GUIDED PRACTICE: NOMINAL & EFFECTIVE INTEREST RATE

Class: ***Engineering Economic Analysis***
Date assigned: Aug. 15, 2018
Date due: Aug. 22, 2018
Time estimate to complete this assignment: 90 Minutes

Overview/Introduction

Solving problems using Nominal and effective Interest Formula:

$i_a = [\ 1 + (r / m)\ ]^{m} - 1$

Where "i" is the effective interest rate, "r" is the nominal interest rate, and "m" is the number of periodic payment.

These functions are used to find F (future worth) or P (present worth) of an investment in the following forms that are related to interest rate:

$F = P(F/P,i\%,n) = P(1 + i)^{n}$

$P = F\ (P/F,i\%,n) = F(1 + i)^{-n}$

In this lesson, we'll learn about the interest rate formula, how to use it to solve an interest rate and some real-word applications, and think about how we can use the interest formula to predict how many solutions the interest rate formula has.

Definitions you need to know:

- **Nominal interest rate** refers to the **interest rate** before taking inflation into account. **Nominal** can also refer to the advertised or stated **interest rate** on a loan, without taking into account any fees or compounding of **interest**.

- The **effective** annual **interest rate** is the **interest rate** that is actually earned or paid on an investment, loan or other financial product due to the result of compounding over a given time period. It is also called the **effective interest rate**, the **effective rate** or the annual equivalent **rate**.

What is the difference between nominal and real interest rates?

A **nominal interest rate** is the **interest rate** that does not take inflation into account. It is the **interest rate** that is quoted on bonds and loans. ... As opposed to the **nominal interest rate**, the **real effective interest rate** adjusts for the inflation and gives the **real rate** of a bond or a loan.

What is the difference between the effective and stated interest rate?

Effective interest rate is the one which caters the compounding periods during a payment plan. It is used to compare the annual **interest between** loans with **different** compounding periods like week, month, year etc. In general **stated** or nominal **interest rate** is less than the **effective** one.

What are the variables in the interest rate formula and compounding interest table?

One very important exponential equation **is the compound-interest formula**: ...where "A" **is the** ending amount, "P" **is the present or** beginning amount (or "principal"), "r" **is the nominal interest** rate (expressed as a decimal), "), "i" **is the effective interest** rate (expressed as a decimal), "**n**" **is the** number of compounding a year, and "t" **is the** total number of years.

Learning Objectives

Basic objectives (to be practiced prior to class):

Upon completion of this lesson, students will be able to:

- Identify nominal and effective interest rate formula.
- State the nominal and effective interest rate formula.
- Use the nominal and effective interest rate to find the amount of interest paid in an investment.
- Compute nominal and effective annual interest rate for investments that have different compounding methods.

Advanced objectives (to be mastered during and after class):

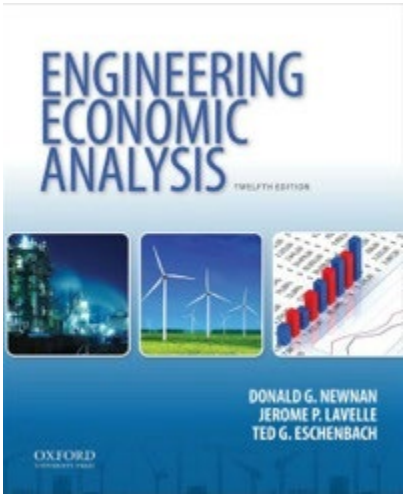
Students will be able to use the formula and/or compounded interest table to:

- Use the nominal interest rate to analyze which investment has to larger yield.
- Identify which of the four quantities P (present worth), F (future worth), i (Interest Rate), and n (the number of compounding interest payment a year at the time of investment) are known and solve the unknown fourth quantity.
- Apply the nominal and effective interest formula, and/or compounding interest table to solve a real-world problem.

Preparatory Activities and Resources for Learning:

The following are suggestions for learning the material in this lesson. You may use these plus any other additional materials you can find.

- Textbook:



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Lesson: See examples of Nominal and Effective Interest Rate section (Chapter 3, Page 89-96).

- Video:

Watch the following videos (total time 15 to 30 minutes)

1. "Intro to Interest Rates":
https://www.youtube.com/watch?v=v_ygUmjuuO0

Following the video take a short quiz (Appendix A) on paper by pencil worth 1 point. Due at the beginning of class.

2. "Interest Rate & Compounding Periods"

https://www.youtube.com/watch?v=P16hIFkNMPY

Following the video take a short quiz (Appendix A) on paper by pencil worth 1 point. Due at the beginning of class.

3. "Interest Rate & Compounding Periods"
www.youtube.com/watch?v=mvP8Lg-iCHY

Following the video take a short quiz (Appendix A) on paper by pencil worth 1 point. Due at the beginning of class.

4. "Different Compounding periods"
https://www.youtube.com/watch?v=7-K0hQParMQ

Following the video take a short quiz (Appendix A) on paper by pencil worth 1 point. Due at the beginning of class.

5. Submit a single "Continuous Interest" question (Appendix A, question 5) on paper by pencil at the beginning of the class, worth 2 points.

All of the quizzes above are to be solved by individual student on student's 8X10 lined papers and pencil.

All quizzes are due at the beginning of the class, before the In-Class Group meeting started.

- Moodle:

  For more practice on the interest rate, a Study Guide with problems and solutions will be posted on Moodle under "ME 3000 Study Guide" "Lesson: Nominal and Effective Interest Rate" (optional).

Exercises: Please complete by Aug. 22, 2018 before 10:00 p.m.

The quizzes for this lesson are found on the Google Form at: Quizzes in Appendix A Nominal and Effective Interest Rate Formula. Work out these exercises in your own notes while you are taking the quizzes and bring the solutions to class to be graded. Remember your work is graded 1 point for each quiz on the basis of completeness, effort, and timeliness only.

In preparation for class, refer to Moodle for "**More Practice Examples for the types of questions you are to find in the quizzes (optional)"**. We will practice solving these examples in class, so if you have familiarized yourselves with the examples you will be ready to start working.

Submission Instructions:

Once you submit the solutions for the quizzes in Appendix A on your papers at the beginning of the class, your work is submitted. You will see the correct answers on projector in class. Submissions are due at the beginning of the class, before the in-class group meeting started. If you do not have access to the internet where you live, make arrangement to come to library on campus.

Note:
All of the quizzes above are to be solved by individual student on student's 8X10 lined papers and pencil.

All quizzes are due at the beginning of the class, before the In-Class Group meeting started.

