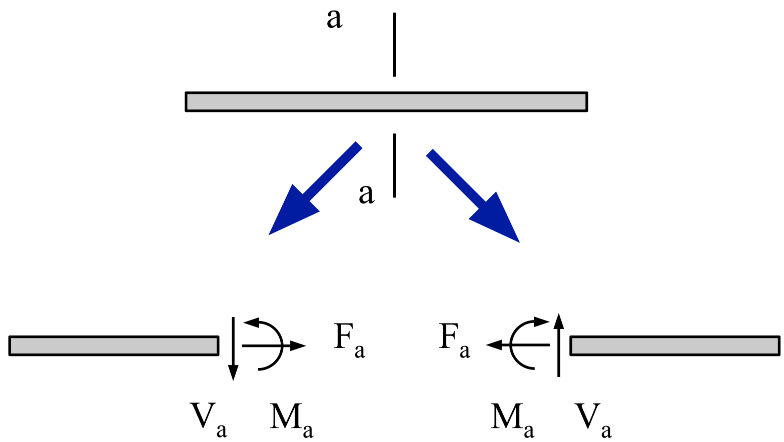
For example, if X = 8 and n = 2, and the structure is stable, then the structure is **statically indeterminate to the 2$^{nd}$ degree**.

$8 > 3(2)$
$8 > 6$
and $8 - 6 = 2$

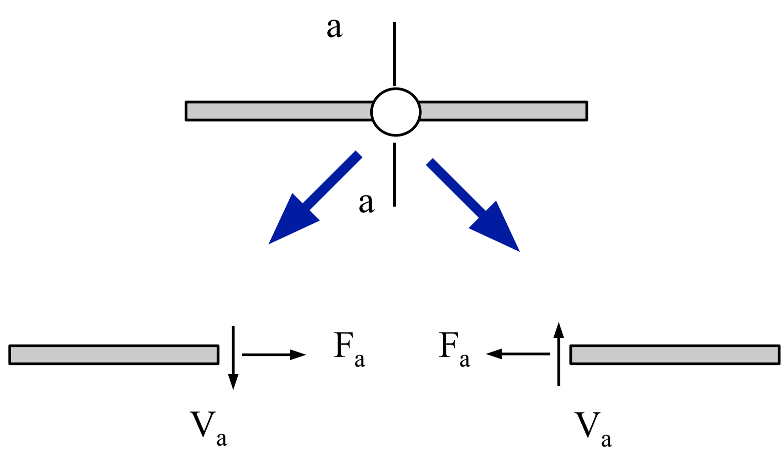
As noted previously, determinacy is only defined for stable structures. We will study how to evaluate structures for stability later.

**Internal forces at an internal hinge**

Recall that there are, in general, three unknown internal forces at each point in a member of a planar structure. For the cut at section a-a below there is a shear force ($V_a$), an axial force ($F_a$), and an internal bending moment ($M_a$). These internal forces are equal and opposite on each side of the cut so that the cut at section a-a introduces three unknowns.

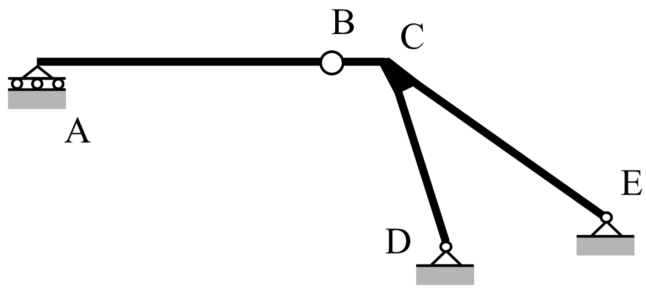



At a point where there is an internal hinge, because there is not resistance to bending at the hinge the internal bending moment is equal to zero. If at point a below, there is an internal hinge, then $M_a = 0$ and there are only two unknowns introduced when we make a cut at the hinge.



