# Guided Pre-Class Practice 

(Page 1-5)

# Department of Mathematics, CSULA 

Math 1040 Pre-Calculus - Fall 2019
Instructor: Hillary Vo
Guided Pre-Class Practice
Topic: Operations with Complex Numbers (Imaginary Numbers)
Date assigned: 08/23/2019
Date due: 08/30/2019
Time estimate to complete this assignment: 50 minutes
Materials needed: an account on MyOpenMath.com, any devices connected to the Internet, online videos (see Appendix 1), pre-class worksheets (see Appendix 2), pencils, and papers.

## Overview/ Introduction

## What is this lesson about?

Introduce to Lesson: We have learned that some quadratic equations have no real solutions. For instance, the quadratic equation $x^{2}=-1$ has no real solutions because the square of any real number x is never a negative number. To overcome this deficiency, mathematicians created an expanded system of numbers using the imaginary unit $i$, defined as $i=\sqrt{-1}$. Note that $i^{2}=-1$.

This guided pre-class practice is about the basic ideas of complex numbers that you need to learn such as: definitions of complex numbers and complex conjugates, plotting complex numbers on a complex plane, and operations of complex numbers such as adding and subtracting complex numbers.

Why do we care?
Background: This is a Pre-calculus course and a main prerequisite for Calculus courses. While participating in this lesson, you have already learned about the operations with real numbers such as adding, subtracting, multiplying, and dividing. Thus, it is a perfect time for you to discover a new concept of the number system which is called a complex number and its operations. The complex number in a rectangular form is one of the most important topics in any Pre-calculus and Calculus courses. You need to understand this topic well to be wellprepared for a next lesson of complex numbers in a polar form. Furthermore, the knowledge of complex numbers is very necessary for your future majors including Electrical engineering majors, Mathematics majors, Physics majors, and so on.

Through this guided pre-class practice, you are assigned to read an online text-book from our course website www.MyOpenMath.com (Section 8.1), watch assigned videos, and practice pre-class worksheets. Once completing well all assignments in this guided pre-class practice, you will have a great opportunity to understand basic ideas of complex numbers in a rectangular form. It also trains you to develop an excellent skill of selfdirected learning. You will gain a lot of valuable experience about how to learn, understand, and solve math problems on your own after reading and watching these helpful resources.

Next steps: You will be able to apply the knowledge of complex numbers that you have learned from these preclass activities to prepare, discuss and solve many different kinds of practice problems in the advanced in-class and post-class activities.

## Learning Objectives

## Basic Objectives for preparatory work

Give a definition and examples of complex numbers.
Give examples of complex conjugates.
Plot a point on a complex plane.
Add complex numbers.
Subtract complex numbers.
Advanced Objectives for classwork and after class work
Multiply complex numbers.
Divide complex numbers.
Solve a quadratic equation involving complex numbers.
Find the absolute values of complex numbers.
Use complex numbers to solve real-world problems.
Pre-class Individual Space Activities and Resources [50 minutes]

| Procedure | Purpose | Estimated Time | Learning Objective |
| :---: | :---: | :---: | :---: |
| Step 1: Read Section 8.1 on www.MyOpenMath.com course website. | Introduce students to some basic ideas about complex numbers. | 12 minutes | \#1 (basic) through \#4 (basic) |
| Step 2: Watch the video 1 "Introduction to Complex Numbers" (see Appendix 1), then answer question 1 (see Appendix 2): Give a definition and three examples of complex numbers. | Provide students with the definition of complex numbers and give some examples of complex numbers. | 7 minutes | \#1 (basic) |
| Step 3: Answer question 2 (see Appendix 2): Express an imaginary number in standard form: <br> a) $\sqrt{-16}$ <br> b) $\sqrt{-20}$ | Check on students' understanding of writing a complex number in standard form of $a+b i$. | 3 minutes | \#1 (basic) |
| Step 4: Go over Section 8.1 again (if necessary), then answer question 3 (see Appendix 2): Give three examples of complex conjugates. | Introduce students to some basic ideas of complex conjugates. | 5 minutes | \#2 (basic) |