Appendix A
Pre-class Quiz Questions
Individual Assignment

Notation:

i = interest rate per compounding period
n = number of compounding periods
P = a present sum of money
F = a future sum of money

Single Payment Present Worth Formula

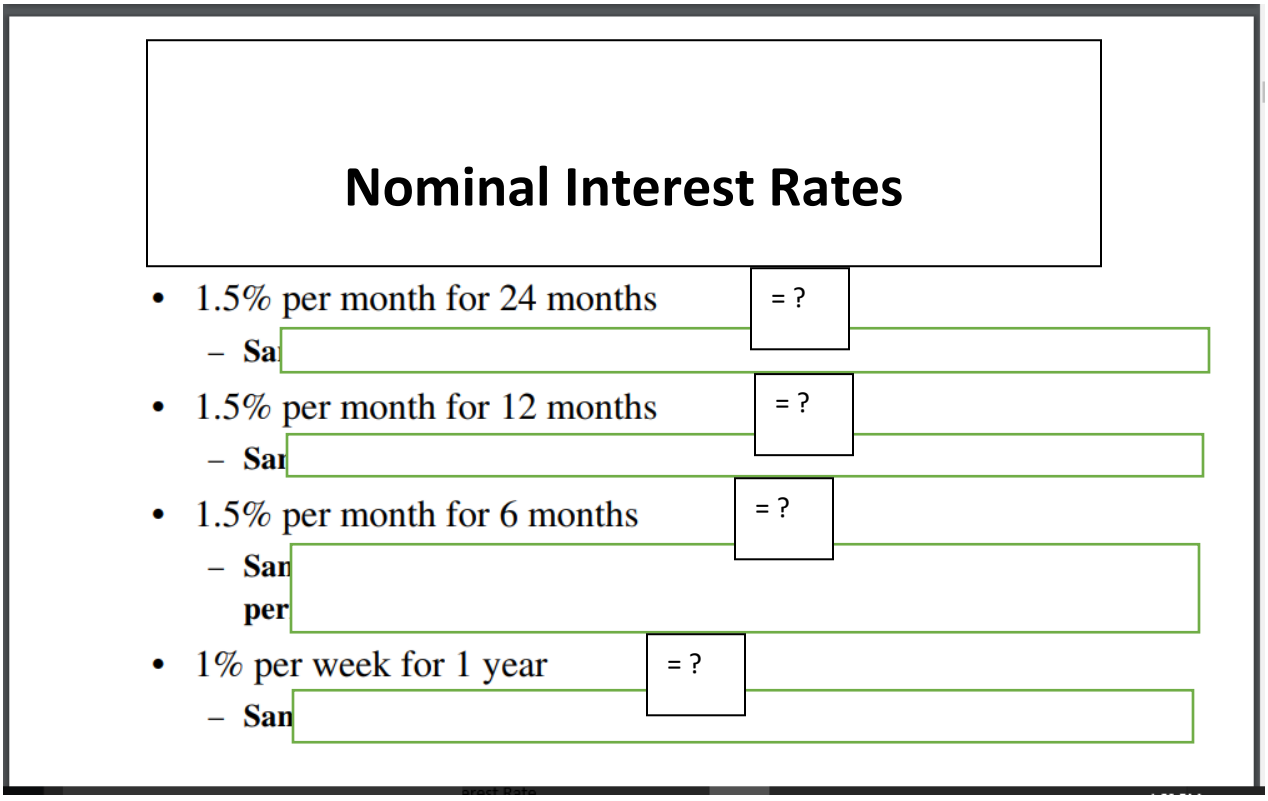
$$P = F(1+i)^{-n} \quad (3\text{-}5)$$

$$P = F(P/F, i, n) \quad (3\text{-}6)$$

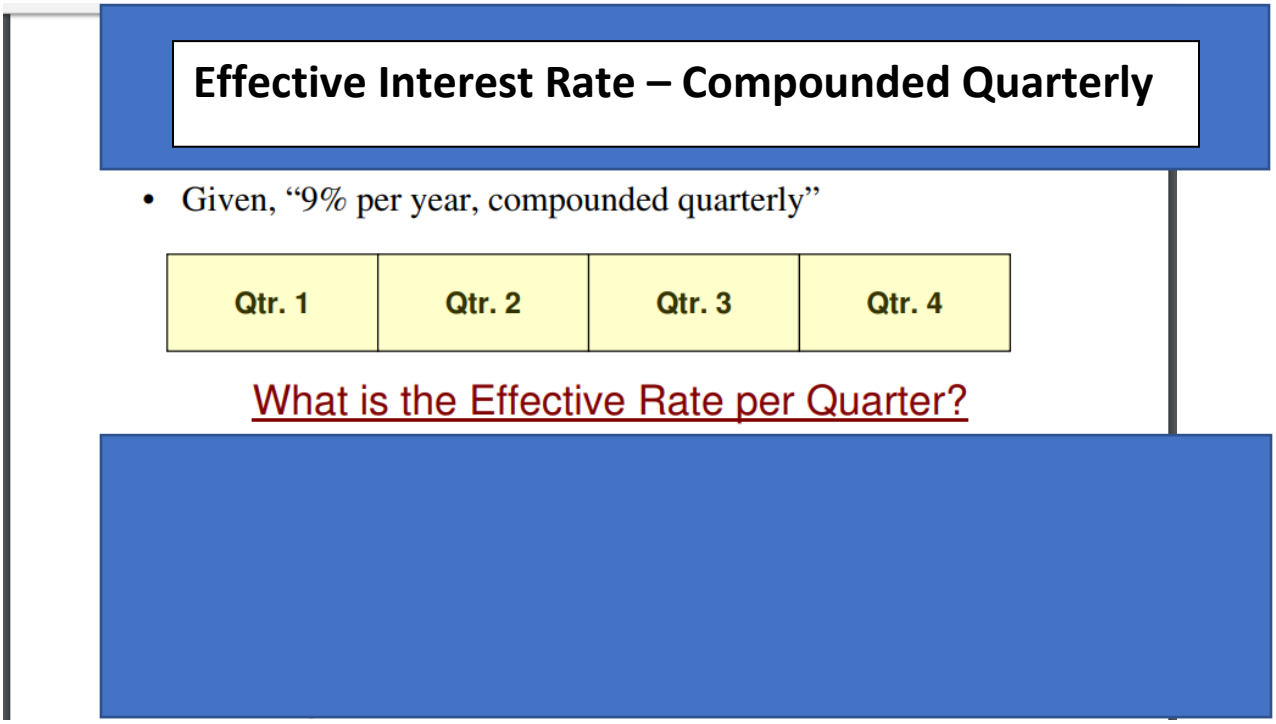
Find P, given F, at i, over n

This is the Google quiz.

Question 1 *1 point*



Question 2 *1 point*



Question 3 *1 points*

A credit card company charges 21% interest per year, compounded monthly. What effective annual interest rate does the company charge?