**Example**

Consider the following idealized planar structure:




Recall that in order to accurately count the total number of unknowns in a structure we must, at the minimum, cut the structure at the following locations:

1. The structure must be cut at all supports (e.g. pins, rollers, fixed supports);

   **In this example, points A, D, and E are supports**

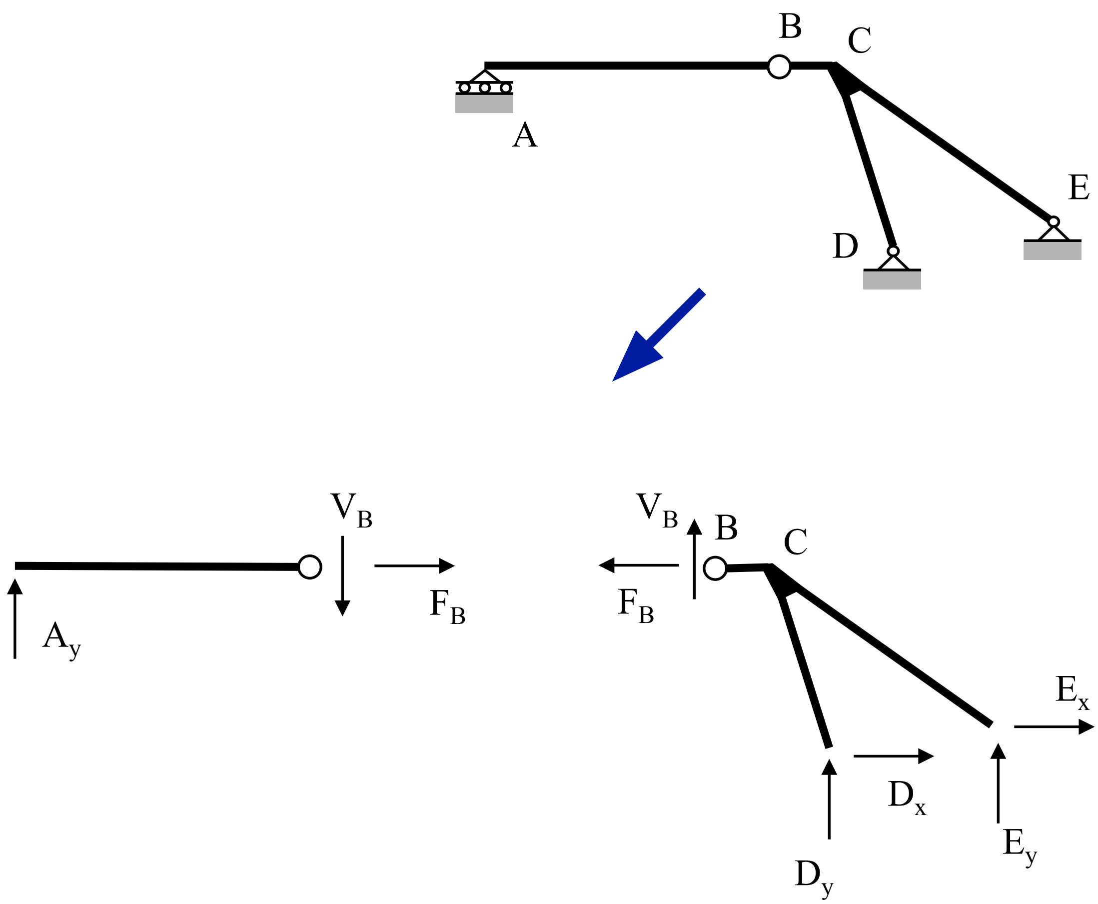
2. The structure must be cut at all locations of known internal force (e.g. internal hinges);

   **In this example, there is an internal hinge at B where we know the internal moment is equal to zero**

3. The structure must be cut in such a way to "open" all closed rigid loops (we will define a closed rigid loop later).

   **In this example, there are no rigid closed loops**

Therefore, we must cut the structure at points A, B, D, and E and draw FBDs of each “piece”.




