Guided Practice: Converting a System of Linear Equations to Matlab Form

Overview: MATLAB makes extensive use of linear systems for a variety of its algorithms. Therefore it is essential to be well versed in linear algebra.

LEARNING OBJECTIVES

Basic objectives (to be practiced prior to class):

BASIC (Knowledge, Comprehension and Application)

1. Students shall recognize the characteristics that distinguish a system of linear equations from other systems of equations.
2. Students shall use MATLAB operations to manipulate arrays to solve systems of linear equations.

Advanced objectives: (to be mastered during and after class):

ADVANCED (Application, Analysis, Synthesis and Evaluation)

1. Students shall analyze a given engineering statics problem to determine if it is solvable using MATLAB techniques covered in this lesson and distinguish the appropriate MATLAB technique to solve the given problem.

Background:

This course is required lower division course for mechanical engineering students. The catalog description says that the course focuses on equipping students with the basic computing skills students will need throughout their engineering disciplines. Introduction to basic engineering problems, algorithm development, and implementation into a computer program. The prerequisites or co-requisites include Math 2550 or ME 2120. The specific student outcomes include criterion 3 ABET a: an ability to apply knowledge of mathematics, science, and engineering, ABET e: an ability to identify, formulate, and solve engineering problems, and ABET k: an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

RESOURCES FOR LEARNING

The following are suggestions for learning the material in this lesson. You may use these plus any other additional materials you can find.

Text:

Textbook: MATLAB, An Introduction with Applications, 5th Edition, Amos Gilat, Wiley

or, alternatively, Wiley E-Text version, www.wiley.com/college/gilat

Today's Lesson

Chapter 2. Creating Arrays

Chapter 3. Mathematical Operations with Arrays

Another excellent reference (optional) is:

http://www.wi.tu-darmstadt.de/media/vwl1/downloads/team_2/eschenhof/LinearAlgebranew.pdf

Videos:

https://www.youtube.com/watch?v=TICOi14fa6I

https://www.youtube.com/watch?v=VFCRq88OpK0

STEP BY STEP INSTRUCTIONS

Step	**Estimated Time**
Step 1: Watch the video, "Linear Algebra: Systems of Linear Equations" from YouTube https://www.youtube.com/watch?v=TICOi14fa6I	15 mins
Step 2: Following the video, take a short quiz on Canvas. Quiz 2.1. (Worth 5 points)	15 mins
Step 3: Read from Text: Chapter 2 "Creating Arrays" Chapter 3 "Mathematical Operations with Arrays"	30 mins
Step 4: Watch the video, "Linear Algebra in Matlab" from YouTube https://www.youtube.com/watch?v=VFCRq88OpK0	15 mins
Step 5: Following the video, take a short quiz on Canvas. Quiz 2.2 (Worth 5 points)	15 mins

EXERCISES

The exercises/quizzes for this lesson are found on CANVAS at in the Lesson 1 Folder.

Work out these exercises on your own, and carefully write the solutions in your own notes so that you will have a record of your work and then take the quiz. Remember you work is graded Pass/Fail on the basis of completeness, effort, and timeliness only.

In preparation for class, please also read through the assigned text sections and the supplementary review material on Linear Algebra, which is posted on CANVAS. We will practice solving word problems using these steps in class, so if you have familiarized yourselves with the steps you will be ready to start working!

SUBMISSION INSTRUCTIONS: Submit your work on CANVAS. You should see the correct answers once you have submitted. Submissions are due at midnight the day before the class.

Stretch Problem: (Optional – Check your understanding. Solution will be posted.)

Petroleum Industry Management (ref: http://wps.prenhall.com/am_kolman_ela_9/73/18883/4834098.cw/-/4834100/index.html)

Introduction

Your group provides consulting services to a petroleum company. The task is to advise them on how to meet the demands of their customers for motor oil, diesel oil and gasoline. They have, at the moment, three plants. They have decided not to store any excess production for a variety of reasons, including added insurance costs, environmental factors, and deterioration of gasoline over time

Specifics and Work to be done

From a barrel of crude oil, factory #1 can produce 20 gallons of motor oil, 10 gallons of diesel oil and 5 gallons of gasoline. There is also waste in the form of paraffin, among other things. Similarly, factory #2 can produce 4 gallons , 14 gallons, and 5 gallons, respectively, while factory #3 can produce 4 gallons,5 gallons, and 12 gallons, respectively of motor, diesel, and gasoline. Factory #1 has 3 gallons of paraffin to dispose of per barrel of crude, factory #2 5 gallons, and factory #3 2 gallons.

Suppose the current daily demand from distributors is 5000 gallons of motor oil, 8500 gallons of diesel oil and 10000 of gasoline.