| Step 5: Watch the video 2 "Plot numbers <br> on the complex plane" (see Appendix 1). | Describe how to plot a point on a <br> complex plane. | 3 minutes | \#3 (basic) |
| :--- | :--- | :---: | :---: |
| Step 6: Answer question 4 (see Appendix <br> 2): Plot this complex number on the <br> complex plane: $3-4 i$ | Check on students' understanding of <br> plotting a complex number on the <br> complex plane. | 3 minutes | \#3 (basic) |
| Step 7: Watch the video 3 "Adding and <br> Subtracting Complex Number". | Show students how to add and subtract <br> complex numbers. | 4 minutes | \#4 (basic) |
| Step 8: <br> Practice question 5 (see Appendix 2): <br> a) Add 5 - 4i and 2 + 7i <br> b) Subtract 2 + 5i from 3-6i | Check on students' understanding of <br> adding and subtracting complex <br> numbers. | 3 minutes | \#4 (basic) |
| Step 9: Double check your answers before <br> submitting your papers. | Build students confidence of their work <br> and their carefulness in solving <br> mathematics problems. | 3 minutes | \#1 (basic) <br> through \#4 <br> (basic) |
| Step 10: Watch the Video 4: "Multiplying <br> and Dividing Complex Numbers" and then <br> practice question 6 (see Appendix 2). | Prerequisite for starting the advanced <br> objectives for classwork and after <br> classwork later. | 7 minutes | \#1 <br> (advanced) |

## Appendix 1

## Video Resources

Video 1: "Introduction to Complex Numbers"
https://www.youtube.com/watch?time_continue=191\&v=NeTRNpBI17I
Video 2: "Plotting Numbers on the Complex Plane"
https://www.khanacademy.org/math/algebra2/introduction-to-complex-numbers-algebra-2/the-complex-plane-algebra-2/v/plotting-complex-numbers-on-the-complex-plane

Video 3: "Adding and Subtracting Complex Numbers" https://www.youtube.com/watch? v=SGhTjioGqqA
Video 4: "Multiplying and Dividing Complex Numbers" https://www.youtube.com/watch?v=O9xQaIi0NX0

## Appendix 2

## Pre-class Activities

Part A: (Question \#1 to \#5) Please complete this pre-class assignment by 12:00 pm, on Friday 08/30/2019, then submit it online through our course website (Math 1040/ Section 8.1 Complex Numbers):

- You can choose either to type your answers by selecting Text Entry box or to upload your papers under files of: .pdf, .doc, and/or .docx.
- For questions 3 and 5, you must provide your own examples.
- If you do not have access to the Internet at your area, please let me know in advance and we will make alternative arrangements.
- This pre-class assignment will be graded for a total of 5 participation points.

Question 1 (1pts): Give a definition and three examples of complex numbers

Question 2 (1pts): Express the imaginary number in standard form
a) $\sqrt{-16}$
b) $\sqrt{-20}$

Question 3 (1pts): Give three examples of complex conjugates

Question 4 (1pts): Plot this complex number on the complex plane: $3-4 i$

Question 5 (1pts): Add and subtract these complex numbers
a) Add $5-4 \mathrm{i}$ and $2+7 \mathrm{i}$
b) Subtract $2+5 \mathrm{i}$ from $3-6 \mathrm{i}$

Part B: (Entry ticket) Please practice and bring this question \#6 to class for group discussion on 08/30/2019 Question 6: Multiply and divide complex numbers
a) Multiply $(4-3 i)(5+2 \mathrm{i})$
b) Divide $(2+5 \mathrm{i})$ by $(3-2 \mathrm{i})$

Question? If you have any questions, please come to my office hours (Monday and Wednesday 12:00 pm - 1:00 pm at ST-F213) or contact me via e-mail: hvo14@calstatela.edu.

# Flipped IN-CLASS Lesson Plan 

(Page 6-12)

## Flipped IN-CLASS Lesson Plan

Topic: Operations with Complex Numbers (Imaginary Numbers)
Date: 08/30/2019
Time estimate to complete this assignment: 85 minutes
Materials needed: an account on MyOpenMath.com, any devices connected to the Internet, online videos (see Appendix 1), pre-class worksheets (see Appendix 2), in-class worksheets (see Appendix 3 and 4), pencils, papers, chalks, and scientific calculators (if necessary).

| Learning <br> Objectives | Basic Objectives for preparatory work | Advanced Objectives for classwork and after <br> classwork |
| :---: | :--- | :--- |
| $\mathbf{1}$ | Give a definition and examples of complex <br> numbers. | Multiply complex numbers. |
| $\mathbf{2}$ | Give examples complex conjugates. | Divide complex numbers. |
| $\mathbf{3}$ | Plot a point on a complex plane. | Solve a quadratic equation involving complex numbers. |
| $\mathbf{4}$ | Add complex numbers. | Find the absolute values of complex numbers. |
| $\mathbf{5 .}$ | Subtract complex numbers. | Use complex numbers to solve real-world problems. |

