

ENGR 1540: Physics Workshop (Engineering Special Topics)
Drawing and Analyzing Vectors
Lesson Plan
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Overview: Physics deals with the laws that govern our physical world and links the study of energy, matter, force, and motion. In fact, a large part of physics involves using visual and mathematical models to describe the motion of objects as they move through space over a duration of time. When analyzing the motion of an object, we need a way to describe the magnitude (size) and direction of quantities such as displacement (change in position), velocity (change in position with respect to time), acceleration (change in velocity with respect to time), and force (a natural influence that causes the motion or shape of an object to change). Vectors allow for such a description by indicating size and direction and play an important role in physics problem-solving applications.

Introduction

The analysis of vectors is dependent on trigonometry. Trigonometry, which unifies geometry, algebra, and physics, plays an important role in vector analysis especially as it concerns vector addition. We will use an x-y-z coordinate system as a graphical tool for laying a foundation for the study of vectors and their components. Vector components simplify vector analysis because they are based on existing knowledge of the x-y-z coordinate system. Thus, the purpose of this lesson is to: 1) develop an understanding of vectors based on prerequisite knowledge of the x-y-z coordinate system, 2) recognize how trigonometry is coupled with vector physics, and 3) gain practice on how to use trigonometry to solve vector problems.

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

Basic Objectives (To be mastered prior to class)

1. Explain the difference between scalars and vectors.
2. Add and subtract vectors graphically.
3. Apply knowledge of the x-y coordinate system and properties of right triangles to decompose a vector into its x-and y-components.
4. Recognize that the Pythagorean Theorem is a mathematical model that defines the relationship between the lengths of the sides of a right triangle and can be used to find the length of one side of a right triangle given the lengths of the other two sides.
5. Add two vectors at right angles.
6. Apply knowledge of how to use the lengths of the sides of a right triangle to find its angles via SOH CAH TOA.
7. Apply knowledge of the x-y-z coordinate system to visualize what unit vectors are.

Advanced Objectives (To be mastered during and after class)

Apply knowledge of right triangles and their related trigonometric functions to:

8. Perform vector calculations using components (Adding vectors using their components).
9. Perform a simple vector addition in three dimensions
10. Find the x-and y-components of a vector given its magnitude and angle.
11. Perform vector calculations using unit vectors. (Adding vectors using unit vectors).
12. Calculate the scalar product using components.
13. Calculate the vector products using components.

RESOURCES FOR LEARNING

Vector Worksheet: http://www.wou.edu/~walshk/ph_211_f10/Vector%20Worksheet%202.pdf

Text: Read *University Physics (Young & Freedman)*, Sections 1.7-1.9

EXERCISES:

In preparation for class, read Sections 1.7 – 1.9 to gain a general understanding of vectors and their components. The videos do a good job at summarizing vector analysis and will help reinforce what you read in the textbook but do not rely on them alone. Reading the textbook provides you with more detailed information that may not be included in the videos so it is important to do both. We will focus on physics problem-solving during class so if you familiarize yourself with the information delivered through this guided practice you will be ready to start working!

For each of the below exercises, prepare a neat set of notes based on the assigned reading (Section 1.7-1.9) and each video lecture. Remember to use a ruler to draw straight lines. Work out these exercises in your own notes so you'll have a record of your work. Remember your work is graded on a Pass/Fail based on completeness, effort, and timeliness.

There will be a quick quiz at the beginning of the class to assess your learning. The above document titled "Vector Worksheet" under 'Resources for Learning' contains practice problems starting on page 11. I will randomly choose two to three questions from there so be prepared.

Step	Learning Activity	Estimated Time
1	<p>a) Using graph paper, draw an x-y-z coordinate system. Use the following videos for references:</p> <p>https://www.youtube.com/watch?v=VA5AmjhTA3A https://www.youtube.com/watch?v=vZNtoloJbEI</p> <p>Note that although most of the work in this lesson related to vectors and their components will be drawn using an x-y (rectangular) coordinate system, it is important to build a good understanding of the x-y-z coordinate system for more advanced 3D problem-solving applications especially as it concerns calculating vector (or cross) products.</p>	15 minutes
2	<p>a) Watch the following Introductory videos about vectors (and scalars):</p> <p>https://www.youtube.com/watch?v=ihNZlp7iUHE https://www.youtube.com/watch?v=1G5E_x0MgLC</p> <p>b) Using graph paper, begin with an x-y (rectangular) coordinate system then practice drawing vectors and adding them graphically. Use the following video as a reference:</p> <p>https://www.youtube.com/watch?v=5Zfe4kfcZio</p>	25 minutes

3	a) Using graph paper, begin with an x-y (rectangular) coordinate system, then practice graphing the x- (horizontal) component and the y- (vertical) component of a vector. Use the following video as a reference: https://www.youtube.com/watch?v=C7iVmueqkZ0	15 minutes
4	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding the resultant of two vectors at right angles using the Pythagorean Theorem noting that the distance from the starting point to the ending point is equal to the length of the hypotenuse. Use the following video as a reference: https://www.youtube.com/watch?v=szgQQ5l1SNw b) For added practice, do example 1.5 (Young and Friedman) on your own.	15 minutes
5	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding vector components given the magnitude of a vector and its angle. Use the following video as a reference: https://www.youtube.com/watch?v=r382kfkqAF4 b) For added practice, do example 1.6 (Young and Friedman) on your own.	15 minutes

Procedure (In-class activities)

Step	Learning Activity	Estimated Time
6	a) Take quick quiz (See Worksheet Questions 1-5 b) Using graph paper, begin with an x-y (rectangular) coordinate system, practice adding vectors using their components in accordance with vector addition problem solving strategy 1.3. Use the following video as a reference: https://www.youtube.com/watch?v=g_TnqKX5ybY c) For added practice, do examples 1.7 and 1.8 (Young and Friedman) on your own. Compare your results with textbook solution.	25 minutes
7	a) Using graph paper, begin with an x-y (rectangular) coordinate system, recognize unit vector notation and practice using unit vectors in accordance with vector addition problem solving strategy 1.3. Use the following videos as references: https://www.youtube.com/watch?v=FaF3v-ezbSk https://www.youtube.com/watch?v=595Tiga1glg b) For added practice, do example 1.9 (Young and Friedman) on your own. Compare your results with textbook solution.	25 minutes