For this example, the required cuts define two “pieces” and the total number of unknowns for the entire structure are seven, thus:

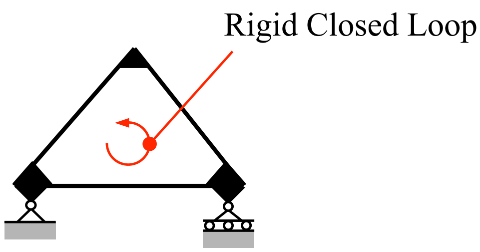
$n = 2$
$X = 7$

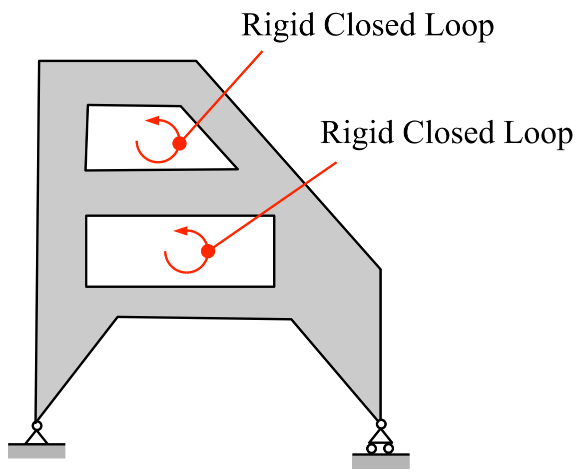
So this structure is **statically indeterminate to the 1$^{st}$ degree**.

$7 > 3(2)$
$7 > 6$
and $7 - 6 = 1$

**Rigidly Connected Closed Loops**

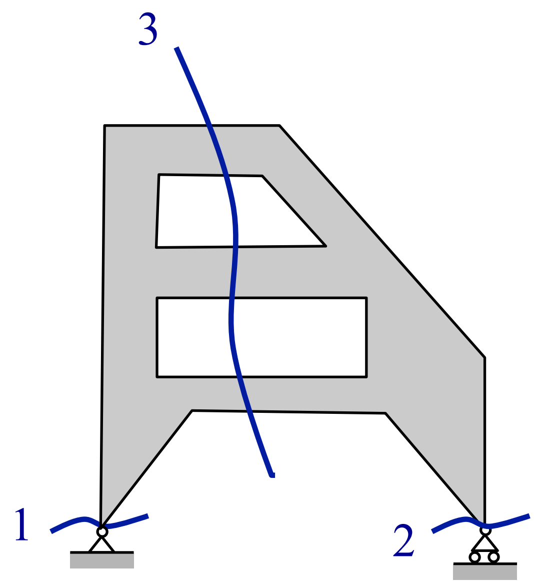
If we can circle a loop in a planar structure and all of the connections around the loop are rigid, then that defined a rigid closed loop and we must cut through the loop in order to accurately access the determinacy of the structure.  Below are examples of structures that contain rigid closed loops.



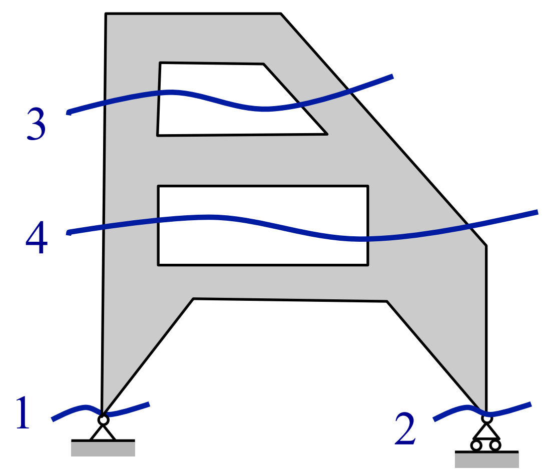



Below are examples of how the second structure can be cut to “open” the rigid closed loops. Each of the two options is an acceptable choice to “open” the rigid loops.

Note that the structure must be always be cut at the supports (cuts 1 and 2) and at any place where internal forces are known (none for this structure).




Option A (n = 2)




Option B (n = 3)

**Exercise**

All students should complete the following exercise on a sheet of paper and turn it in at the beginning of the next class meeting.