$i_a = [\ 1 + (r / m)\ ]^{m} - 1$

r = 0.21 per year

m = 12 months per year

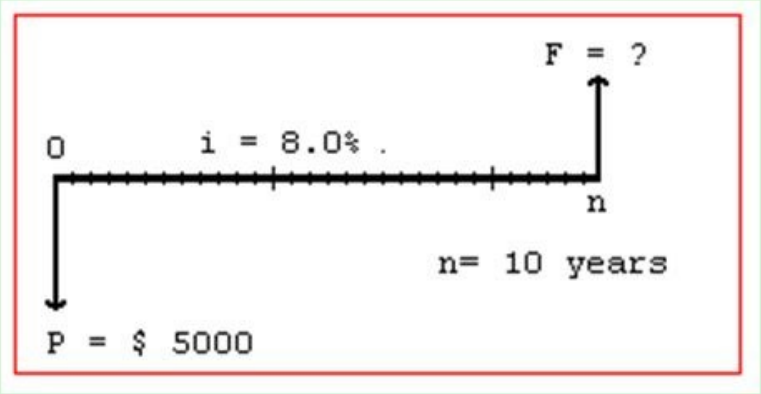
Question 4 *1 points*

Example Finding Future Value

A person deposits $5000 into a money market account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to :

(A) $2,792 (B) $ 9,000 (C) $ 10,795 (D) $12,165

The cash flow diagram is as follows:



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Question 5 *1 points*

Continuous Interest:

If $100 is invested at 8% interest per year, compounded continuously, how much will be in the account after 5 years?

$F = P\, e^{r\,n}$ F/P

$P = F\, e^{-r\,n}$ P/F

$i_a = e^{r} - 1$ Actual interest rate for the time unit (compounded continuously).

Moodle:

More Practice Examples with solutions to solving word problems for the types of examples you are to find in quizzes (optional):

Problem 1:

Sam, a friend of Pam, made an investment of $20,000 at a much later date when he turned 35. Now that he is also 50, what is his investment worth if his investment also an earned an interest rate of 6.5% compounded semi-annually.

A. $52,207.37 B. $44,491.96 C. $32,500 D. None of these

Solution:

Since the number of semi-annual periods in 15 years is 30, F can be determined as below**.**

F = P (F/P, 3.25%, 30)

= 20,000 (1 + 0.0325) ^30

= $52,207.37

The answer is "**A**".

Problem 2:

Mr. Ray deposited $200,000 in the Old and Third National Bank. If the bank pays 8% interest, how much will he have in the account and the end of 10 years?

Solution:

$F = 200{,}000(F/P, 8\%, 10) = \$413{,}800$

Problem 3:

Don Krump wants to triple his investment in 6 years. An investment firm offers him an attractive interest rate. If the interest is compounded monthly, determine the nominal interest for this investment.

A. 20.09% B. 15.76% C. 18.45% D. 16.67%

Solution:

3P = P (1 + i) ^ 72

(1 + i) = 1.015375509

i = 0.015375509 per month

Nominal interest per year = 0.015375509 × 12 = 0.1845 or 18.45%

The answer is **"C**".

Problem 4:

If you can earn 6% interest on your money, how much is $1,000 paid to you 12 years in the future worth to you now?

Solution:

$P = 1{,}000\left(\frac{P}{F}, 6\%, 12\right) = \$497.00$

Problem 5:

How long will it take for an investment to double at a 3% per year?

a) Simple interest rate
b) Compound interest rate

a) P = P + Pin
   2 = 1 + 0.03n
   n = 1/0.03 = 33.3 years
b) $2P = P(1+0.03)^n$
   $2 = 1.03^n$
   ln (2) = n ln(1.03)
   n = ln (2)/ln(1.03) =23.45 years

Note:

- If interest rate "i", and the number of compounding a year (n) is in monthly, then n will come out number of months not years, and you have to divide the result by 12 months to get "n" in years.

- Change "n" and "i" according to payment period (the number of compounding a year) when solving problems.

Good Site to refer (optional):

http://www.aui.ma/personal/~A.Berrado/EGR2302/EGR2302_Ch04.pdf

Questions?

If you have any questions email me at adanesh3@calstatela.edu.

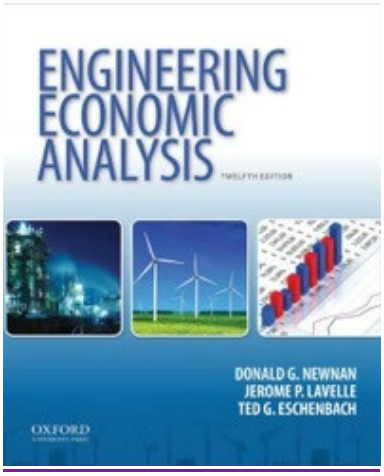
Lesson Plan
Engineering Economic Analysis
Akram Amy Daneshgaran
Engineering Department, ME

Lesson: Nominal and Effective Interest Rate (Chapter 3, Page 89-96).

Timeframe: Approximately 75 minutes

Materials needed: Students: Textbook, papers and pencils for answering quiz questions, calculator. Instructor: Projector for presenting PowerPoint slides.

Required Textbook:



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Objectives:

Upon successful completion of this unit (Nominal and Effective Compounded Interest Rate), the student will be able to do the following:

Basic:

1. State the formula for Nominal and Effective Interest Rate (see the front page of the textbook).

2. Recognize various types of the "Compounded Interest Rate" (see the Table at the back of textbook).

3. Identify and label different variables in the interest rate formula.

4. Identify different compounding periods (n).

<u>Advanced:</u>

1. *Apply the principles of nominal and effective interest rate formula to predict the amount of interest.*
2. *Practice the different types of interest rates and situations in which they occur (periodic payment).*
3. *Use the formula and table to solve interest rate problems.*

Background:

The course consists of primarily students in the engineering field and in this part of the class they are learning to use nominal and effective interest rate formula and compounded interest table to solve problems and prepare for Fundamental Engineering (FE) exam. These students often have difficulties understanding how to use different types of formulas (nominal, effective, continuous, etc.) in solving real life problems they can encounter. They practice on assigned homework problems from the textbook which are mostly real-life problems. Solving the problems needs creativity and differentiating which formulas to use related to the periodic payments, for example if periodic payment (n) is semi-annual they recognize to use n=2 and if n is quarterly they use n=4 in the same formula.

Introduction to Lesson:

During the week before the class the students will refer to guided practice on Moodle that explains nominal and effective interest rates and describes the different types of periodic payments. The guided practice also covers various types of examples of periodic payments (n=2, n=4, n=infinite, etc.). Students will distinguish which formulas to apply in different situations and when to use them. Students will also learn to use the formulas and table to calculate present and future values. All of the examples in the guided practice are followed by a short (2 question) quiz; see Appendix A for sample questions. During the in-class group activity the students will apply what they've learned to solve the in-class examples (see Appendix B). Finally, as a post-class homework exercise they will use their textbook and guided practice to solve the problems assigned.