Procedure (During and after class activities)

Step	Learning Activity	Estimated time
8	a) Calculate the scalar product using components. Use the following videos as references: https://www.youtube.com/watch?v=tObaMIGgiI4	25 minutes

	<p>https://www.youtube.com/watch?v=tObaMIGgiI4</p> <p>b) For added practice, do examples 1.10 and 1.11 (Young and Friedman) on your own. Compare your results with textbook solution.</p>	
9	<p>a) Calculate the vector products using components. Use the following video as a reference.</p> <p>https://www.youtube.com/watch?v=qsgK1d-8ik</p> <p>b) For added practice, do example 1.12 (Young and Friedman) on your own. Compare your results with textbook solution.</p>	25 minutes

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Lesson: Drawing and Analyzing Vectors

Overview: Students apply basic knowledge about coordinate systems, geometry, and trigonometry (right triangles) to understand how to analyze vectors.

Timeframe: 1.5 weeks

Materials needed:

- Computer with internet access
- Graph paper
- Pencils
- (x3) 12 inch/30 cm rulers

Learning Objectives (LO):

By the end of this lesson, you will be able to:

1. Explain the difference between scalars and vectors.

Key Concepts

- a. A scalar is described by a single number.
- b. Calculations that combine scalar quantities use the operations of ordinary arithmetic.
- c. A vector quantity has both a magnitude and a direction in space.
- d. Vectors are drawn as a line with an arrowhead at its tip
- e. The combination of vectors requires a different set of operations that require knowledge of the x-y-z coordinate system and trigonometry.

2. Add and subtract vectors graphically.

Key Concepts

- a. Vectors may be added graphically using the tip-to-tail method
- b. Vectors may be added graphically using the parallelogram method.

3. Apply knowledge of the x-y coordinate system and properties of right triangles to decompose a vector into its x-and y-components.

Key Concepts

- a. It is important to note that the base and height of a right triangle represents the x- and y-components of a vector and that the hypotenuse of a right triangle represents the resultant vector of these two components.
- b. Components are not vectors- they are quantities/numbers that simplify vector analysis by allowing us to use trigonometry to add vectors.

4. Recognize that the Pythagorean Theorem is a mathematical model that defines the relationship between the lengths of the sides of a right triangle and can be used to find the length of one side of a right triangle given the lengths of the other two sides.

Key Concepts

- a. Given the side of the triangle parallel to the x- and y-axes, we can use the distance formula to find the length of the hypotenuse which is the distance between two points on an x-y coordinate system.
5. Add two vectors at right angles.
 - a. When adding two vectors at right angles, the distance from the starting point of the first vector and the endpoint of the second vector is equal to the length of the hypotenuse which is the same as using the distance formula discussed above (See LO 4)
6. Apply knowledge of how to use the lengths of the sides of a right triangle to find its angles via SOH CAH TOA.

Key Concepts

- a. It is important to note that knowing how to find the angles of a right triangle can be used to find the resultant direction of a combination of vectors.
7. Apply knowledge of the x-y-z coordinate system to visualize what unit vectors are.
 - a. Beginning with a rectangular coordinate system, we draw the tail at the origin and extend it over the x, y, and z axes. As the name implies, the magnitude of a unit vector is equal to one.
 - b. A unit vector describes a direction in space. They are used to express the relationship between component vectors and components.

Advanced Objectives (To be mastered during and after class)

Apply knowledge of right triangles and their related trigonometric functions to:

8. Perform vector calculations using components (Adding vectors using their components).
9. Perform a simple vector addition in three dimensions
10. Find the x-and y-components of a vector given its magnitude and angle.
11. Perform vector calculations using unit vectors. (Adding vectors using unit vectors).
12. Calculate the scalar product using components.
13. Calculate the vector products using components.

Background

Students pursuing engineering degrees must pass a series of physics courses which serve as a foundation for engineering design and problem-solving applications. Oftentimes, students struggle with physics because they lack understanding of basic concepts and principles. If mastered early on, the difficulties with physics learning will be greatly reduced. A key concept in physics is vectors. They are used in a variety of physics problem solving applications and require the following prerequisite knowledge:

1. How to draw, label, and interpret an x-y-z coordinate system.
 - a. Basics of number lines
 - b. Plotting points in a two-dimensional coordinate system
 - c. Plotting points in a three-dimensional coordinate system.
2. How to calculate the lengths of the sides of a right triangle using the Pythagorean Theorem.
3. How to calculate the angles of a right triangle using SOH CAH TOA.

Introduction to Lesson

During the week prior to class, students will be required to read their textbook and develop a set of notes (a.k.a. learning toolkit) based on assigned readings and videos to gain a basic understanding of vectors and their components. Students must complete pre-class activities prior to meeting in class. Students will receive a grade for completing each pre-class activity. To receive full credit, 100% of the activities must be completed.

Procedure (Prior to class activities)

In preparation for class, follow Steps 1-5 to gain a basic understanding of vectors. During class, we will apply what you learn to solve problems related to HW and Exam Questions.

Step	Activity	Learning Objectives	Estimated Time
	For each of the below activities, prepare a neat set of notes based on each video. Use a ruler to draw straight lines.		
1	a) Using graph paper, draw an x-y-z coordinate system. Use the following videos for references: https://www.youtube.com/watch?v=VA5AmjhTA3A https://www.youtube.com/watch?v=vZNtoloJbEI Note that although most of the work in this lesson related to vectors and their components will be drawn using an x-y (rectangular) coordinate system, it is important to build a good understanding of the x-y-z coordinate system for more advanced 3D problem-solving applications especially as it concerns calculating vector (or cross) products.	Prerequisite	15
2	a) Using graph paper, begin with an x-y (rectangular) coordinate system then practice drawing vectors and adding them graphically. Use the following video as a reference: https://www.youtube.com/watch?v=5Zfe4kfcZio	2	15
3	a) Using graph paper, begin with an x-y (rectangular) coordinate system, then practice graphing the x- (horizontal) component and the y- (vertical) component of a vector. Use the following video as a reference: https://www.youtube.com/watch?v=C7iVmueqkZ0	2,3	15
4	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding the resultant of two vectors at right angles using the Pythagorean Theorem noting that the distance from the starting point to the ending point is equal to the length of the hypotenuse. Use the following video as a reference: https://www.youtube.com/watch?v=szgQQ5l1SNw b) For added practice, do example 1.5 (Young and Friedman) on your own.	4,5	15
5	a) Using graph paper, begin with an x-y (rectangular) coordinate system, practice finding vector components given the magnitude of a vector and its angle. Use the following video as a reference: https://www.youtube.com/watch?v=r382kfkqAF4 b) For added practice, do example 1.6 (Young and Friedman) on your own.	6	15

