GUIDED PRACTICE

Class: Precalculus
Date assigned: July 16, 2018
Date due: July 30, 2018
Time estimate to complete this assignment: 90 – 120 minutes

Overview/Introduction

Finding zeros of a polynomial function in the form of $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, where $a_n, a_{n-1}, \ldots a_2, a_1, a_0$ are real numbers and n is a positive integer, will probably take more work than finding zeros of a quadratic function. If the polynomial is a polynomial with degree 2, then we have a quadratic function, for which we can easily find the zeros by factoring or using the quadratic formula. However, when we have a polynomial function with degree 3 or higher, it might be a polynomial that's not factorable. Also, the quadratic formula only works for polynomial with degree 2. Polynomial functions can be used to describe curves of various types. Polynomials are used in different areas, like engineering, business, physics, etc. For example, engineers use polynomials to graph the curves of roller coasters. In this lesson, we will learn about properties of polynomial functions, how to find zeros of a polynomial function, and how to solve a polynomial equation.

Learning Objectives

Basic objectives

1. Identify whether the given function is a polynomial function
2. Identify the degree, the leading coefficient, and the constant term of the polynomial function
3. Identify how many zeroes the given polynomial function has
4. Verify if the given number is a zero of the function
5. Identify all possible rational zeros of a given polynomial function using the Rational Zero Theorem

Advanced objectives

1. Finding zeros of a polynomial function
2. Solve real-world problems that involves polynomial function

Preparatory Activities and Resources:

1. ***Read*** the following definitions and examples.
    - A **polynomial function of degree *n*** is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, where $a_n, a_{n-1}, \ldots a_2, a_1, a_0$ are real numbers and n is a nonnegative integer.
        - n is the **degree of the polynomial**, which is the highest power of x in the polynomial.
        - $a_n x^n$ is the **leading term,** which is the term containing the highest power of the variable, or the term with the highest degree.
        - a_n is the **leading coefficient**, which is the coefficient of the leading term.
        - a_0 is the **constant term**, which is the term that has 0 degree.

Example of polynomial function	Degree of the polynomial function	Leading Coefficient	Constant Term
$f(x) = 3x^4 - 5x^3 + 2x^2 + x + 7$	4	3	7
$g(x) = -\frac{1}{2}x^2 - x$	2	$-\frac{1}{2}$	0
$h(x) = x^{10} - 3x^9 + 4x^4 - 5$	10	1	-5
$F(x) = 5x + 2$	1	5	2

- Examples of functions that are not polynomial functions:

$$f(x) = x^{1/2} - 2x, \qquad g(x) = 4x^2 - \frac{3}{x}$$

- The number a is a **zero** of the polynomial function $f(x)$ if $f(a) = 0$. Also, if a is a zero of $f(x)$, $x - a$ is a factor of $f(x)$.
  - Example
    $x = -1$ is a zero of $f(x) = 5x^4 + 2x^3 - x - 4$ because
    $$f(-1) = 5(-1)^4 + 2(-1)^3 - (-1) - 4 = 0$$
- If a polynomial $f(x)$ has $(x - a)$ as a factor exactly k times, then a is a **zero of multiplicity k** of the polynomial $f(x)$.
  - Example
    $$f(x) = x^2(x - 5)^3(x + 2)^4(x + 4)$$
    This polynomial has:
    - 0 as a zero of multiplicity of 2
    - 5 as a zero of multiplicity of 3
    - -2 as a zero of multiplicity of 4
    - -4 as a zero of multiplicity of 1

2. **Read:** Please take notes as you read Section ____ from the textbook on Page _____ (or on the website listed below) for the following topics.
   - Remainder Theorem
   - Factor Theorem
   - Fundamental Theorem of Algebra
   - Rational Zero Theorem [How to find a list of all possible rational zeros?]

   https://cnx.org/contents/_VPq4foj@7.52:BJGigUVi@12/Zeros-of-Polynomial-Functions

*****Things that you should take notes of: definition, theorem, examples, questions you have in mind

3. **Watch:** Please watch these videos and take notes when you are watching these videos.
   - Is the Function is a Polynomial? [3:30]
     https://www.youtube.com/watch?v=XwloximjjlA&index=58&list=PLHRatQsym1_jx2R1_vSKX7oMdTlKnybNU
   - Determine the maximum number of zeros of a polynomial function [2:23]
     https://www.youtube.com/watch?v=xNRla8KcdtM

- Lesson 6.8 – Finding Possible Rational Zeros [7:01]
  https://www.youtube.com/watch?v=U3e951jgI4A

Exercises: Please complete by 7/30.