Procedure:

Pre-Class Individual Activities and Resources [90 minutes]:

Steps	Purpose	Estimated Time	Learning Objective
Step 1: Watch the video "Intro to Interest Rates" https://www.youtube.com/watch?v=v_ygUmjuuO0 Following the video take a short quiz (Appendix A) by paper and pencil, worth 1 point. Due at the beginning of class.	Introduce students to basic interest formulas and table as it applies nominal interest rate in an investment.	20 min.	#1 (Basic)
Step 2: Watch the video "Interest Rate & Compounding Periods" https://www.youtube.com/watch?v=P16hIFkNMPY Following the video take a short quiz (Appendix A) by paper and pencil, worth 1 point. Due at the beginning of class.	Explain the basic amount of interests affected by the various periodic payments, and effective interest rate in an investment.	20 min.	#2 & #3 (Basic)
Step 3: Watch the video "Interest Rate & Compounding Periods" www.youtube.com/watch?v=mvP8Lg-iCHY Following the video take a short quiz (Appendix A) by paper and pencil, worth 1 point. Due at the beginning of class.	Explain how different periodic payments, and interest rates affect the amount of interests in an investment.	20 min.	#2 & #3 (Basic)

Step 4: Watch the video "Different Compounding periods" https://www.youtube.com/watch?v=7-K0hQParMQ Following the video take a short quiz (Appendix A) by paper and pencil, worth 1 point. Due at the beginning of class.	Introduce students how to apply different compounding periods (n) to find future and present values.	20 min.	#4 (Basic)
Step 5: Submit a single "Continuous Interest" question (Appendix A) by paper and pencil at the beginning of the class, worth 1 point. *All of the quizzes above are to be solved by individual student on student's 8X10 lined papers with pencil.* *All quizzes are due at the beginning of the class, before the In-Class Group meeting started.*	Prerequisite for starting the discussion in the group activity.	10 min.	All Basic LOs

In-Class Group Activities and Resources [75 minutes]:

Steps	Purpose	Estimated Time	Learning Objective
Step 1: Answer a selection of the "Nominal and Effective Interest Rate" questions submitted before class.	Clear up any general confusion or misconceptions.	5 min.	All Basic LOs
Step 2: Short quiz (1-2 questions) conducted in the "Think, Pair, Share" format. Questions are similar to those in the quizzes taken by paper and pencil during the pre-class Individual activities. Optional: Collect and grade.	Review and reinforce the formula and table introduced in the individual activity.	15 min.	All Basic LOs
Step 3: Working in small (3-4 person) groups, the students try to solve simple Interest Rate questions. The challenge is to predict what a simple solution will be when using formulas or table. Each team will work on questions presented on the projector and they solve the question as a group. See Appendix B for details.	Have students apply the concepts they learned in the individual activity to solve problems.	15 min.	#1, #2, #3 (Advanced)
Step 4: Using the projector, instructor shows the students what the solutions, using formulas or table, should be and discusses the key features. Students are given a chance to discuss and ask questions.	Allow students to self-assess their work and discuss it with peers.	10 min.	#1, #2, #3 (Advanced)
Step 5: Steps 3 and 4 are repeated for two more questions, each of increasing complexity (e.g., periodic payments (n) are unknown and the other variables are known). Optional: For the last questions, before solutions are shown (Step 4) the student teams turn in their solutions by paper and pencil for grading.	Reinforce the concepts by applying them to successively more difficult problems.	30 min.	All Advanced LOs

Closure/Evaluation [5 minutes]:

Describe the upcoming homework assignment (see below and Appendix C) and discuss how it relates to the group activities that were completed in class.

Analysis:

Students in this class are primarily majoring in the Engineering Field (Civil, Electrical, Mechanical, etc.) and they are accustomed to solving word problems as part of their work. They tend to classify themselves as "engineer learners" so whenever they have the opportunity to learn by paper and pencil they consider that a big plus. The in-class group activity is designed to allow students to apply the general concepts they learn in the pre-class individual activities in a mode that they are comfortable working in, namely, solving problems by paper and pencil. Engineering Economic Analysis is an extremely powerful concept but also complex and confusing. Having the students' first practice applying the concepts of interest rate manually should make it easier for them to later apply in solving homework problems, which they do as the post-class individual activity.

Post-Class Individual Activities:

Students will apply their knowledge of compounded interest rate in a homework assignment (see Appendix C). Specifically, they will be given problems in word or diagrams to solve. Their assignment will be to solve more complex real life cases, such as arithmetic and geometric gradients. To achieve an acceptable solution they will need to use different formulas to solve unknown variables in a given formula. In order to solve the problems correctly they will also have to use previous concepts and results. Note that in a previous homework assignment the students become familiarized with the interest rates by solving simple problems using formulas or table.

Connections to Future Lesson Plan(s):

The topics that come next are arithmetic and geometric gradients. For these topics an understanding of basic compounded interest rate is an essential foundation. The individual activities (watching videos; doing quizzes; submitting “compounding interest rate” question) would be similar. The group activities could also be similar; however, the small team challenge exercises would have to be tailored for each topic.

Appendix A
Pre-class Quiz Questions
Individual Assignment

Notation:

i = interest rate per compounding period
n = number of compounding periods
P = a present sum of money
F = a future sum of money

