

## GUIDED PRACTICE – DEFINE LIMITS AND APPLY THE CONCEPT TO PROBLEMS IN BUSINESS

Class: Math 71: Calculus for Business and Aviation

Date assigned:

Date due:

Time estimate to complete this assignment: Approximately 180 minutes

### Overview

Practical problems in business, economics and sciences can often be expressed using functions (both simple and complicated) Calculus is a branch that lets you optimize functions, analyze rates of change. The idea of the limit of a function forms the basis for a more rigorous development for the laws and procedures of Calculus. The purpose of this lesson is to provide an introduction to this concept. Our approach will be gradual starting with intuitive and leading to formal

### Learning Objectives

Basic objectives (To be completed before class)

1. Describe the notation of a limit of a function. (For example,  $x \rightarrow 2$  means  $x$  approaches 2)
2. Develop methods to find limits (numerical, graphical)
3. Discuss different kinds of outcomes/answers as to the value the limit could be

Advanced objectives (To be completed during and after the class)

1. Connect various methods to find limits.
2. Compile a list of properties of limits.
3. Determine/Decide the most efficient way to compute limits
4. Solve limit application problems

### Preparatory Activities and Resources:

1. **Watch the videos (Khan Academy)**
2. **Work out the 4 problems related to each video, you would need to work the problems on paper and bring solutions to class.**
3. Answer the muddiest points question in canvas: **What was the unclear or confusing part of the work assigned to you before class?**

### Questions?

Please complete the assignment before class. If you need help, please free to e-mail me via Canvas, use the chat feature or discussion board on canvas. You may also e-mail me ([medha.bodas@sjsu.edu](mailto:medha.bodas@sjsu.edu)).

**Lesson Plan**  
**Limits (Math 71)**  
**Lesson Plan**  
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**Lesson:** Define Limits and Apply the concepts to Business problems

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**Timeframe:** Note how long will it take the learner to complete all of the activities from pre-class to post-class activities.

**Approximately 180 mins**\_\_\_\_\_

**Materials needed:** Describe what items will be needed to complete the in-class activities.

Course textbook, on-line videos, Graphing Calculator or Desmos (graphing utility on computer), Pre-class worksheet, in-class exercises, HW set.

**Objectives:** List out the basic objectives tied to pre-class activities and the advanced objectives tied to in-class and post-class activities.

*Basic:*

- 1. Describe the notation of a limit of a function.*
- 2. Develop methods to find limits (numerical, graphical)*
- 3. Discuss different kinds of outcomes/answers as to the value the limit could be*

*Advanced:*

- 1. Connect various methods to find limits.*
- 2. Compile a list of properties of limits*
- 3. Determine/Decide the most efficient way to compute limits*
- 4. Solve limit application problems.*

**Background to the Lesson:** Note the typical composition of learners in the class, how this lesson fits into the course design/schedule, prerequisite knowledge required, and typical challenges that learners face with this content area.

**This Calculus course is designed for students taking business. The focus is to introduce the students to the concepts in calculus, use these to solve application problems. This class has a mix of students who have taken Calculus in high school and most of them who have finished Pre-Calculus or Intermediate Algebra. In many cases, the students tend to be intimidated by the subject.**

**Introduction to Lesson:** Describe the purpose of this content area for learners and an overview of the activities and resources for the flipped lesson.

**This lesson on limits starts with the visual introduction of the concept of limit gently progressing to the pedagogy of algebraic ways to find it. Students will watch a video that introduces the mathematical notation along with what a “limit” actually is. The video is accompanied by examples that focus on finding limits by graph, moves on to introduce one-sided limits. During the in-class part, we touch on algebraic method (a more sophisticated approach). Having all the tools to compute limits, we move onto the word problems (limits in real life situations) with emphasis on business applications**

**Procedure [50 minutes].**

**Pre-Class Individual Space Activities and Resources:** Outline the major steps for the preparatory activities and be sure to tie the steps to the basic learning objectives you have noted above. Note resources required for learner preparation.