Procedure (In-class activities)

Step	Activity	Learning Objectives	Estimated Time
6	a) Take quick quiz (See Worksheet Questions 1-5 b) Using graph paper, begin with an x-y (rectangular) coordinate system, practice adding vectors using their components in accordance with vector addition problem solving strategy 1.3. Use the following video as a reference: https://www.youtube.com/watch?v=g_TnqKX5ybY c) For added practice, do examples 1.7 and 1.8 (Young and Friedman) on your own. Compare your results with textbook solution.	7	25
7	a) Using graph paper, begin with an x-y (rectangular) coordinate system, recognize unit vector notation and practice using unit vectors in accordance with vector addition problem solving strategy 1.3 (Young and Friedman). Use the following videos as references: https://www.youtube.com/watch?v=FaF3v-ezbSk https://www.youtube.com/watch?v=595Tiga1glg b) For added practice, do example 1.9 (Young and Friedman) on your own. Compare your results with textbook solution.	8-11	25

Procedure (During and after class activities)

Step	Activity	Learning Objectives	Estimated Time
8	a) Calculate the scalar product using components. Use the following videos as references: https://www.youtube.com/watch?v=tObaMIGgiI4 https://www.youtube.com/watch?v=tObaMIGgiI4 b) For added practice, do examples 1.10 and 1.11 (Young and Friedman) on your own. Compare your results with textbook solution.	12	25
9	a) Calculate the vector products using components. Use the following video as a reference: https://www.youtube.com/watch?v=qsgK1d-8ik b) For added practice, do example 1.12 (Young and Friedman) on your own. Compare your results with textbook solution.	13	25

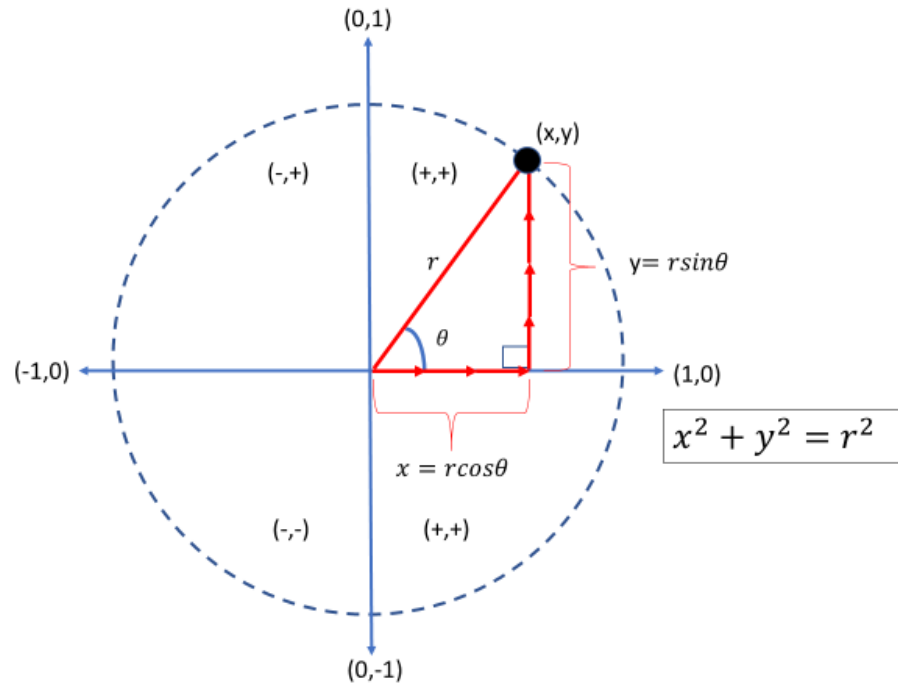
Additional Notes

Vector analysis represents the intersection between geometry, trigonometry, algebra, and physics. Vectors are merely extensions of previously learned concepts and principles involving coordinate geometry wherein the location of points can be determined using a pair of numbers and the distance between two numbers can be found using the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Upon careful inspection it is not difficult to see that the above formula is merely a variation of $c^2 = a^2 + b^2$. Moreover, finding the vertical and horizontal components of a vector (or a combination of vectors) involves using prior knowledge of SOH CAH TOA. It is important to note that all the knowledge

needed to perform vector analysis is encapsulated in the diagram of a unit circle as shown below where $a=x$, $b=y$, and $c=r$:



EVALUATION

Analysis

Resistance to Change

The students in this class are primarily freshman and sophomore engineering and computer science majors who have completed their pre-calculus requirements (geometry, trigonometry, algebra) and are either taking Calculus I or have completed Calculus I. They typically rely on lecture to acquire knowledge of physics concepts and principles with little or no course preparation in terms of reading and taking notes based on readings. There will be some resistance to reading and developing their notes for problem solving applications (a.k.a learning toolkits) prior to class but with time they will realize the benefits of developing these good study habits and skills.

Prior Knowledge

Also, at first, they may find vectors to be a bit complicated but with time and practice they will realize that vectors are merely extensions of geometry, trigonometry, and algebra that they already have prior knowledge of.

Pre-class Individual Activity

The pre-class activity will give students their first exposure of the content alone so that students are familiar with independent work. The in-class activity will help students deepen their understanding by working out more advanced problem-solving applications. Lastly, the post-class activity will give students the opportunity to practice these concepts individually to prepare them for exams.

Post-Class Individual Activity

Many freshman-level students have a difficult time transferring what they have learned as a group and performing individually on a class exam. Students will apply their newfound knowledge by completing sample test question for each concept that they have learned. They will then be required to post ONE unique test question with the answer on the course's Moodle "Students' Test Questions" discussion board where the instructor will choose student questions for the in-class exam. Students could use the Student Test Questions as a study guide.