Single Payment Present Worth Formula

$$P = F(1+i)^{-n} \quad (3\text{-}5)$$

$$P = F(P/F, i, n) \quad (3\text{-}6)$$

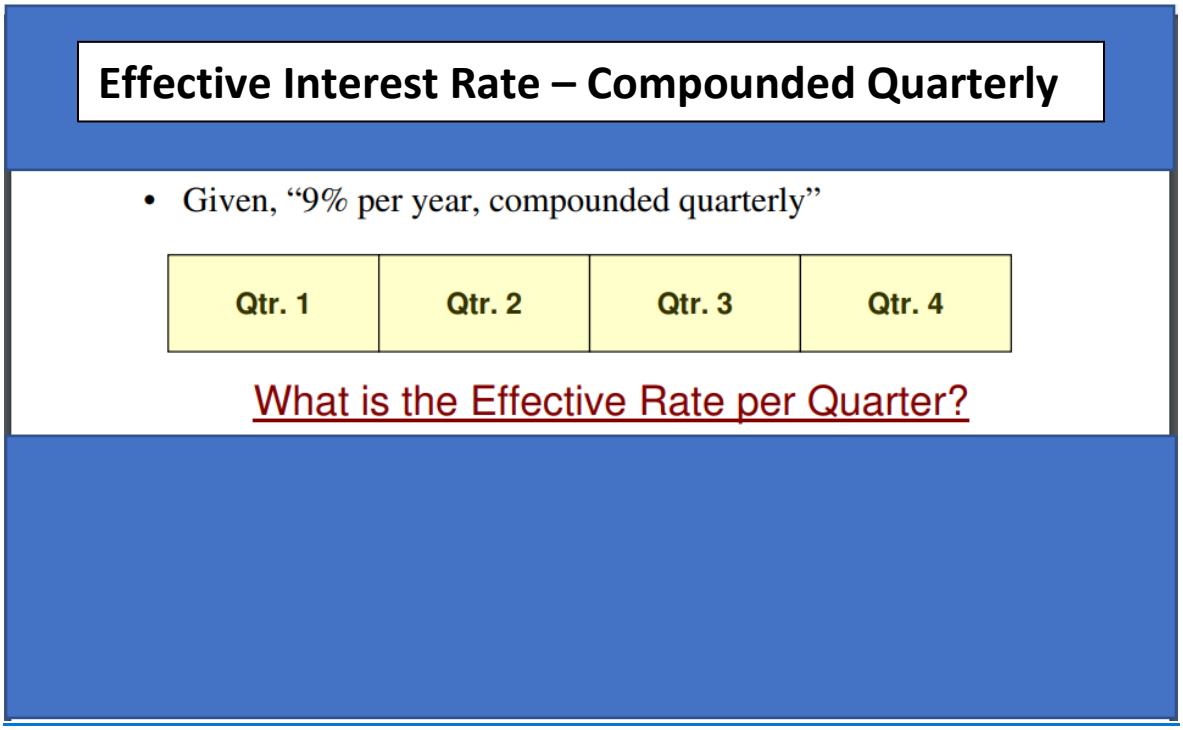
Find P, given F, at i, over n

Question 1 *1 point*

Nominal Interest Rates

- 1.5% per month for 24 months = ?
- 1.5% per month for 12 months = ?
- 1.5% per month for 6 months = ?
- 1% per week for 1 year = ?

Question 2 *1 point*



Question 3 *1 points*

A credit card company charges 21% interest per year, compounded monthly. What effective annual interest rate does the company charge?

$i_a = [1 + (r / m)]^{m} - 1$

r = 0.21 per year

m = 12 months per year

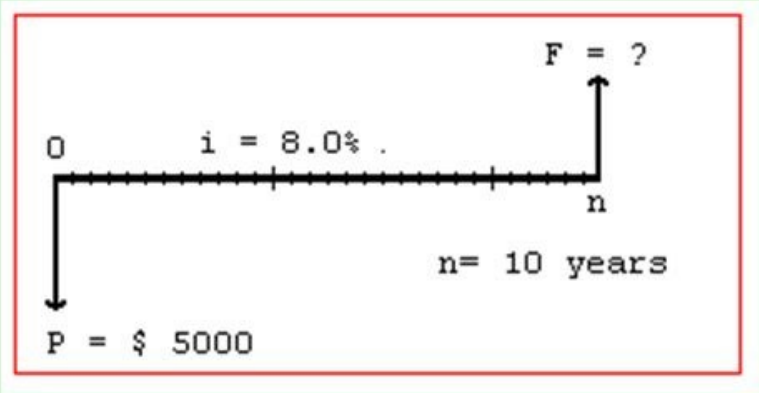
Question 4 *1 points*

Example Finding Future Value

A person deposits $5000 into a money market account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to :

(A) $2,792 (B) $ 9,000 (C) $ 10,795 (D) $12,165

The cash flow diagram is as follows:



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Question 5 *1 points*

Continuous Interest:

If $100 is invested at 8% interest per year, compounded continuously, how much will be in the account after 5 years?

$F = P e^{rn}$ F/P

$P = F e^{-rn}$ P/F

$i_a = e^{r} - 1$ Actual interest rate for the time unit

Appendix B
In-Class Group Activity

Students will see the questions on the projector similar to the one on the following pages for the in-class group activity. The purpose of the activity is for them to apply what they've learned about nominal and effective compounded interest rate to predict what the amount of interest will be in an investment.

The questions on the next pages show the amount of interest on the investment based on nominal and effective interest rate per year. On the following pages we will also have different periodic payments with different interest rates; this will be shown on the projector in the classroom.

Students will work in small (2-3 person) groups to solve the problems to predict what the amount of interests are. Specifically, they should pay attention to periodic payment (n) and change yearly interest rate accordingly. They will be encouraged to solve accurately by discussing in their group. Student from group will come to document camera and explain their group solution. Other group members can help out. Other student groups with different solution can share theirs.

After about 10 minutes of working on their own papers the solution will be shown to the class. The instructor will point out various key features in the solution that students should look for and check in their papers.

This activity is then repeated for two more times, each time with more complicated problems. Optional: For the last problem, before the solution is presented, the teams turn in their papers for grading.

Appendix B
In-Class Group Activity
(Providing Activity on Projector)

Students' name in each group:

1. _____
2. _____
3. _____
4. _____

Group Solution (Optional to be collected for grading):

Problem 1

Practice Problem

- Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have?

Problem 2

Practice Problem

- If your credit card calculates the interest based on 12.5% APR, what is your monthly interest rate and annual effective interest rate, respectively?
- Your current outstanding balance is $2,000 and skips payments for 2 months. What would be the total balance 2 months from now?

