Guided Practice: Flow through Convergent-Divergent Nozzles

OVERVIEW: A high-quality supersonic wind tunnel relies on the proper sizing of the nozzle section, thus the preliminary design of this component is of utmost importance. This lesson will guide students toward a solid understanding of the phenomenology of supersonic flows through ducts, it will allow them to be able to size any nozzles and it will put an important foundation for the diffuser's design to be discussed in later lessons.

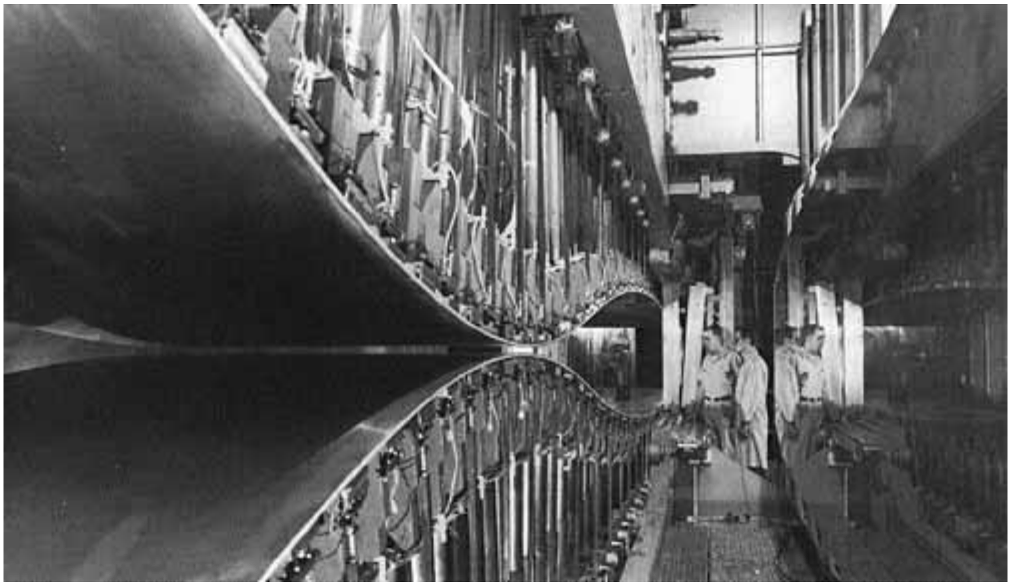

Figure 1. Hypersonic Wind Tunnel Nozzle at AEDC (Source: https://history.nasa.gov/SP-440/ch5-5.htm)

LEARNING OBJECTIVES

Basic objectives (to be practiced prior to class):

- Define quasi-one dimensional flows and identify basic equations and assumptions
- Apply the Area-Mach relation and the isentropic flow equations to calculate Mach number, pressure and temperature at each location of a duct of given geometry.
- Explain the need for a convergent-divergent duct to accelerate a flow to supersonic speeds.
- Identify the role of pressure in the process of accelerating a flow.
- Define the working regimes of a supersonic nozzle.

Advanced objectives (to be mastered during and after class):

- Calculate the plenum pressure range necessary to obtain a fully subsonic solution throughout a nozzle.
- Calculate the plenum pressure that assures a perfectly expanded nozzle.
- Calculate the plenum pressure that assures a normal shock at the exit.
- Calculate the position of the normal shock in the divergent section of the nozzle if the pressure is just below that needed to assure a normal shock at the exit.

RESOURCES FOR LEARNING

The following are default suggestions for learning the material in this lesson. You may use these plus any other additional materials you can find.

- Textbook: Read Chapter 10 of "Fundamentals of Aerodynamics" (pages 617 – 644).
- Videos: Watch the following videos (total time 8:52)
    - UQx HYPERS301x 2.5.3 Difference Between Subsonic and Supersonic Nozzles (5:07)
    - Mach 2.0 Free Jet Nozzle Flow Visualization (3:45)
- Problems: Using the textbook as a guide and work out Examples 10.1 to 10.4. Then, solve the following problems:

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    - Problem 10.2, page 655.
    - Problem 10.4, page 655.
    - Problem 10.7, page 655.
    - Problem 10.9, page 656.
    - Problem 10.12, page 656.
- Professor: Contact the instructor with any questions you may have about the material.

SUBMISSION INSTRUCTIONS: The solution of the problems listed in the previous section should be submitted before class starts by emailing it to the instructor. The report can be hand-written; every step must be justified/explained and must be clearly readable.

Aerothermodynamics (AE 164)
Flow through Convergent-Divergent Nozzles: Lesson Plan

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Lesson: Supersonic Nozzle Flows

Timeframe: 6 hours

Materials needed: Textbook, Laptop with coding software (Matlab)

Objectives:

Basic:

1. *Define quasi-one dimensional flows and identify basic equations and assumptions.*

2. *Apply the Area-Mach relation and the isentropic flow equations to calculate Mach number, pressure and temperature at each location of a duct of given geometry.*

3. *Explain the need for a convergent-divergent duct to accelerate a flow to supersonic speeds.*

4. *Identify the role of pressure in the process of accelerating a flow.*

5. *Define the working regimes of a supersonic nozzle.*

Advanced:

1. *Calculate the plenum pressure range necessary to obtain a fully subsonic solution throughout a nozzle.*

2. *Calculate the plenum pressure that assures a perfectly expanded nozzle.*

3. *Calculate the plenum pressure that assures a normal shock at the exit.*

4. *Calculate the position of the normal shock in the divergent section of the nozzle if the pressure is just below that needed to assure a normal shock at the exit.*

Background to the Lesson: the material covered in the lesson will help students to complete the preliminary design a supersonic nozzle and will allow them to be able to determine the working conditions of a supersonic wind tunnel. Emphasis will be given to the following aspects of the topic which are often overlooked: working principles of convergent-divergent nozzles, establishment of the flow and nozzle states.