Steps	Purpose	Estimated Time	Learning Objective
<p><b>Step 1:</b> Watch the video (0:00-7:34)</p> <p><a href="https://www.khanacademy.org/math/calculus-home/limits-and-continuity-calc/limits-introduction-calc/v/introduction-to-limits">https://www.khanacademy.org/math/calculus-home/limits-and-continuity-calc/limits-introduction-calc/v/introduction-to-limits</a>.</p> <p>Turn in the problems (# 5,6,7,8) in class</p> <p><a href="http://tutorial.math.lamar.edu/ProblemsNS/Calci/TheLimit.aspx">http://tutorial.math.lamar.edu/ProblemsNS/Calci/TheLimit.aspx</a> (5 points)</p>	<p><b>Introduction to the notation and idea of limit</b></p>	<p><b>20 mins</b></p>	<p><b>Basic #1</b></p>
<p><b>Step 2:</b> Watch the video (0:00-9:15)</p> <p><a href="https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-3/v/one-sided-limits-from-graphs">https://www.khanacademy.org/math/ap-calculus-ab/ab-limits-new/ab-1-3/v/one-sided-limits-from-graphs</a></p> <p>Turn in problems (#1,2,8,9) in class</p> <p><a href="http://tutorial.math.lamar.edu/ProblemsNS/Calci/OneSidedLimits.aspx">http://tutorial.math.lamar.edu/ProblemsNS/Calci/OneSidedLimits.aspx</a> (5 points)</p>	<p><b>Introduce the concept of one-sided limits, (limit may not exist)</b></p>	<p><b>20 mins</b></p>	<p><b>Basic LO #2, #3</b></p>

<p><b>Step 3:</b>  <b>Submit a single “muddiest points” question on canvas (2 points)</b></p> <p><b>What was the unclear or confusing part of the work assigned to you before class?</b></p>		<p><b>10 min</b></p>	<p><b>All 1-3</b></p>
<p><b>Step 4:</b></p>			
<p><b>Step 5:</b></p>			

***In-Class Group Space Activities and Resources.*** Outline the major steps for the in-class activities and be sure to tie the steps to the advanced learning objectives you have noted above. Also note any resources needed/developed to provide effective active learning activities within class.

<b>Steps</b>	<b>Purpose</b>	<b>Estimated Time</b>	<b>Learning Objective</b>
<p><b>Step 1:</b>  <b>: Answer a selection of “muddiest point” questions submitted before class</b></p>	<p><b>Get concepts cleared in the class</b></p>	<p><b>5 mins</b></p>	

<p><b>Step 2:</b></p> <p>“Skeletal notes” Create a set of incomplete lecture notes that include the definition of a limit and examples similar to the ones in the video (3 points)</p>	<p>Reinforce the previous concept, examples. Brainstorm if there was an algebraic way to do the same examples</p>	15 mins	All basic LO
<p><b>Step 3:</b></p> <p>Introduce the limit of a rational function.</p>	<p>Discover that this limit can be found algebraically rather than the methods learnt above.</p>	10 mins	Advanced #1, #2
<p><b>Step 4: Form groups (Turn to a partner)</b> Worksheet to find limits, algebraic examples. Students will work on word problems. Interpret the Answer</p>	<p>Read the problem and recognize the limit that needs to be computed. Compute the limit.</p>	20 mins	All advanced
<p><b>Step 5:</b> Each group presents the solution to 1 word problem on the board. Students discuss the method used to find limits, if there was another way (faster or efficient) to get to the answer. (5 points) (all advanced)</p>	<p>Students discuss the method used to find limits, if there was another way (faster or efficient) to get to the answer.</p>	15 mins	all advanced

**Post-Class Individual Space Activities and Resources.** Outline the major steps for the post-class activities and be sure to tie the steps to the advanced learning objectives you have noted above. Also note any resources learners will need to complete any post-class activities assigned after the group space activities.

Steps	Purpose	Estimated	Learning
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		Time	Objective
<b>Step 1:</b> Students will turn in Hw (problems assigned in the textbook)  Section 7.1: 30,33,36,42,45,48,49,54,57,60,63,65.  By hand the next class. (30 points)	<b>The intention is for the students to get familiar with writing mathematics, understanding the notations (notations are generally typed in online systems like Web Assign)</b>	40 mins	Advanced LO #4
<b>Step 2:</b>			

**Evaluation:**

**Analysis.** In this section, note what you think will work, and what challenges you think you may face in implementation.

*The non- algebraic introduction to a concept like limits will reduce the algebra anxiety that most face. A logical progression onto why algebra is needed and ultimately powerful tool to compute limits will make the pedagogy relevant. Word problems are challenging in math.*

*I hope to use the familiarity and fundamental idea of a limit, done in the pre-class activity, emphasized in the class to catapult the students into dealing the word problems with ease.*

**Connections to Future Lessons.** In this section, note how you think this lesson plan connects to your next topics in the course.

We will explore the concept of limits further to define Continuity. In business applications: taxes, cell-phone rate plans, hotel charges, postal charges can be modeled as step functions (not continuous everywhere)

Limits are also used to find derivatives or rates of change, for example: how does Cost change with production?