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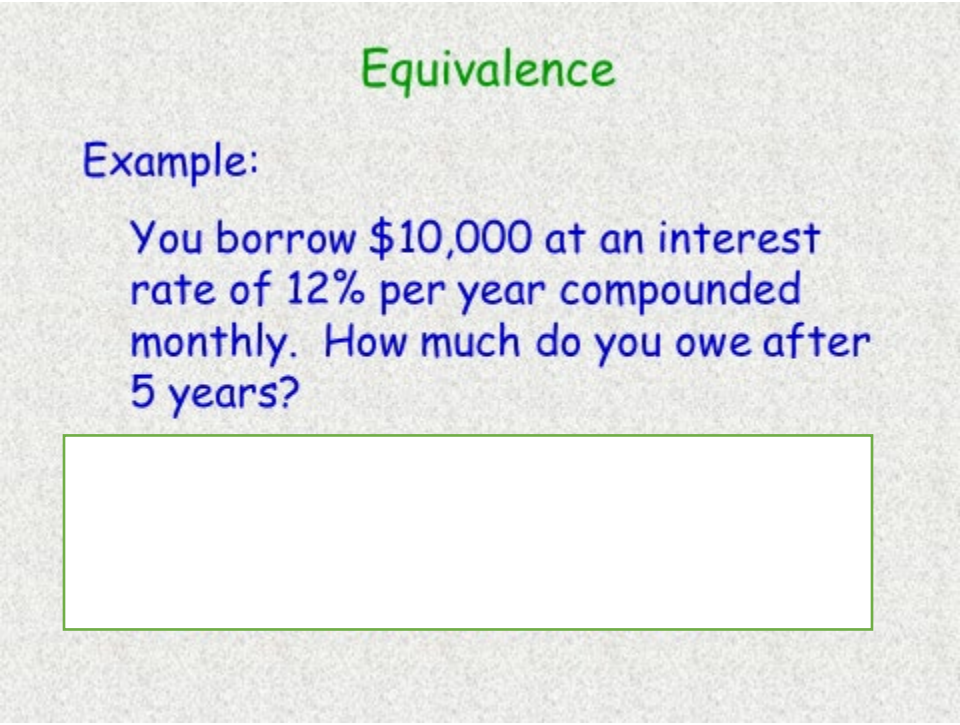
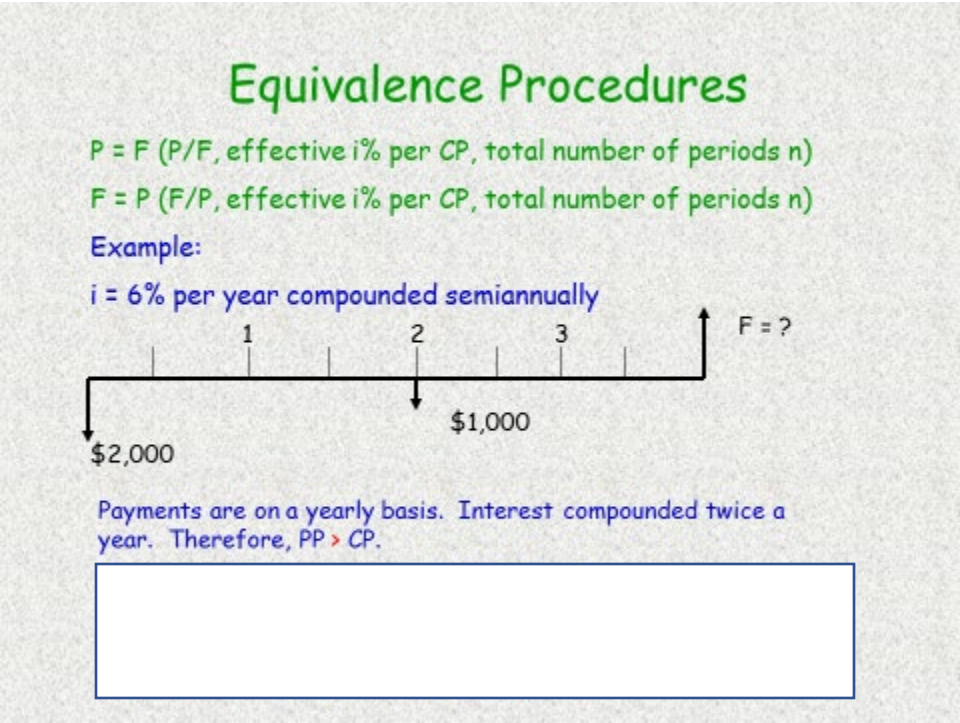
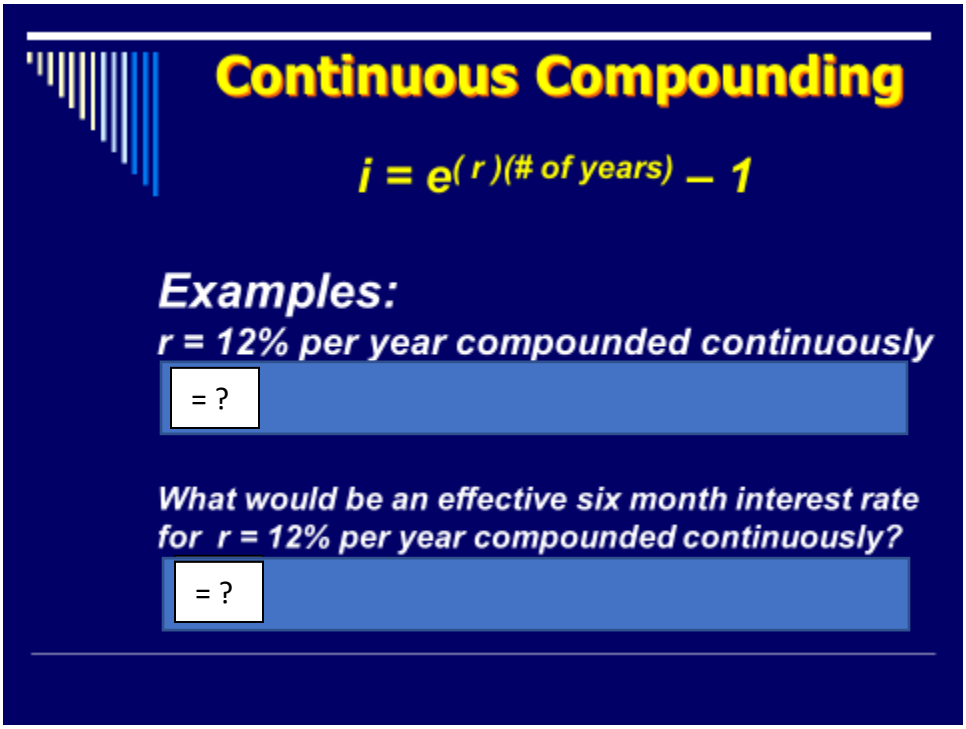
Problem 3


Equivalence
Example:
You borrow $10,000 at an interest rate of 12% per year compounded monthly. How much do you owe after 5 years?

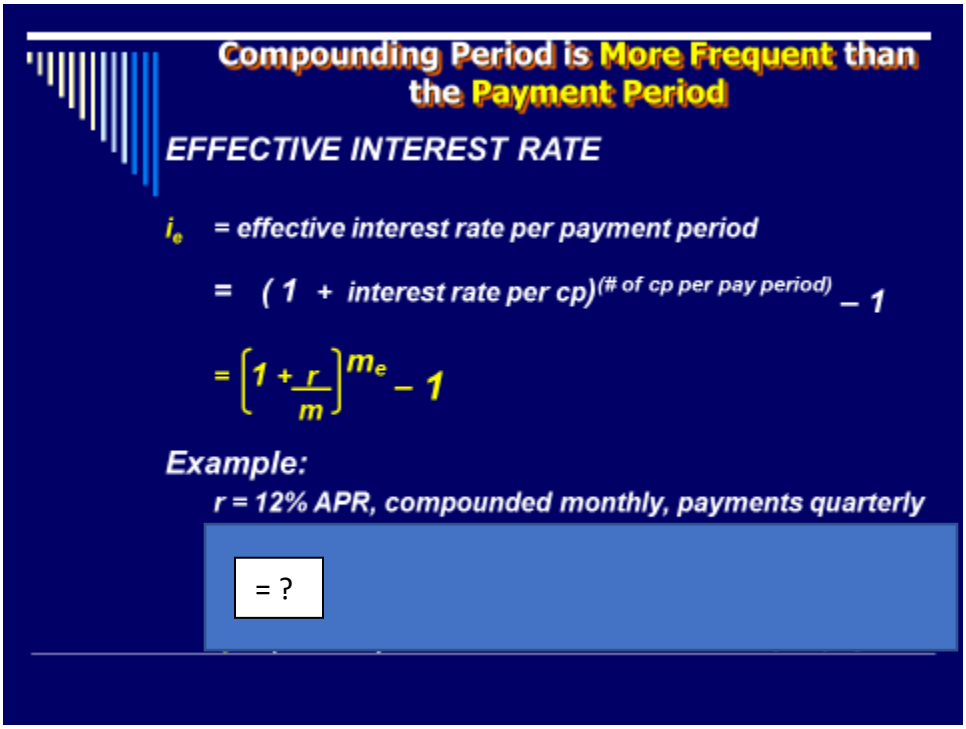
Problem 4


Equivalence Procedures
P = F (P/F, effective i% per CP, total number of periods n)
F = P (F/P, effective i% per CP, total number of periods n)
Example:
i = 6% per year compounded semiannually
1
2
3
F = ?
$1,000
$2,000
Payments are on a yearly basis. Interest compounded twice a year. Therefore, PP > CP.