Please set up the system of equations which describes the above situation. Include units.

Next, decide how many barrels of crude oil each plant should get in order to meet the demand as a group. Remember that you can only provide each plant with an integral number of barrels.

Suppose the total demand for all products doubled. What would your solution now be? How does it compare to the original solution? Why, mathematically, should this have been expected?

Suppose that the company acquires another group of distributors and that the demand of this group is 2000 gallons of motor oil, 4000 gallons of gasoline, and 4000 gallons of diesel oil. How would you set up production of just this supply? Are there any options (more than one way)?

Next, calculate the needs of each factory (in barrels of crude, as usual) to meet the total demand of both groups of distributors. When you have done this, compare your answer to results already obtained. What mathematical conclusion can you draw?

Numerical Methods (ME 2800)

Lesson Plan

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Lesson: **Introduction to MATLAB for Application to Problems in Linear Algebra**

Timeframe: Approximately 90 minutes (pre-class); 150 minutes (in-class); 60 minutes (post-class)

Materials needed:

1. Textbook: MATLAB, An Introduction with Applications, 5th Edition, Amos Gilat, Wiley or, alternatively, Wiley E-Text version, www.wiley.com/college/gilat

2. Software: MATLAB 2017B, available on Cal State LA computers in ECST; student version available for purchase; or alternatively, you can use GNU Octave software, available for free download at www.gnu.org/software/octave

3. Computer/Internet Access

Objectives:

BASIC (Knowledge, Comprehension and Application)

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ADVANCED (Application, Analysis, Synthesis and Evaluation)

3. Students shall analyze a given engineering statics problem to determine if it is solvable using MATLAB techniques covered in this lesson and distinguish the appropriate MATLAB technique to solve the given problem.

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ability to apply knowledge of mathematics, science, and engineering, ABET e: an ability to identify, formulate, and solve engineering problems, and ABET k: an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

Introduction to Lesson:

In this lesson we discuss how Matlab, or other similar systems, can be used to solve linear systems of equations of use in many applications in engineering. A review of the basic characteristics of linear equations and systems of linear equations is presented and the use of Matlab to solve the resulting system of equations is presented.

Procedure:

Pre-Class Individual Space Activities and Resources (90 minutes):

Step	Purpose	Estimated Time	Learning Objective
Step 1: Watch the video, "Linear Algebra: Systems of Linear Equations" from YouTube https://www.youtube.com/watch?v=TICOi14fa6I	Students watch video to review the characteristics that distinguish a system of linear equations from other systems of equations.	15 mins	#1
Step 2: Following the video, take a short quiz on Canvas. Quiz 2.1. (Worth 5 points) See Appendix A.	Test your Learning	15 mins	#1
Step 3: Read from Text: Chapter 2 "Creating Arrays" Chapter 3 "Mathematical Operations with Arrays"	Introduce students to elementary Matlab scripts and the development of MATLAB scripts to solve a given problem involving a system of linear equations.	30 mins	#2
Step 4: Watch the video, "Linear Algebra in Matlab" from YouTube https://www.youtube.com/watch?v=VFCRq88OpK0	Reinforce student reading	15 mins	#2
Step 5: Following the video, take a short quiz on Canvas. Quiz 2.2 (Worth 5 points) See Appendix A.	Test your Learning	15 mins	#2

In-Class Group Space Activities and Resources (150 minutes):

Step	**Purpose**	**Estimated Time**	**Learning Objective**
Step 1: Problem – Solution Outline. "The Big Picture. Application Involving Linear Systems." Instructor will pose an engineering problem that can be solved using the techniques covered in this lesson. Students will be work in pairs (assigned by instructor) on an applied problem with objective to identify the key elements of the linear system and prepare the problem for solution. See Appendix B.	Check for student's comprehension of material and ability to apply the material on a problem.	20 mins	#1
Step 2: Mini-Quiz Student Teams will complete a short multiple-choice quiz to check elements of their solution	Provide immediate feedback on level of student comprehension	10 mins	#1
Step 3: Student Presentation A team will volunteer (or be selected) to present their solution. Instructor will provide answer	Provide opportunity for students to express their logical approach and to hear the approach of others.	10 mins	#1
Step 4: Coding in Matlab Student Teams will use Matlab to solve the problem presented above. Students will record their on Canvas.	Demonstrate mastery of, and practice skills with, the analysis and synthesis of today's lesson, through the proper use of Matlab commands.	10 mins	#2
Step 5: Skeletal Notes Instructor will provide skeletal notes and discuss them	Set context for remainder of class	10 mins	#2