- The exercises for this lesson are found on the Google Form at the website listed below (or on Canvas). Work out these exercises on your own paper while taking the quiz online. Your work is graded Pass/Fail on the basis of completeness, effort, and timeliness only.
  Link

- In preparation for class, please do the following, which will help you familiarize the topics that we will be working in class on 7/30.
  1. **Read** Section ___ from the textbook on page _____ and take notes on the following topic
     - Finding Zeros of Polynomial Functions

  https://cnx.org/contents/_VPq4foj@7.52:BJGigUVi@12/Zeros-of-Polynomial-Functions

*****Things that you should take notes of: theorem, examples, questions you have in mind

  2. **Watch** the video and take notes
     - Algebra 2 – The Rational Zero Theorem (part 1 of 2) [13:23]
       https://www.youtube.com/watch?v=koaUgQ90kd0

Questions?

Please email me at ycho13@calstatela.edu or come to my office hour if you need help on this assignment.

Online Quiz

1. Let $f(x) = (x-3)^2(x+5)$. Fill in the blanks.
   a. $f(x)$ is a polynomial of degree ____.
   b. $f(x)$ has at most _____ zeros.
   c. $f(x)$ has 3 as a zero of multiplicity ____.
   d. $f(x)$ has -5 as a zero of multiplicity ____.

2. Let $f(x) = x^3(2x+1)(3x-12)^2$. Fill in the blanks.
   a. $f(x)$ is a polynomial of degree ____.
   b. $f(x)$ has at most _____ zeros.
   c. $f(x)$ has _____ as a zero of multiplicity 3.
   d. $f(x)$ has _____ as a zero of multiplicity 2.
   e. $f(x)$ has _____ as a zero of multiplicity 1.

3. Let $g(x) = 2x^3 + 9x^2 - 2x - 9$. Fill in the blanks.
   a. The leading coefficient of $g(x)$ is _____.
   b. The constant term of $g(x)$ is _____.
   c. The factors of the leading coefficient are _____.

d. The factors of the constant term are _____.

e. List all the possible rational zeros for $g(x)$.

_____

4. Is $x = -2$ a zero of $f(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$?

5. Is $x = 1$ a zero of $f(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$?

Lesson Plan

Lesson: Finding zeros of a polynomial function

Timeframe:

Pre-class activities: assign 2 weeks ahead of time (90 – 120 minutes)
In-class activities: Two 50-min class or One 100-min class
Post-class activities: 1 week

Materials needed:

- Sorting Strips (6 – 8 sets)
- 3 polynomial functions (for Think-Pair-Share activity)
- 4 – 5 application problems involving polynomial functions (for Peer Lesson activity)
- Scientific calculator
- Chalks (if using chalk board); Markers (if using white board)

Objectives:

Basic objectives

1. Identify whether the given function is a polynomial function
2. Identify the degree, the leading coefficient, and the constant term of the polynomial function
3. Identify how many zeroes the given polynomial function has
4. Verify if the given number is a zero of the function
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Advanced objectives

1. Finding zeros of a polynomial function
2. Solve real-world problems that involves polynomial function

Background to the Lesson:

This is a Precalculus class, which is a prerequisite for Calculus. In a Precalculus class, different types of functions and their properties that the students will encounter in a Calculus class are introduced. The polynomial function is one of the types of functions that students in a Calculus class will see. Linear and quadratic functions are already covered in the previous sections or chapters. Note that they are also polynomial functions. From those sections, students should learn how to solve linear equations and quadratic equations and what a zero of a function is. Students should know how to perform synthetic division.

Introduction to Lesson:

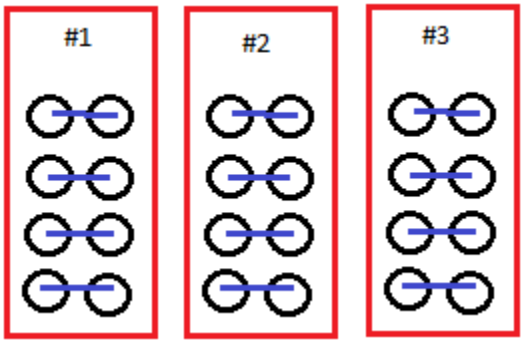
Students are required to read and watch videos about polynomial functions in the individual space. They will be assessed using an online quiz for understanding of the basic objectives and completeness of the individual space activities. The reading and videos also prepare the students for the in-class activities. The in-class activities help students learn how to find the zeros of a polynomial function with the rational zero theorem and how to solve the application that involves polynomial functions. In the post-class activity, students will be able to practice how to find the zeros of a polynomial function and how to solve the application that involves polynomial functions. This will prepare them for the next topic, which is graphing polynomial functions.