Connections to Future Lesson Plans:

The topics covered in this activity will serve as a foundation for understanding future topics involving 2D motion (linear and circular), force, momentum, and torque. It also reinforces how to organize work involving physics problem solving by drawing neat diagrams which are used to visualize interactions between objects in the physical world.

Vector Worksheet

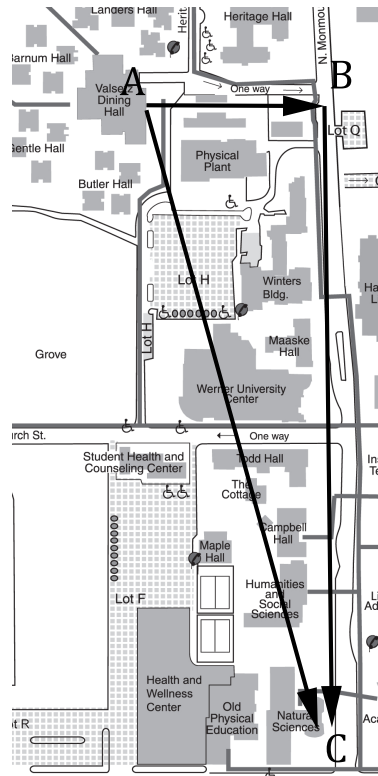
Much of the physical world can be described in terms of numbers. Examples of this are the mass of an object, its temperature and its volume. These are called scalar quantities. But some quantities also need a direction to fully describe it. What if you wanted directions to the nearest physics lab? The answer "Go three miles" is not very helpful. More useful would be "Go three miles south." A quantity having both size (magnitude) and direction is called a vector quantity. Vectors are not just about position. Such quantities as velocity, acceleration, force and momentum all have both a size and a direction. For example, what we know as "speed" is just the magnitude of the velocity vector. To fully describe velocity, we must have both the speed and the direction in which we are going.

We represent vectors geometrically with an arrow. This arrow will have a length that represents magnitude and point in a direction. This worksheet will walk you through some basic vector operations. In your textbooks, you will see vectors denoted in boldface (\mathbf{v}), but when writing a vector, we denote it by writing an arrow above the letter (\vec{v}). We can also, and will do here, use this notation: \vec{v}_{AB} to denote a vector that has its tail at a point A and the tip of its arrow at another point B . In this case we say "The vector from A to B ."

Vector Facts

There are many different types of vector quantities, but they all have similar properties. To explore these properties, let's look at one type of vector quantity, position vectors.

Suppose you are trying to get from Valsetz Dining Hall to the Natural Sciences Building. On the map below, you begin at Valsetz (point A) and you walk to Monmouth Ave., due east, about 400 feet (point B). The vector \vec{v}_{AB} describes this displacement by starting where you started and ending at the place you will turn. It points in the direction you will go, and has a length (magnitude) that represents the 400 feet you will walk. When you reach point B , you turn and walk down Monmouth Avenue about 1500 feet, and end up at point C , in front of the Natural Sciences Building where you will have your favorite class, physics. Vector \vec{v}_{BC} represents this displacement in the same way as \vec{v}_{AB} , but with the new direction and magnitude. If you were a bird and you wanted to fly from Valsetz to NSS, you would probably fly along vector \vec{v}_{AC} , which describes your flight path, stating point, ending point, direction, and magnitude in much the same way.



As you can see, vectors represent some very important information and, once you get used to using them, will be instrumental in helping you to visualize and solve most physics problems.

Vector Addition and Subtraction

Back to the map above. You start out at point, A , and move to point B . Then you turn and go to point C . If you were the bird, you could have accomplished the same net direction by flying directly from A to C . The resulting displacement vector \vec{v}_{AC} is the sum of \vec{v}_{AB} and \vec{v}_{BC} and is written $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$. This is illustrated in Figure 2.

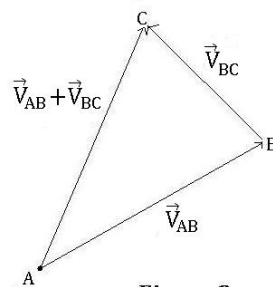
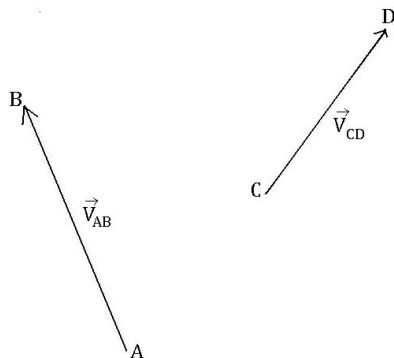


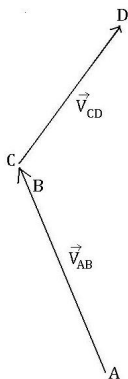
Figure 2

Figure two is useful when we are presented with vectors that are placed with the tip of one

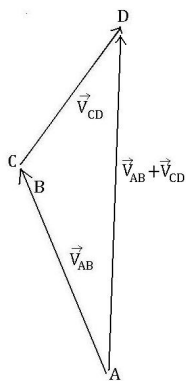
vector at the tail of another. This placement is common when we are discussing position, but more often we want to add vectors that look more like this:



What do we do then? Well, we can move vectors, as long as we don't change their magnitude or direction so that the tip of one is at the tail of the other, like this:



Now we can draw the vector that is $\vec{V}_{AB} + \vec{V}_{CD}$:



We can also subtract vectors. This is not very different from vector addition. Really, if we have two vectors, \vec{v}_A and \vec{v}_B , and we want to know $\vec{v}_A - \vec{v}_B$, all we do is add to \vec{v}_A a vector that is in the direction exactly opposite of \vec{v}_B . Thus, $\vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$. Figure 3 illustrates this point.

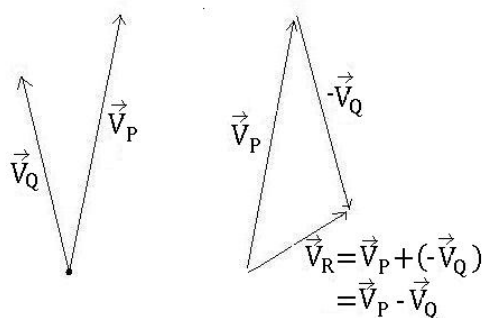


Figure 3

In subtracting vectors, like adding them, if we are presented with vectors that are not placed "tip to tail", we can move them so they are, and then perform our calculations.