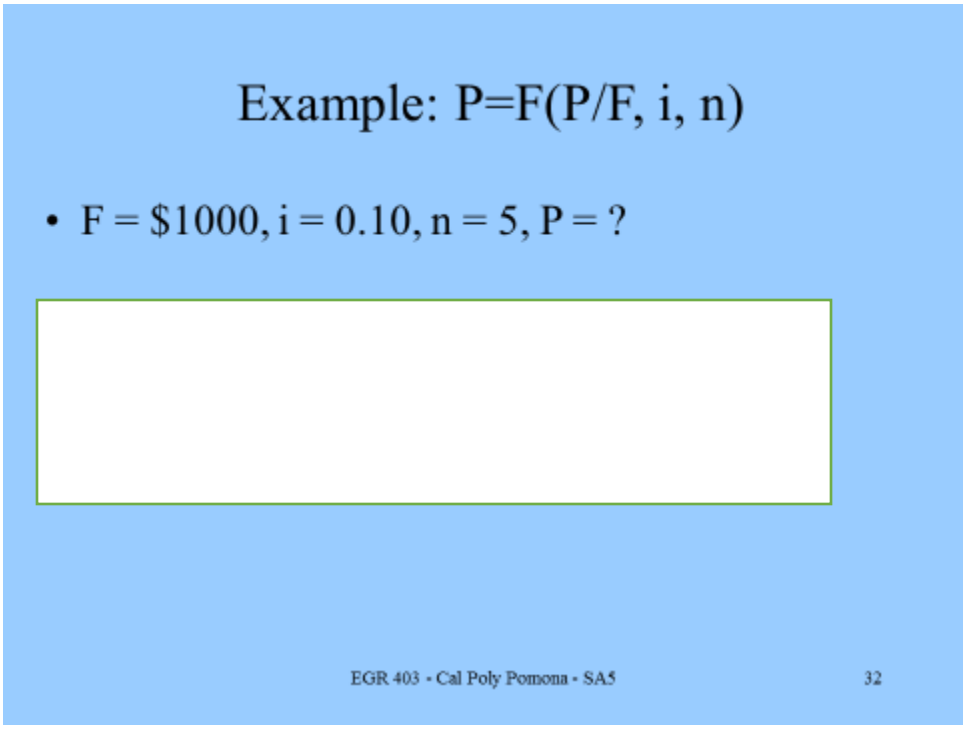
Problem 5


Continuous Compounding
i = e(r)(# of years) – 1
Examples:
r = 12% per year compounded continuously
= ?
What would be an effective six month interest rate
for r = 12% per year compounded continuously?
= ?

Problem 6


Compounding Period is More Frequent than the Payment Period
EFFECTIVE INTEREST RATE
i_e = effective interest rate per payment period
= (1 + interest rate per cp)^(# of cp per pay period) – 1
$= \left[1 + \frac{r}{m}\right]^{m_e} - 1$
Example:
r = 12% APR, compounded monthly, payments quarterly
= ?

Problem 7

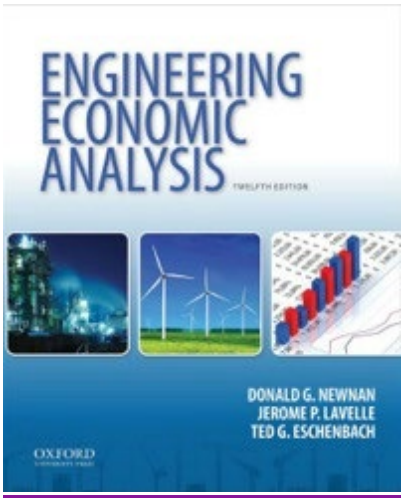


See the solutions to all pre-class quizzes and all in-class activity questions in separate attachment.

Appendix C
Post-Class Individual Assignment

The following homework assignment will allow students to apply what they learned in the Group activities.

Textbook:



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Chapter 3 Homework:

- 4, 12, 15, 18, 19, 20, 24, 25
- 30, 38

- Total: 10

10 points for completed homework, due at the beginning of the class (5 points if late).

Work individually or with you group on your homework assignment.

Here are some Interest Rate tutorials that you may find helpful:

https://www.youtube.com/watch?v=EVp6mSG9fUg (Links to an external site.)

https://www.youtube.com/watch?v=pQ5rRfU0cOg (Links to an external site.)

Here are more good examples to find present value and future value of an investment:

https://youtu.be/ks33lMoxst0?t=8m18s

https://www.youtube.com/watch?v=OXx2lqrWzfY

http://ise.tamu.edu/people/faculty/butenko/inen303/chap4.pdf

http://www.aui.ma/personal/~A.Berrado/EGR2302/EGR2302_Ch04.pdf

http://global.oup.com/us/companion.websites/9780190296902/sr/interactive/formulas/continuous/#q2

https://www.engineeringtoolbox.com/effective-nominal-interest-rates-d_1468.html

If you have a question, please email me: adanesh3@calstatela.edu

Lesson Plan

Appendix A

Sample Pre-class Quiz Questions

(Solutions)

(Providing Activity on Projector)

Question 1 *1 point*

4.1 Examples – Nominal Interest Rates

- 1.5% per month for 24 months
  - **Same as: (1.5%)(24) = 36% per 24 months**
- 1.5% per month for 12 months
  - **Same as (1.5%)(12 months) = 18%/year**
- 1.5% per month for 6 months
  - **Same as: (1.5%)(6 months) = 9%/6 months or semiannual period**
- 1% per week for 1 year
  - **Same as: (1%)(52 weeks) = 52% per year**

Question 2 *1 point*

4.1 Example 4.1 (9%/yr: Compounded quarterly)

- Given, "9% per year, compounded quarterly"

Qtr. 1	Qtr. 2	Qtr. 3	Qtr. 4

What is the Effective Rate per Quarter?

- $i_{Qtr.} = 0.09/4 = 0.0225 = 2.25\%/quarter$
- **9% rate is a nominal rate;**
- **The 2.25% rate is a true effective monthly rate**

Question 3 *1 points*

http://global.oup.com/us/companion.websites/9780190296902/sr/interactive/formulas/nominal/

A credit card company charges 21% interest per year, compounded monthly. What effective annual interest rate does the company charge?