Procedure [Time needed, include additional steps if needed].

Pre-Class Individual Space Activities and Resources: Outline the major steps for the preparatory activities and be sure to tie the steps to the basic learning objectives you have noted above. Note resources required for learner preparation.

Steps	**Purpose**	**Estimated Time**	**Learning Objective**
Step 1: Read the definitions and examples on the guided practice.	Review some basic definitions of a polynomial. Provides the definitions that are needed to solve a polynomial equation.	10 min.	Basic #1, 2, 4
Step 2: Read the textbook from the online website.	Introduce the theorems needed to solve a polynomial equation.	20 min.	Basic #3, 4, 5
Step 3: Watch the videos listed on the guided practice.	Review what they read. Provide more examples that helps the students to learn the basic objectives.	20 min.	Basic #1, 2, 3, 5
Step 4: Short quiz on the basic objectives.	To check for understanding of the basic objective. Use to check for the preparedness.	20 min.	Basic #2 - 5
Step 5: Read and watch the videos about how to find the zeros of a polynomial function using rational zero theorem.	To prepare for the in-class activity	20 min.	Advanced #1

In-Class Group Space Activities and Resources. Outline the major steps for the in-class activities and be sure to tie the steps to the advanced learning objectives you have noted above. Also note any resources needed/developed to provide effective active learning activities within class.

Steps	**Purpose**	**Estimated Time**	**Learning Objective**
Step 1: #1 Sorting Strips in groups #2 Quick Write (Have the students write out the steps on how to find the zeros of a polynomial function.) [If it is two 50 minutes class, #1 will be use at the beginning of the first class and #2 will be use at the beginning of the second class. If it is one 100 minutes class, #1 will be used at the beginning of class.]	#1 Check if they completed the individual space activities #2 Check if they know the steps on how to find the zeros of a polynomial function	10 min.	Basic #1 -5
Step 2: Think-Pair-Share (Divide the class into 3-4 groups. There will be about 6-8 students in each group. Each group (or every 6 or 8 students) is given a different polynomial function. Each student will find the zeros of the assigned polynomial function individually. Then they will be paired with someone that has the same polynomial function to discuss the problems. After they discuss the problem in pairs, we will discuss all three or four problems as a class.)  [If it is two 50 minute class, this step will be on the 1st day.]	Have student attempt the problem individually to see how much they get from the video. Share ideas with their partner. Ensure the students learn how to find the zeros of a polynomial function in the whole class discussion.	10 min – individually 10 min – in pairs 15 min – whole class discussion	Advanced #1
Step 3: Peer Lessons (Divide the students into groups of 3. Two groups will be assign the same application problem that involves a polynomial function. After they completed the problems, the two groups that have the same problems will switch their work to check on correctness. I will randomly choose 1 group to show the work on the board and the other group will explain the problem to the class.)	Have students work together to discuss/share ideas on how to solve the application problem. Have them explain their thought processes and methods out loud.	10 min – working in groups 5 min switch work and discuss 20 min - presentation	Advanced #1, 2

[If it is two 50 minutes class, this step will be Step 2 on the 2nd day.]			
Step 4: #1 Muddiest point #2 Peer Performance Evaluation Form [If it is two 50 minutes class, #1 will be use at the end of the first class and #2 will be use at the end of the second class. If it is one 100 minutes class, #1 will be use at the end of class.]	#1 Check if they have any questions #2 Ensure everyone in the group contribute to the group discussion	5 min	All objectives

Post-Class Individual Space Activities and Resources. Outline the major steps for the post-class activities and be sure to tie the steps to the advanced learning objectives you have noted above. Also note any resources learners will need to complete any post-class activities assigned after the group space activities.

Steps	**Purpose**	**Estimated Time**	**Learning Objective**
Step 1: Complete the homework assignment associated with the advanced objectives	Apply what they learned in class. Practice to gain confidence for the upcoming quiz or exam	45 – 90 minutes	All objectives

Evaluation:

Analysis.

The students will enjoy being able to discuss and share ideas on the problems related to the advanced objectives. Also, they can make sure if they are on the right track during the small group discussion and in the whole class discussion. The challenges that I may face is whether the students can recall the previous knowledges, like how to find factors of a number and how to perform the synthetic division. If the students have a hard time recalling these, it will take them longer to solve the assigned problems. The students that are not familiar with synthetic division will be asked to watch a short video on synthetic division.

Connections to Future Lessons.

This lesson will connect to the next topic for the class, which is graphing polynomial functions. Zeros of a polynomial function are x-intercepts. The multiplicity of the zeros also indicates the behaviors of the graph near the x-intercepts.