Vector Components

Drawing pictures of vectors is great to get a sense of what we have been learning conceptually, but to obtain accurate results, we have to use some math, specifically trigonometry and geometry. Going back to our map above, we can place a coordinate system over the map, with the origin at Valsetz, and see that \vec{v}_{AB} represents motion along the x -axis, and \vec{v}_{BC} represents displacement along the y -axis. The resultant vector \vec{v}_C has *components* in both the x and y directions. Thus, if \vec{v}_x and \vec{v}_y are the x and y components of \vec{v} , then $\vec{v} = \vec{v}_x + \vec{v}_y$. \vec{v}_x and \vec{v}_y are called *component vectors*. The *x-component vector* is the projection of \vec{v} along the x -axis, and the *y-component vector* is the projection of \vec{v} along the y -axis. To visualize a projection, imagine a flashlight on the vector pointing from top to bottom will leave a shadow, or projection, on the x -axis. Figure 4 will be of use to shed some light on this idea.

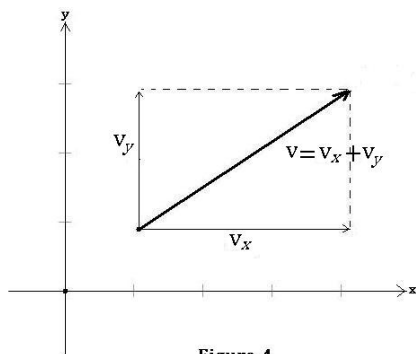
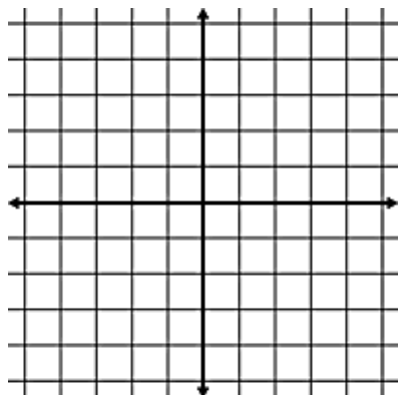


Figure 4

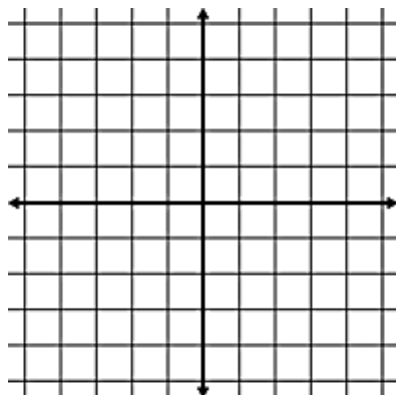
Here's how you determine the components of a vector, \vec{v} :

1. The absolute value $|v_x|$ of the x -component of \vec{v} is the magnitude of \vec{v}_x .
2. The *sign* of $|v_x|$ is positive if \vec{v}_x points in the positive x direction, and negative if \vec{v}_x points in the negative x direction.
3. The y -component \vec{v}_y is determined in the same way.

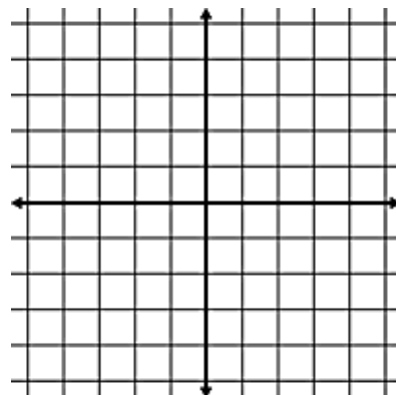
Let's practice drawing some vectors. On the axes below, start at the origin and draw the given component vectors and the resultant vector. Each line is one unit.



(a) $|v_x| = 2, |v_y| = 3$



(b) $|v_x| = -4, |v_y| = 4$



(c) $|v_x| = -3, |v_y| = -5$

Now, let's talk about how one finds the exact numerical values for the magnitudes of vectors. Consider the vector in figure 5.

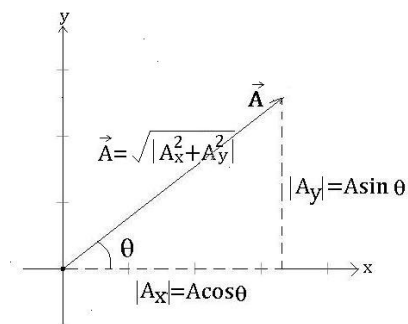


Figure 5

Let's place our \vec{A}_x component vector (see figure 5) along the x -axis. Let's also place our \vec{A}_y component vector so that it runs from the x -axis to the tip of \vec{A} . Why did we do this? We did this because this placement of the component vectors makes the computations for finding the length of them (and the length of \vec{A}) much easier. Notice also that \vec{A} has an angle of θ above the x -axis. Trigonometry gives us the lengths of our component vectors, and since our component vectors form a right triangle with \vec{A} as its hypotenuse, the Pythagorean Theorem can be used to find the length of \vec{A} . An example may help.

Example:

Find the length of the x and y components of vector \vec{R} if \vec{R} is at an angle of 30° above the x -axis and has a magnitude of 25 cm.

Solution:

$$|R_x| = R \cos \theta = 25 \cos(30) = 21.65 \text{ cm}$$

$$|R_y| = R \sin \theta = 25 \sin(30) = 12.5 \text{ cm}$$

Is this solution correct? We can check it using the Pythagorean Theorem:

$$\vec{R} = \sqrt{|R_x|^2 + |R_y|^2}, \text{ so } \sqrt{21.65^2 + 12.5^2} = 25 \text{ cm. Check!}$$

Going the other way, we can also determine the length and angle of a vector from its x and y components. As in the check of the example above, we can use Pythagorean Theorem to find the length of a vector. To find the angle, we can use the fact that the tangent of angle θ is the ratio of the opposite side to the adjacent side (SOHCAHTOA, remember?), so $\theta = \tan^{-1} \left(\frac{|R_y|}{|R_x|} \right)$.

