GUIDED PRACTICE

Class: Math 71 Calculus for Business and Aviation — Linda Roper
Date assigned: To be determined
Date due: To be determined
Time estimate to complete this assignment: 160 minutes

Overview/Introduction

The Derivative and the Slope of the Graph

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

This section describes how and why the slope function in algebra is expanded into the limit function in Calculus. The limit function in calculus, the derivative, will allow us to find a new slope function that will only need to use one point on a graph to find the slope. The algebra slope equation only worked accurately for a straight line, since you needed two points to find the slope. But most often in life a graph is not a straight line. If we drive a car we cannot go forever at 60 miles per hour - we need to start from a stop position - we need to speed up to get to 60 miles an hour - then at some point we will need to slow down and eventually stop. If we graph this adventure it would not be a straight line. So we need to expand our algebraic slope function to the limit function used in calculus. We will use this new limit function to find the slope function on some simple equations such as the line and the parabola. It gets very cumbersome to find the slope function of more complicated equations using the limit equation, but luckily the next few lessons after this will show us quicker ways to find the slope function than using the limit equation. It is important though to see the transition from the algebraic slope equation to the calculus limit equation. This transition will allow us find the slope using one point on a smooth curve. Wow!! That opens up a whole lot of answers to a variety of questions in Business, Aviation and Technology!!

Learning Objectives

Basic objectives

1. Define the concept of slope.
2. Describe how to graph the slope of a straight line given 2 points.
3. Describe how to graph an equation given in slope-intercept form.
4. Express an algebraic equation in slope-intercept form.
5. Interpret the formula for the slope of a straight line.
6. Calculate the slope of a straight line given 2 points.
7. Identify the definition of the derivative.

Advanced objectives

1. Describe a tangent line and how it can be used to approximate the slope at a particular point on a parabola.
2. Connect how the algebraic slope function expands to the limit function used in calculus.
3. Calculate the slope of a straight line and a parabola using the limit function.

Preparatory Activities and Resources:

Do the following to prepare for class:

- Text: Read Section 7.3 The Derivative and the Slope of a Graph
- Video: Watch the following algebra review videos (time: 24 minutes)
    - Understanding Slope (run time 7:34 minutes)
    - Graphing Linear Equations – Best Explanation – (time: 11:06 minutes)
    - Finding the Slope of a Line from two points (time: 4:32 minutes)
- Video: Watch the following video introducing the derivative (total time 15 minutes)
    - Derivative..what? (time 14:30)

Exercises: Please complete by 24 hours before class.

- Submit a "Muddiest Point" question through Canvas worth 5 points. Please let me know in advance if you do not have access to the internet where you live so we can make other arrangements.

Questions?

Contact me through email or come by my office. My email and office number are on the Syllabus and on the Mathematics Department website.

Lesson Plan

Linda Roper

Lesson: The Derivative and the slope of a Graph

Timeframe: From Pre to Post: 160 minutes

Materials needed: Worksheets, Calculators, Rulers, Projector system for presenting slides.

Objectives:

Basic:

1. *Define the concept of slope.*
2. *Describe how to graph the slope of a straight line given 2 points.*
3. *Describe how to graph an equation given in slope-intercept form.*
4. *Express an algebraic equation in slope-intercept form.*
5. *Interpret the formula for the slope of a straight line.*
6. *Calculate the slope of a straight line given 2 points.*
7. *Identify the definition of the derivative.*

Advanced:

1. *Describe a tangent line and how it can be used to approximate the slope at a particular point on a parabola.*
2. *Connect how the algebraic slope function expands to the limit function in Calculus.*
3. *Find the slope of a straight line and the slope function of the parabola using the limit function.*

Background to the Lesson: Students taking Math 71 are generally from the following majors: Business, Aviation and Technology. Generally the students coming into this course do not have a strong mathematical background. The students may not have had algebra for some time and many of their difficulties are not so much from the calculus that they learn in this course but from the algebra that is undergirding the calculus. Given an application problem, sometimes students focus so much on the calculations that they loose sight of what they are trying to find or how to find it. If they are able to visualize the bigger picture they will be able to see the steps that need to be taken to find the solution to the problem. Many application problems in Business, Aviation and Technology, if visualized on a graph, do not involve a straight line as in algebra but a smooth curve. If one wants to find the slope on a smooth curve and have it be accurate, we need to be able to find the slope using one point not two as in the slope formula used in algebra.

Introduction to Lesson: In preparation for the class the students will watch 3 short videos that review how to calculate and graph the slope used in algebra. The students will then watch a 4th short video that expands the algebraic slope formula to the limit formula used in Calculus. Using this limit function in calculus will give us another function, a slope function that will allow us to use one point on the graph to find the slope. Thus, we no longer need two points as we do in algebra. Being able to use only one point using our new function allows us to find the slope on smooth curves, and thus the answers to many more application problems. For example, if the graph of the stock market was always an upward straight line, it would have one continuous slope which you could find with the algebraic slope formula, but that is not the case. As in all things in life, the stock market has its ups and downs, which involves curves, thus we need to be able to use one point on the graph to find the slope not two.