$i_a = [1 + (r / m)]^{m} - 1$

r = 0.21 per year

m = 12 months per year

$i_a = [1 + (.21 / 12)]^{12} - 1$

$= [1 + 0.0175]^{12} - 1$

$= (1.0175)^{12} - 1 = 1.2314 - 1$

= 0.2314 = 23.14%

Question 4 *1 points*

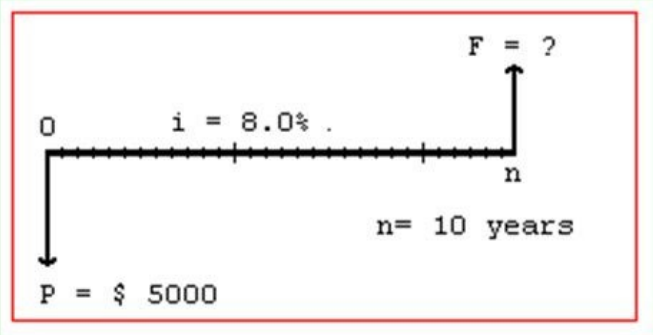
http://slideplayer.com/slide/5254786/16/images/5/Example+Finding+Future+Value.jpg

Example Finding Future Value

A person deposits $5000 into a money market account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to :

(A) $2,792 (B) $ 9,000 (C) $ 10,795 (D) $12,165

The cash flow diagram is as follows:



Solution:

F = P (F/P, i, n)

= 5000 (F/P, 8%,10)

= 5000 (2.1589)

= $ 10,794.50

Answer is (C)

2-5

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Question 5 *1 points*

http://global.oup.com/us/companion.websites/9780190296902/sr/interactive/formulas/continuous/

Continuous Interest:

If $100 is invested at 8% interest per year, compounded continuously, how much will be in the account after 5 years?

$F = P e^{r n}$ F/P

$P = F e^{- r n}$ P/F

$i_a = e^{r} - 1$ Actual interest rate for the time unit

P = $100

r = 8%

n = 5 years

$F = P e^{r n} = (\$100) e^{(.08)(5)}$

$= (\$100) e^{0.4} = (\$100)(1.4918) = \$149.18$

Appendix B
In-Class Group Activity

(Solutions)

(Providing Activity on Projector)

Practice Problem

- Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have?

(a) Interest rate per quarter:

$$i = \frac{9\%}{4} = 2.25\%$$

(b) Annual effective interest rate:

$$i_a = (1 + 0.0225)^4 - 1 = 9.31\%$$

(c) Balance at the end of one year (after 4 quarters)

$$F = \$10,000(F/P, 2.25\%, 4)$$

$$= \$10,000(F/P, 9.31\%, 1)$$

$$= \$10,931$$

Practice Problem

- If your credit card calculates the interest based on 12.5% APR, what is your monthly interest rate and annual effective interest rate, respectively?
- Your current outstanding balance is $2,000 and skips payments for 2 months. What would be the total balance 2 months from now?

Solution

Monthly Interest Rate:

$$i = \frac{12.5\%}{12} = 1.0417\%$$

Annual Effective Interest Rate:

$$i_a = (1+0.010417)^{12} = 13.24\%$$

Total Outstanding Balance:

$$F = B_2 = \$2,000(F/P, 1.0417\%, 2)$$
$$= \$2,041.88$$

Equivalence

Example:

You borrow $10,000 at an interest rate of 12% per year compounded monthly. How much do you owe after 5 years?

F = P (F/P, i, 5)

1) i_a = 12.683% per year compounded yearly

F = \$10,000 $(1.12683)^5$ = \$18,167

Equivalence

Or 1% per month for 5(12) = 60 months

2) i_a = r/m = 12%/12 = 1 % per month
compounded monthly

F = \$10,000 $(1.01)^{60}$ = \$18,167

Therefore we can conclude that 1% per month compounded monthly for 60 months is equivalent to 12% per year compounded monthly for 5 years. Both statements imply effective interest rates!

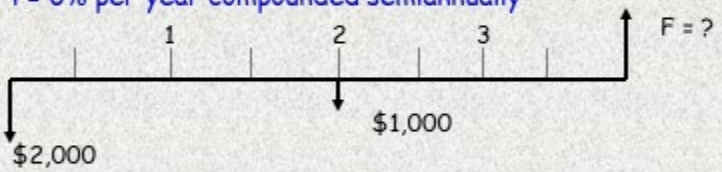
Equivalence Procedures

P = F (P/F, effective i% per CP, total number of periods n)

F = P (F/P, effective i% per CP, total number of periods n)

Example:

i = 6% per year compounded semiannually



Payments are on a yearly basis. Interest compounded twice a year. Therefore, PP > CP.

Effective i% per CP = r/m = 6%/2 = 3% per 6 months

Total number of periods = m(n) = 2(4) = 8 semiannual periods

F = \$2,000(F/P, 3%, 8) + \$1,000 (F/P, 3%, 4)

Equivalence Procedures

F = $2,000(F/P, 3%, 8) + $1,000 (F/P, 3%, 4)

Please note that the interest rate is quoted over a 6-month period which corresponds with the total number of 6-month periods.

F = $2,000(1.2668) + $1,000(1.1255)

F = $3,659

Equivalence Procedures

Method 2:

Determine the effective interest rate for the time period t of the nominal rate, and set n equal to the total number of periods using this same time period.

Example:

i = 6% per year compounded semiannually

Effective i% per year = $(1 + 0.06/2)^2 - 1 = 6.09\%$ per year

F = $2,000(F/P, 6.09%, 4) + $1,000 (F/P, 6.09%, 2)

F = $2,000(1.0609)^4 + $1,000(1.0609)^2

F = $3,659 ($3,659 from Method 1)

Method 1 is preferred over Method 2 since tables are easier to use.

Continuous Compounding

$$i = e^{(r)(\#\ of\ years)} - 1$$

Examples:

r = 12% per year compounded continuously

$$i_a = e^{(.12)(1)} - 1 = 12.75\%$$

What would be an effective six month interest rate for r = 12% per year compounded continuously?

$$i_{6\ month} = e^{(.12)(.5)} - 1 = 6.184\%$$

Compounding Period is More Frequent than the Payment Period

EFFECTIVE INTEREST RATE

i_e = effective interest rate per payment period

$$= (1 + \text{interest rate per cp})^{(\#\ of\ cp\ per\ pay\ period)} - 1$$

$$= \left[1 + \frac{r}{m}\right]^{m_e} - 1$$

Example:

r = 12% APR, compounded monthly, payments quarterly

$i_{month} = \frac{12\%\ yearly}{12\ months} = 1\%$ compounded monthly

$i_e = (1 + .01)^3 - 1 = .0303$ – or – 3.03% per payment

Example: P=F(P/F, i, n)

- F = $1000, i = 0.10, n = 5, P = ?
- Using notation: P = F(P/F, 10%, 5)
  = $1000(.6209) = $620.90