1. Draw FBDs of each “piece” for both Option A and Option B
2. Compare X to 3n for both Option A and Option B
3. Briefly comment on your results

## Introduction to Lesson:

**Collect the short homework assignment based on the pre-class activities**

**Review the important points of the pre-class activities:**

- **Review of planar statics;**
- **FBD construction;**
- **Process for evaluating determinacy of planar structures.**

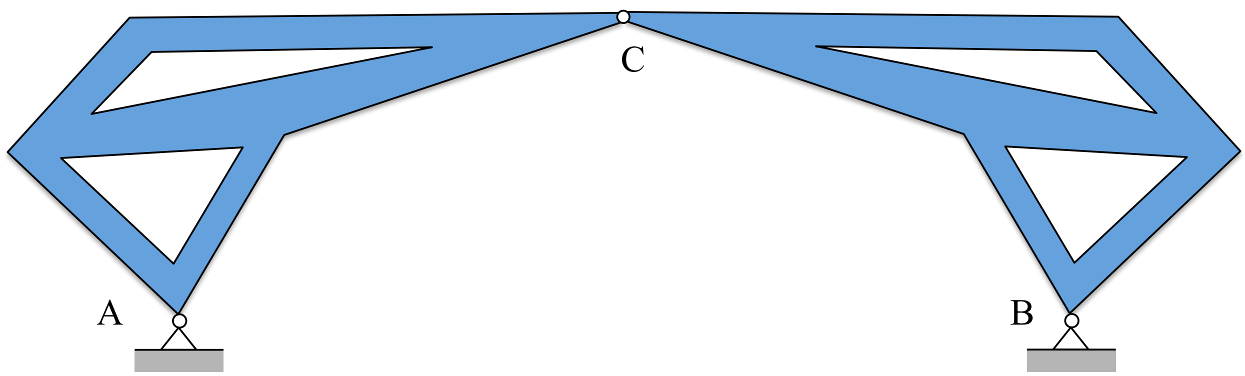
***In-Class Group Space Activities and Resources:***

| Steps | Purpose | Estimated Time | Learning Objective |
|---|---|---|---|
| **Step 1:**<br><br>**Collect individual space activity exercise** | **Confirmation of completion of Individual space activity** | **3 min** | **Advanced Objective 1** |
| **Step 2:**<br><br>**Discuss results from individual space activity exercise and review basic concepts** | **Review individual space activity** | **8 min** | **Advanced Objective 1** |
| **Step 3:**<br><br>**Each student works on a problem based activity (see attached -- a more complex example problem)** | **Learn the basic advanced learning objective of the lesson** | **25 min** | **Advanced Objective 2** |
| **Step 4:**<br><br>**Think/Pair/Share activity**<br>**(see attached – extending the planar process to a general three-dimensional structure)** | **Extend the basic advanced learning objective of the lesson to a more general problem** | **8 min** | **Advanced Objective 3** |

# Determinacy of Planar Structures
# In-Class Activities

**Problem Solving Activity**

Consider the following idealized planar structure:




The stable structure is pin supported at points A and B with an internal hinge at point C.

Draw the appropriate Free Body Diagrams and determine if the structure is statically determinate or indeterminate when subjected to any general loading.

This is an individual activity to be performed by all students.

**Think/Pair/Share Activity**

Pair up with one of your neighboring classmates and answer the following question.

We have just studied how to evaluate the determinacy of stable general planar structures. How would the process change for evaluating the determinacy of stable general three-dimensional structures?

**Closure/Evaluation:**

***Analysis:***

**Answer student questions from activities**

**Re-emphasize important concepts/skills from the lesson**

***Post-Class Individual Space Activities:***

**Homework assignment to complete the relevant problems from the Leet textbook**

***Connections to Future Lesson Plan(s):***

**The next lesson builds on the FBD drawing and analysis from this lesson – Stability of General Planar Structures**

**The lesson after Stability of General Planar Structure also builds on the FBD drawing and analysis from this lesson – Stability and Determinacy of Planar Trusses. We will apply the determinacy and stability analyses of general planar structures to a specific type of structure – a planar truss.**