Introduction to Lesson: The purpose of this lesson is to introduce students to the challenges of supersonic nozzle design, with direct connections to wind tunnel operations and practices. The students will be trained to be able to calculate the flow conditions in ducts of any given geometry and the pressure and temperature needed to "start" a nozzle.

Procedure: 6hrs

Pre-Class Individual Space Activities and Resources:

Steps	Purpose	Estimated Time	Learning Objective
Step 1: • Read the textbook pp. 618-629 • Watch the following video: https://www.youtube.com/watch?v=RvMBwJo67ZI	Become familiar with the concept of quasi 1-D flow	30min	1, 2 (basic)
Step 2: • Read the textbook pp. 629-638 • Watch the following video: https://www.youtube.com/watch?v=9UeVK1t5BgU	Become familiar with the nozzle flow states	30min	3, 4, 5 (basic)
Step 3: Solve the following problems from the textbook: 1. Problem 10.2, page 655 (easy) 2. Problem 10.4, page 655 (easy) 3. Problem 10.7, page 655 (average) 4. Problem 10.9, page 656 (complex) 5. Problem 10.12, page 656 (complex) Students are encouraged to contact the instructor for guidance in the solution of the problems.	Calculate flow conditions at any location within a duct of given geometry	1hr, 15min	2, 5 (basic) [May also help with: 1-3 (advanced)]

Step 4: • Form groups of four students, assigning a specific task to each member. The detailed distribution of the workload must be emailed to the instructor before class time. • Carefully read the in-class problem statement (uploaded on Canvas two days before class starts and attached to this lesson plan). • Formalize the problem to be solved during the in-class group space. Use the examples provided in Chapter 10 of the textbook to facilitate the task.	Identify what equations will be needed to solve the proposed problem	30min	4 (advanced)

In-Class Group Space Activities and Resources:

Steps	Purpose	Estimated Time	Learning Objective
Step 1: • Multiple choice and True/False quiz (10 questions) on quasi 1-D flows, Area-Mach relation and supersonic nozzle states based on the readings and the videos watched during the pre-class activities (5 min). • Go over the answers to the quiz (5 min).	Verify that LOs 1-5 (basic) have been reached.	10min	1-5 (basic)
Step 2: • Interactive problem solving. Students will guide the instructor through the solution of the most relevant problem of those assigned as pre-class individual space. The problems will be those related to the advanced LOs 2 and 3 (i.e., Problem 4 and Problem 5).	Prepare the students to the problems to be solved in groups during class time	10min	1-5 (basic) 1-3 (advanced)

Step 3: Part I of the solution of the in-class problem: • Discuss the problem to be solved within the group. • Propose a draft of the steps to be taken in the solution and the set of equations to be used to the instructor.	Facilitate the solution of the problem through a constructive discussion based on the individual pre-class work.	15 min	4 (advanced)
Step 4: Part II of the solution of the in-class problem: • Based on the feedback received from each group, the instructor will work out at the board part of the problem, providing the details of the steps to be taken in the iterative approach to solve it.	Facilitate the solution of the problem through a constructive discussion based on the individual pre-class work.	15 min	4 (advanced)
Step 5: Part III of the solution of the in-class problem: • The students should have a clear idea of how to approach the solution; if not they should contact the instructor right away for clarifications. • Students may begin using their computer to code the solution or to support their calculations.	Solve the in-class problem	25 min	4 (advanced)

Post-Class Individual Space Activities and Resources.

Steps	**Purpose**	**Estimated Time**	**Learning Objective**
Step 1: • Using the iterative approach discussed in class, each student (no groups) will have to calculate the position of the normal shock in the divergent section of the supersonic nozzle described in the In-class problem for the following plenum pressures: 1. 5.5atm (advanced) 2. 6.5atm (advanced) 3. 7atm (advanced) • The work will have to be submitted to the instructor within 24 hours from the end of the class.	Fulfill the requirements of LO 4 (advanced)	2hrs	4 (advanced)

Evaluation:

Analysis. The most effective steps will probably be the following: Step 4 of the individual space activities as it will allow the students to visualize a solution for the problem even before class time; Step 2 and Step 4 of the in-class activities, where the instructor will show the correct approach to the solution of this class of problems. The main challenge is related to the accountability of the learner: he/she needs to be willing to devote 4 hours and 30 minutes of individual time to really master the concepts.

Connections to Future Lessons. The lesson will help each student gain the ability to create a preliminary design for a nozzle and a diffuser section of a supersonic wind tunnel, two of the most delicate and fundamental components of the facility which will be introduced in the next two lectures.

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WORKOUT #5 – In-Class Problem

NOTE:

In order to solve the problem, you have to identify in what state the nozzle works: underexpanded, perfectly expanded, overexpanded, shock at the exit, subsonic-sonic at the throat-subsonic, shock in the divergent.

A nozzle with exit-to-throat area ratio of 25 is operated with a plenum pressure of 6atm and exhausts in a room at ambient conditions (i.e., p_b = 1atm).

a) What is the working condition of the nozzle? (i.e., underexpanded, perfectly expanded, overexpanded, shock at the exit, subsonic-sonic at the throat-subsonic, shock in the divergent).

b) Calculate the Mach number and the pressure at the exit section of the nozzle (note that if the solution admits a normal shock in the divergent section, you must calculate its location to provide your answer).