Using components, we can now add (and subtract) vectors exactly. Thus, if $\vec{R} = \vec{A} + \vec{B}$, then $|R_x| = |A_x + B_x|$ and $|R_y| = |A_y + B_y|$. See figure 6.

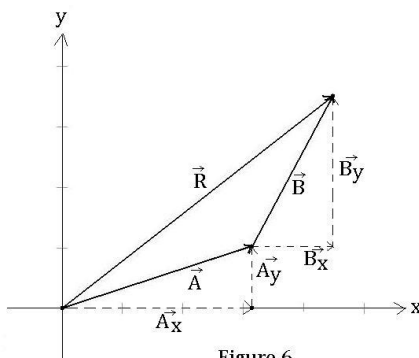


Figure 6

Here's an example:

Example:

Find $\vec{R} = \vec{A} + \vec{B}$, given that \vec{A} has magnitude 4 and is at an angle of 30° above the positive x -axis and \vec{B} has magnitude 2 and is at an angle of 55° above the positive x -axis.

Solution:

First find the x and y components of \vec{A} and \vec{B} .

$$|A_x| = A \cos \theta_A = 4 \cos(30) = 3.46$$

$$|A_y| = A \sin \theta_A = 4 \sin(30) = 2$$

$$|B_x| = B \cos \theta_B = 2 \cos(55) = 1.15$$

$$|B_y| = B \sin \theta_B = 2 \sin(55) = 1.64$$

Now add them to find the x and y components of \vec{R} .

$$|R_x| = |A_x + B_x| = 4.61 \qquad |R_y| = |A_y + B_y| = 3.64$$

Use the Pythagorean Theorem to find the length of \vec{R} .

$$|R| = \sqrt{|R_x^2 + R_y^2|} = \sqrt{(4.61)^2 + (3.64)^2} = 5.87$$

Now use \tan to find the measure of \vec{R} above the x -axis.

$$\tan \theta = \frac{|R_y|}{|R_x|} = \frac{3.64}{4.61} = 0.790.$$

$$\tan^{-1}(0.790) = 38.3^\circ.$$

Thus, \vec{R} has a magnitude 5.87 and is at an angle of 38.3° above the positive x -axis.

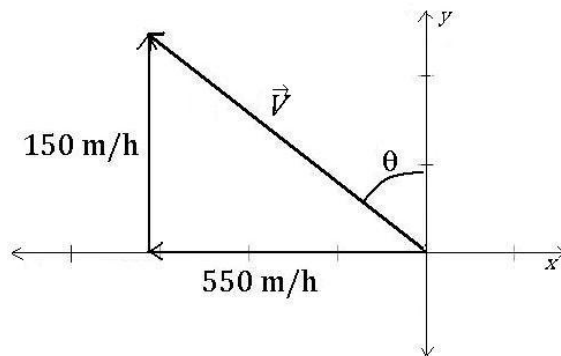
So far, we have only been measuring our angle θ from the positive x -axis. However, using the laws of Trigonometry, we can easily measure θ from any axis we choose. This is simply a matter of keeping track of the component vectors and where they are in relation to the angle of measurement.

Lets say, for example, that you are on a jet plane that takes off from PDX. Its velocity is 550 miles per hour due east. There is a wind blowing with a velocity of 150 miles per hour from the south.

A) Use vector addition to diagram the two vectors and calculate the resultant vector.

B) What is the direction of the jet's velocity vector measured east of north?

The first step in solving any physics problem is to draw a diagram including all of the relevant information. So lets do that:



This diagram gives us a clear picture of the information we have been given in the problem and the information we are looking for.

To solve part (A), we use what we know about adding vector components.

$$|V| = \sqrt{|V_x^2 + V_y^2|} = \sqrt{(-550)^2 + 150^2} = 570m/h$$

To solve part (B), we need to be careful. Notice that our angle is being measured from the

y -axis toward the negative x -axis. This means that our x component is the side opposite θ and the y component is adjacent to θ . Thus, our calculation for the angle measured east of north looks like this:

$$\tan \theta = \left(\frac{V_x}{V_y} \right)$$

Thus,

$$\theta = \tan^{-1} \left(\frac{V_x}{V_y} \right) = 74.74^\circ$$

So we can conclude that the jet is moving at 570 m/h at an angle of 74.74° east of north.

Let's tie all of these ideas up in a nice, fun physics problem.

You may be familiar with Newton's Second Law of Motion, $\Sigma \vec{F} = m\vec{a}$. In English, this equation says: The sum, or net, (Σ) of the forces (\vec{F}) acting upon an object equals (=) the mass (m) of the object multiplied by the object's acceleration (\vec{a}). Even if you are familiar with this famous equation, did you notice the arrows above F and a before? If so, you may have wondered what they mean. They are vectors! This means that they have both a magnitude and a direction and can be expressed in terms of their components. These arrows are a flashing red light that alerts you to the fact that these quantities imply *two* equations; one in the x direction and one in the y direction.

We'll work through a problem together:

A pack of four Arctic wolves are exerting four different forces upon the carcass of a 500 kg dead polar bear. Wolf one is pulling on the bear with a force of 20 Newtons (N) due north. Wolf two is pulling on the bear with a force of 30 N, 30 degrees north of west. Wolf three is pulling on the bear with a force of 25 N due west, and wolf four is pulling on the bear with a force of 35 N, 10 degrees west of south. Determine the net force acting upon the polar bear and the direction of that force measured north of west. Then compute the magnitude of the acceleration of the polar bear.

Wow. This much information can be scary at first but we are just going to use the techniques we have learned above and work our way through. Take a deep breath. You will be fine.

Your first step should always be to draw a diagram of the situation. Do so here: (Hint: Draw this as though you are looking down on it from directly above with the bear at the origin of your coordinate system, like a two-dimensional tug-o'-war.

We are trying to find the length (magnitude) of the total force vector and its direction. To find the length, we will need to find the x and y components of each of the forces being exerted on the bear. Lets take it a vector at a time.

Wolf One

$$|F_x|=$$

$$|F_y|=$$

Wolf Two

$$|F_x|=$$

$$|F_y|=$$

Wolf Three

$$|F_x|=$$

$$|F_y|=$$

Wolf Four

$$|F_x|=$$

$$|F_y|=$$

Now add up all of the x components: (Careful of minus signs)

Add up all of the y components: (Careful of minus signs)

Now use Pythagorean Theorem to find the magnitude of the total force vector:

What direction is the total force in?