Step 6. Following the Outline Students, individually, will follow the skeletal notes and results will be recorded on Canvas	Provide the students with time to reflect on the lesson and step through several problems to ensure understanding and mastery through practice	30 mins	#3
Step 7. Mini-Lecture "Common Problems" Instructor will review common errors and corrections.	Provide the student some immediate feedback through and explanation of the kinds of common mistakes and examples	10 mins	#3
Step 8. Think Pair Share Students will complete Skeletal Outlines (10 mins), Pair with teammate to discuss (5 mins) and then Class will gather to Share (10 mins)	Give the students time to correct errors in their own work and then discuss with one another. Give the students an opportunity to demonstrate their understanding and address their questions with the entire class.	25 mins	#3
Step 9. Recap Instructor will recap key concepts in lesson	Quick reinforcement of key lesson topics.	5 mins	ALL
Step 10. Mini-Lecture. "Framing the Topic" Instructor will frame the topic in the context of the goals for the course.	Reinforcement of the key elements of the lesson and their place in the larger objectives of the class.	10 mins	ALL

Closure/Evaluation (10 minutes):

Instructor will provide an explanation of the upcoming homework and a preview of the next lesson.

Students will upload comments regarding areas of greatest confusion or difficulty.

Analysis:

TBD

Post-Class Individual Space Activities:

Students will complete a homework assignment and upload their solution to Canvas before the stated deadline.

See Appendix C.

Connections to Future Lesson Plan(s):

In the next few lectures, the topics will be:

1. File Input/Output
2. Plotting
3. Polynomical Curve Fitting
4. Programming with Matlab

It is important to realize that Matlab makes use of Matrices in each of these topics. In general, the files that are read for input are in the format of matrices (as are the output files.) Matlab also makes use of pairs of one-dimensional (Nx1) vectors when calling plotting routines. The fundamentals presented in today's lecture is related directly to those applications.

We will also show how polynomial fitting, which is an extremely useful statistical interpolation technique, can be done most simply through the use of simple matrix ideas. All of this sets the stage for the more advanced Matlab programming which will be addressed in subsequent lessons this semester.

Appendix A

Sample Pre-class Quiz Questions

Problem 1. Write the following system of linear equations in matrix form A x = b.

$$3x + 4y - 6z + w = 1$$
$$2x - 3z = 8$$
$$2z + 3w = 2$$
$$3x + 3y - 4z + w = 7$$

The solution is:

(Multiple Choices to be provided on CANVAS)

Problem 2. Write the following system of linear equations in matrix form A x = b.

$$3x + 4y - 6z + w = 1$$
$$2x - 3z = 8$$
$$2z + 3w = 2$$
$$3x + 3y - 4z + w = 7$$

The following are correct MATLAB expressions for A and b:

(Multiple Choices to be provided on CANVAS)

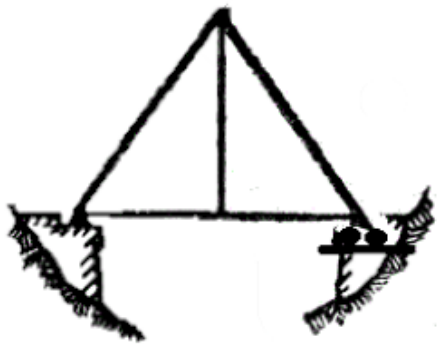
Problem 3. Identify each of the following that are linear equations.

(a) $a^2 + b^2 = c^2$
(b) $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$
(c) $y = 3x - 5$
(d) $z = 4x - 3y$
(e) $ax + by + cz + e = 0$, where a, b, c , e are constants

Appendix B

Group Space Activity

Given the simple truss structure below, set up the statics problem as a linear set of linear equations. (Hint: decompose the forces in the vertical and horizontal direction and consider whether the truss members are in tension or compression.)



Appendix C

Example Question for Post-Class Individual Space Assignment

Problem 1. Use MATLAB to solve for $\vec{x}$.

Provide the MATLAB commands and the solution for $\vec{x}$.

$$A\vec{x} = \vec{y}$$

with

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Problem 2. Solve for C = A*B – B*A. Provide the matlab commands and the solution.

with A defined above and

$$B = \begin{bmatrix} 3 & -2 & -1 & 0 & 0 \\ -2 & 3 & -2 & -1 & 0 \\ 0 & -2 & 3 & -2 & -1 \\ 0 & 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & -2 & 3 \end{bmatrix}$$

Problem 3.

Which commands create the matrix:

$a = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Which of the following are correct? (find all that are correct.)

(a) [1 2; 3 4]
(b) [1,2;3,4]
(c) [1,2,3,4]
(d) (1 2; 3 4)
(e) (1,2; 3,4)
(f) {1,2;3,4}

Problem 4. Create the matrices:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

$B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$.

Compute: $C = A^{-1}B$