Procedure:

Pre-Class Individual Space Activities and Resources (50 minutes):

Steps	Purpose	Estimated Time	Learning Objective
Step 1: Watch the following video youtube.com "Understanding Slope" – Eric Buffington	Review the concept of slope for a straight line	7:34 min.	#1 & #2 (Basic)
Step 2: Watch the video youtube.com "Graphing Linear Equations – Best Explanation" – Better than your Prof	Review how to graph a linear equation in slope intercept form y=mx+b	11:06 min.	#3 & #4 (Basic)
Step 3: Watch the Video youtube.com "Finding the Slope of a Line from two points" – ThinkwellVids	Review how to find the slope of a line from two points	4:32 min.	#5 & #6 (Basic)

Step 4: Watch the video youtube.com “Derivative..what? (mathbff)” - NancyPi	Introduce students to the derivative or how to find the slope function of a curved line provided the limit exist.	14:30 min.	All Basic LO’s
Step 5: Submit a “Muddiest Point” question by Canvas worth 5 points. The Muddiest Point question is due 24 hours before the In-Class Group Space meeting	Prerequisite for starting the discussion in the group space.	12 min.	All Basic LO’s

In-Class Group Space Activities and Resources. (75 minutes)

Steps	Purpose	Estimated Time	Learning Objective
Step 1: Answer a selection of the “Muddiest Point” questions submitted before class.	Clear up any general confusion or misconceptions.	10 minutes	All Basic LO’s
Step 2: Working in small (3-4 person) groups the students will work through the first 3 problems on the worksheet reviewing concepts of the slope used in algebra. Students will need rulers and will be given the attached worksheet. Groups will be self-determined.	This part of the worksheet will provide a review of the slope used in algebra to better prepare the students to see the connection to the limit function needed in calculus.	15 minutes	All Basic LO’s

Step 3: Working in small (3-4 person) groups the students will discuss and work through the problems 4, 5 and 6 where they will graph a parabola and discuss why the algebraic slope formula is not very affective on a curve. After discussing and answering problems 4, 5 and 6, the students will write down the limit formula (derivative). This formula will be used to find a new slope formula that will supply an answer to the last three questions on the attached worksheet.	Discussing and working through problems 4, 5 and 6 in the worksheet the students will realize the limitation of the algebraic slope formula and see a need for the limit process.	15 minutes	All Basic LO's Advanced LO's 1 & 2
Step 4: Working in their (3-4 person) groups the students will use the limit slope formula (derivative) to find the slope formula for a straight line and the slope formula for the parabola.	Working through these problems the students will gain a better understanding of the limit formula and how it provides a new slope function that can give an accurate slope on a line or a curve.	15 minutes	All advanced LO's
Step 5: Working in their (3-4 person) groups the students will discuss and briefly write about how the answers are different on questions 7 and 8.	As the students reflect on how the solutions differ in questions 7 and 8 they will realize the answer to number 7 (the line) is a number, a very simple function whereas the answer to number 8 (the parabola) is not a number but another function. Wow!!	10 minutes	All advanced LO's

Step 6: Groups will be assigned to share their results of a particular question with the rest of the class.	We will also discuss as a class the marvel in that the derivative gives us another function and why that has to be.	10 minutes	All advanced LO's

Post-Class Individual Space Activities and Resources (35 minutes)

Steps	Purpose	Estimated Time	Learning Objective
Step 1: Students will apply their knowledge to estimate the slope on a smooth curved in problems 1, 2 and 3 in the Web Assign homework.	Visually estimating the slope on a smooth curved will provide a deeper understanding of the slope on a smooth curve and why the slope in algebra no longer works.	5 minutes	Advanced LO's 1 & 2
Step 2: Students will apply their knowledge to find the slope of the tangent line to the graph of a function f at a given point in problems 4, 5, 6 and 7 in the Web Assign homework.	To gain a deeper understanding of the formal definition of the derivative and how one can find the slope using one point on the graph of some simple functions.	15 minutes	All Advanced LO's
Step 3: Students will apply their knowledge to find the equation of the tangent line to the graph of f at the given point as well as recognize the correct graph of the function with its tangent line at the point in problems 8 and 9 in the Web Assign homework.	To understand how to find the equation of the tangent line to the graph of f at the given point as well as recognize the correct graph of this situation.	10 minutes	All Advanced LO's

Step 5: Students will apply their knowledge in order to describe which x-values of a given function is differentiable as well as why (multiple choice) in problems 10 and 11 in the Web Assign homework.	It is important for the student to know where on a curved line a function is not differentiable.	5 minutes	All Advanced LO's

Evaluation:

***Analysis*.**

It is difficult for students to transition and see the connection between the slope in algebra and the slope in Calculus. The slope in calculus is not simply a number as in algebra but another function. It is quite a transition going from a straight line where the slope is the same everywhere versus a smooth curved function where the slope varies throughout the curve. The derivative changes the position function, the picture of the graph, into a slope function, provided the slope at every point on the graph. It is not an easy concept to grasp, but if students can take hold of this amazing concept it can make the course much easier to understand – especially with applied problems. Today Mathematics is much easier to visualize using technology to draw more complicated curves. Seeing the different curves makes it easier for students to grasp the slope function concept. To learn these concepts, It is helpful for the student to draw some simple curves such as the straight line as was taught in algebra and a simple smooth curve such as a parabola so that they can see the transition of the slope equation used in algebra to calculus. In later lessons it is encouraged to use a graphing tool either calculator or computer to better visualize more complicated equations.

This lesson may take more than one class. The first time teaching this lesson I will not force the students through this material if they need more time to work through the problems and the discussion. I will slow the lesson down if needed and finish the lesson the following class.

***Connections to Future Lessons*:**
In the next several lessons the students will learn simple techniques to find the slope (derivative) of a function. They will take the derivative of the first function, I will call the position function, to get a second function a slope function. So the second function, the slope function, will allow the student to find the slope of any point on the first function, using the x-value of the point. This will allow the student to solve many application problems in Business, Aviation and Technology.