Now find the magnitude of the acceleration by using the equation $\Sigma|F| = m|a|$ and what you know about solving for stuff.

A word on notation:

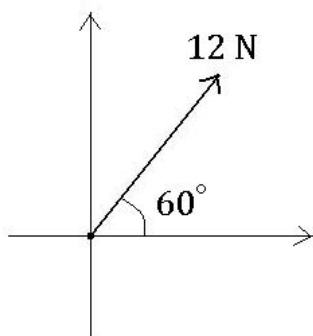
So far we have been expressing vectors and their components as $\vec{v} = \vec{v}_x + \vec{v}_y$. This notation is acceptable but you may run into something that looks like this: $\vec{v} = \langle v_x, v_y \rangle$. This notation is more formal, but accomplishes the same thing. In this new notation, the wolf problem above would look like this:

$$\begin{aligned}\Sigma\vec{F} &= \langle \Sigma F_x, \Sigma F_y \rangle \\ &= \langle F_{1x} + F_{2x} + F_{3x} + F_{4x}, F_{1y} + F_{2y} + F_{3y} + F_{4y} \rangle\end{aligned}$$

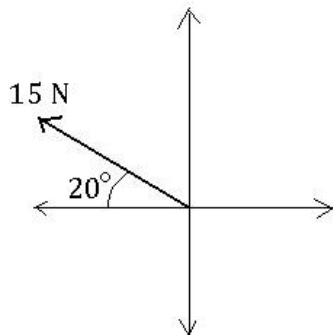
In place of all of the F_{ix} and F_{iy} 's, you would put the numbers you found in the problem and calculate the same way.

Now that you have the idea, here are some practice problems to cement the concepts in your mind.

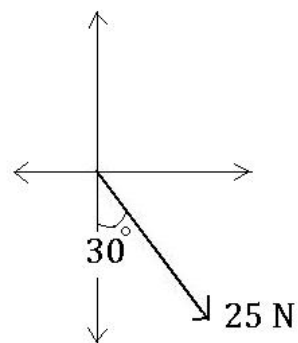
Practice Problems



(a)



(b)



(c)

1. Determine the x and y components of each of the force vectors above.

2. Are the following quantities vectors or scalars? Explain.

- (a) The cost of a theater ticket.
- (b) The current in a river.
- (c) The initial flight path from Houston to Dallas.
- (d) The population of the world.

3. Of the following quantities, which are vectors? If it is, explain why. If it isn't, explain why.

- (a) The speed of a car.
- (b) The velocity of a car.
- (c) How long it takes to do a physics assignment.
- (d) The magnitude of the force required to push a pumpkin off an abandoned overpass.
- (e) The acceleration of said pumpkin.

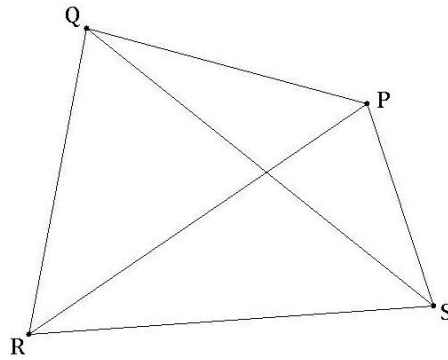
4. Write each combination of vectors as a single vector. See figure below.

(a) $\vec{PQ} + \vec{QR}$

(b) $\vec{RP} + \vec{PS}$

(c) $\vec{QS} - \vec{PS}$

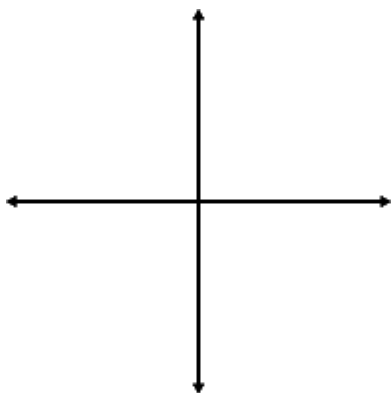
(d) $\vec{RS} + \vec{SP} + \vec{PQ}$



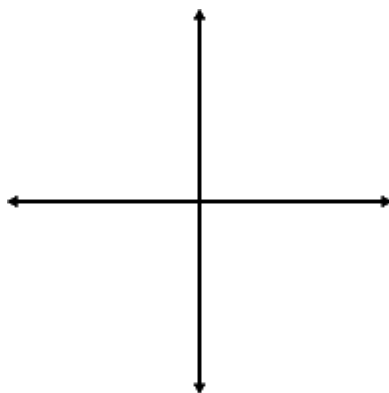
5. If a child pulls a sled through the snow with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

6. A plane flies with a velocity of 52 m/s east through a 12 m/s cross wind blowing the plane south. Find the magnitude and direction (relative to due north) of the resultant velocity at which it travels.

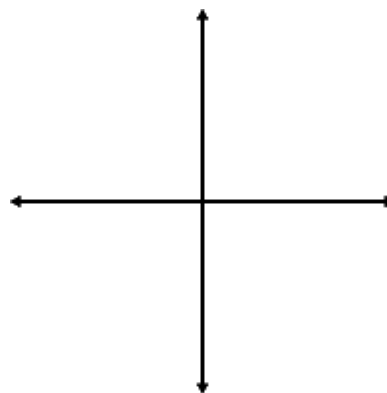
7. Draw the vector \vec{v} and find the magnitude and direction, counter clockwise from the $+x$ -axis.



(a) $v_x=2.5, v_y=1$

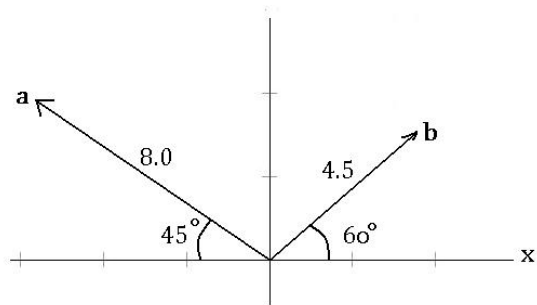


(b) $v_x=-0.5, v_y=1$

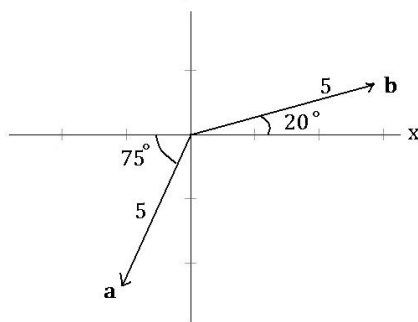


(c) $v_x=-1.5, v_y=-1$

8. Construct $\mathbf{c}=\mathbf{a}+\mathbf{b}$ by drawing and calculating the direction and magnitude of \mathbf{c} . The direction should be measured from the $+x$ -axis.

