## Guided Practice Pre-CLASS Activity

Class: PHY 4330/A
Date assigned: September 16
Date due: September 18
Time estimate to complete this assignment: $\sim 1.5$ hour

## Basic Learning Objective:

After the preparatory work before class, student will be able to:

1. Define probability.
2. Find the probability of simple event such as the probability of getting a head while a fair coin is flipped 5 times.
3. Define microstate and macrostate, and multiplicity.

Please go carefully over the pages 1 through 3 from the document named "Note_Combinatorics" that is available in the course website on blackboard. Answer the following questions before you come to the class. Based on the note available we will have a quiz during the first 5-10 minutes of the class.

1. How many ways are there to arrange the cards in 52-cards?
2. You flip a fair coin 10 times.
(a) Make a list of all the possible outcome.
(b) How many macrostates are possible?
(c) How many microstates are possible?
(d) What is the probability that you will get 2 heads and 8 tails?

Class: PHY 4330/A
Date assigned: September 18
Date due: September 25
Time estimate to complete this assignment: $\sim 1.5$ hour

Based on the lecture and in class activity solve the following problems.

1. Consider an Einstein solid A with $\mathrm{N}_{A}=3$ oscillators and $\mathrm{q}_{A}=4$ units of energy.
(a) Write all possible microstates.

| Oscillator 1 | Oscillator 2 | Oscillator 3 |
| :--- | :--- | :--- |
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(b) Write down the formula for multiplicity of the Einstein solids as a function of N and q and use it to find the multiplicity of solid A?
(c) Suppose the oscillators are labelled red white and blue. What is the probability that the red particle has energy equal to one?
(d) Given that the red particle has energy 1 , what is the probability that the blue particle has energy 2 ?
(e) Suppose that another Einstein solid (solid B) with $\mathrm{N}=2$ oscillators and $\mathrm{q}=6$ unit of energy unit is weakly coupled with solid A. How many different microstates are available to the system of solid A and B now?
(f) If the system is in thermal equilibrium, what is the probability of finding all energy in solid A?
(g) What is the probability of finding exactly half of the energy in the solid A?
2. Consider two Einstein solids with $N_{\mathrm{A}}=4$ and $q_{\mathrm{A}}=10$ and $N_{\mathrm{B}}=4$ and $q_{\mathrm{B}}=2$. The two systems are thermally isolated from each other.
(a) How many macrostate are there in total for the system of solids A and B?
(b) Find the number of microstates accessible to subsystem A.
(c) Find the number of microstates accessible to subsystem B.
(d) The internal constraint is removed so that the two subsystems may exchange energy. Determine the probability $\mathrm{P}_{\mathrm{A}}\left(\mathrm{q}_{\mathrm{A}}\right)$ that system A has energy $\mathrm{q}_{\mathrm{A}}$. Fill up the following table.

| $q_{A}$ | $\Omega_{\mathrm{A}}$ | $q_{B}$ | $\Omega_{\mathrm{B}}$ | $\Omega_{\mathrm{tot}}=\Omega_{\mathrm{A}} \Omega_{\mathrm{B}}$ | $P\left(q_{A}\right)=\Omega_{\mathrm{tot}} / \Omega$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
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| 10 |  |  |  |  |  |
| 11 |  |  |  |  |  |
| 12 |  |  |  |  |  |

(e) What is the probability that energy is transferred from system A to system B?
(f) What is the probability that energy is transferred from system B to system A?
(g) What is the most probable energy for the system?

## IN-CLASS Lesson Plan (PHY 4330 Thermal and statistical Physics-Dr. Krishna Sigdel)

Topic: The mathematics of counting: Combinatorics

## Basic objectives for preparatory work:

After the preparatory work before class, student will be able to:

1. Define probability.
2. Find the probability of simple event such as the probability of getting a head while a coin is flipped 5 times.
3. Define microstate and macrostate, and multiplicity.

## Advanced objectives for classwork \& after class work:

After this class students will be able to:

1. Apply the concept of probability to shed light on various conditions such as what is the chance of finding getting exactly 25 heads and 25 tails when 50 fair coins are flipped.
2. Find the multiplicity and probability of two state system such as two-state paramagnet.
3. Find multiplicity and probability of two state Einstein solid.

|  | Time planned | Activity and rationale | Resources needed |
| :---: | :---: | :---: | :---: |
| Beginning of class period | 10 mins | Quiz: Question of the day based on the preparatory activity and discussion on preparatory reading and answer of the question of the day | Guided practice worksheet <br> Note_Combinatorics (available in black board). |
| Middle of period | 20 mins | Mini-lecture to derive formula of multiplicity of macrostate. This session will be interactive; In each step of derivation students are asked to explain how the next step emerges | Lecture notes |
| Middle of period | 30 mins | Practice solving problems and articulating solutions. Work in a group of 3-4 students. Compare their result with a neighboring group. | In class practice problem sheet |
| End of period | 15 mins | Summarize and explain the advanced work. Students solidify understanding in preparation for doing advanced work at home. | Advanced practice worksheet |

## In-class practice

1. Write the multiplicity for " N choose n " and find the value of " 20 choose 8 ".
2. Suppose you flip 50 fair coins.
(a) How many possible outcomes (microstates) are there?
(b) How many ways are there of getting exactly 25 heads and 25 tails?
(c) What is the probability of getting exactly 25 heads and 25 tails?
(d) What is the probability of getting exactly 30 heads and 20 tails?
(e) What is the probability of getting exactly 25 heads and 25 tails?
(f) What is the probability of getting exactly 50 heads and no tails?
(g) Plot a graph of probability of getting $n$ heads as a function of $n$.