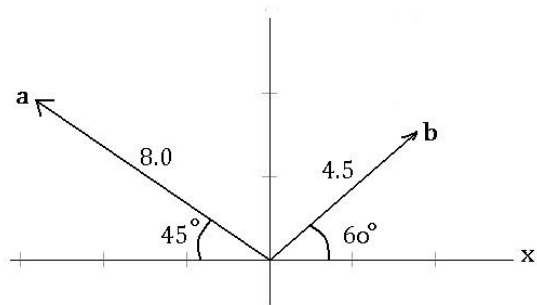


(a)

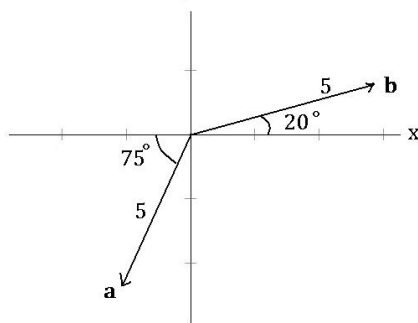


(b)

9. Construct $\mathbf{c}=\mathbf{a}-\mathbf{b}$ by drawing and calculating the direction and magnitude of \mathbf{c} . The direction should be measured from the $+x$ -axis.



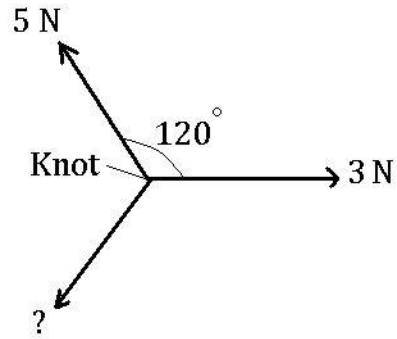
(a)



(b)

10. A racing car is accelerating at 20 m/s^2 70° N of W. Find the acceleration of the car in the north direction and in the west direction.

11. Your uncle Mike's boat can travel 4.0 m/s in still water. One sunny afternoon, you and Unk Mike decide to go fishing. While waiting for a bite, you begin thinking, "If this river is flowing at 5.5 m/s southward, and we are heading eastward, directly across the river, what are the direction and magnitude of our total velocity?" Answer your own question.



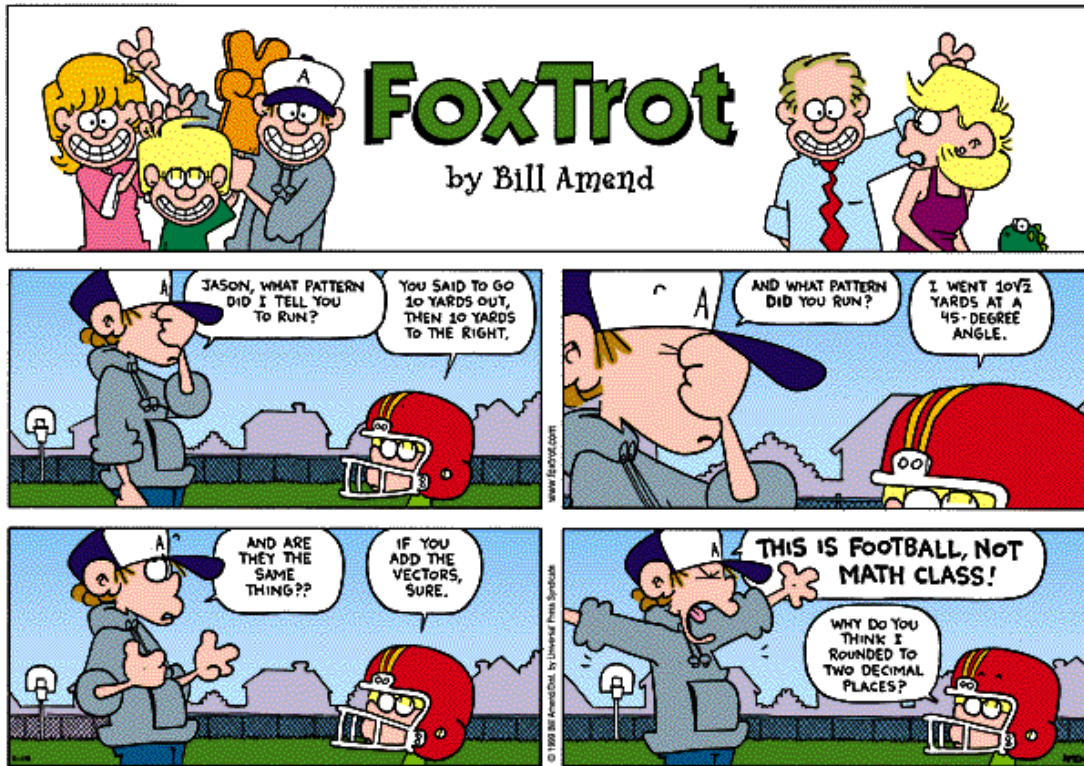
12. The figure above shows three ropes tied together in a knot. One of your friends pulls on a rope with a force of 3 Newtons and another pulls on a second rope with a force of 5 Newtons. How hard and in what direction must you pull on the third rope to keep the knot from moving? (*Hint:* If you don't want the knot to move, the net force must be zero.)

References:

Knight, Randall D., *Physics: A Strategic Approach*, Volume one, Pearson Education, 2004.

Stewart, James, *Calculus: Concepts and Contexts*, Third Edition, Thompson Brooks/Cole, 2005.

The Physics Hypertextbook, Vector Addition and Subtraction, <http://physics.info/vector-addition/>.



Worksheet created by Laura K. Waight