Flipped IN-CLASS Lesson Plan [85 minutes]

|  | Time Planned | Activity | Rationale | Resource Needed |
| :---: | :---: | :---: | :---: | :---: |
| Beginning of class period | 6 minutes | - Review the pre-class assignments (question 1 to 5 from the Pre-class activities, see Appendix 2). <br> - Collect students' entry tickets (question 6 from the Pre-class activities, see Appendix 2). | Check on students' understanding of the basic objectives of learning complex numbers. | - Papers, pencils and chalks. <br> - All videos (see Appendix 1) <br> - Problem sheets (see <br> Appendix 2). <br> - Section 8.1 (Complex <br> Numbers). |
| Middle of period 1 | 8 minutes | - Teach a mini-lecture based on Q\&A from the Pre-class activities. | Make sure students understand well all basic | - Papers, pencils and chalks. <br> - Course website: |


|  |  | - Ask and answer questions for any misconceptions. | objectives of complex numbers. | www.MyOpenMath.com |
| :---: | :---: | :---: | :---: | :---: |
| Middle of period 2 | 15 minutes | - Introduce students to a new lecture on multiplying and dividing complex numbers. <br> - Discuss examples 1 and 2 (see Appendix 3). | - Help students get to know the operations with complex numbers <br> - Clarify misconceptions about complex numbers. | - Papers, pencils and chalks. <br> - Video 4 (see Appendix 1). <br> - Course website: <br> www.MyOpenMath.com |
| Middle of period 3 | 6 minutes | - Assign students to work in group for question 6. <br> - Use think-pair-share strategy: form groups of 3 to 4 students each, let students show their own work first, and then have them share and discuss results with their partners. | - Apply what they have learned into practice problems. <br> - Check on students' understanding of multiplying and dividing complex numbers. | - Papers, pencils and chalks. <br> - Video 4 (see Appendix 1). <br> - Question 6 (see Appendix 2). <br> - Course website: <br> www.MyOpenMath.com |
| Middle of period 4 | 10 minutes | - Introduce new practice problems on the board. <br> - Continue using Think-pair-share technique: <br> - Students make notes on their own, then share notes with their partners. <br> - Each group need to show work on the board, then present answers to the class. | - Make sure each student participates in class activities. <br> - Use physical movement on the board: one of the most useful methods to help students engage more in class. <br> - Check on students' understanding. <br> - Clarify misconceptions (if any). | - Papers, pencils and chalks. <br> - Course website: <br> www.MyOpenMath.com |
| Middle of period 5 | 12 minutes | - Continue explaining advanced objectives \# 2 and \#3. <br> - Practice homework problems. | Allow students to learn more knowledge of complex numbers. | - Papers, pencils, and chalks. <br> - Course website: <br> www.MyOpenMath.com |
| Middle of period 6 | 8 minutes | - Show more advanced examples 3 and 4 (see Appendix 3). <br> - Give students a chance to discuss and ask questions. | Help students to understand deeper by exploring more advanced examples and solving different kinds of quadratic equations | - Papers, pencils, and chalks. <br> - A list of examples for inclass activities. <br> - Course website: |


|  |  | - Arrange students to work in groups. | involving complex numbers. | www.MyOpenMath.com |
| :---: | :---: | :---: | :---: | :---: |
| Middle of period 7 | 10 minutes | - Finish the last advanced objectives: the application of complex numbers in realworld problems. <br> - Discuss example 5 (see Appendix 3). | - Give students many benefits of understanding the application of complex numbers in the real-life. <br> - Encourage students to engage more into the topic of complex numbers. | - Papers, pencils, and chalks. <br> - An example of complex numbers in the real-life. <br> - Course website: <br> www.MyOpenMath.com |
| End of class period | 10 minutes | - Recall all advanced objectives of the complex number topic. <br> - Manage students to selfdirected learning and share results with their partners. <br> - Provide a hand-out of four problems as an Exit-ticket (see Appendix 43). | - Provide students a great time to ask and answer questions before the lecture ends. <br> - Correct and remark on misconceptions (if any) at the last minutes. <br> - Check on students’ understanding at the end of the class. | - Papers, pencils, and chalks. <br> - Course website: <br> www.MyOpenMath.com <br> - Scientific calculators (if necessary). <br> - Practice problem sheets. <br> - Exit-ticket sheets. |

Flipped AFTER CLASS Work Plan [Extra view beforehand]

| Advanced Learning Objective | Activity and Rationale | Instructions to Students |
| :---: | :---: | :---: |
| Multiply complex numbers. <br> Divide complex numbers. <br> Solve a quadratic equation involving complex numbers. <br> Find the absolute values of complex numbers. <br> Use complex numbers to solve real-world problems. | Instructors: <br> - Provide students homework. <br> - Upload all homework online. <br> - Offer homework solutions when students complete it. <br> Students: It is a great opportunity for students to practice and check on their understanding. <br> - Review Section 8.1 on the course website www.MyOpenMath.com <br> - Write one-page summary sheet for Section 8.1. <br> - Complete homework Section 8.1 online on the course website www.MyOpenMath.com. | - Homework Section 8.1 is due online on the course website www.MyOpenMath.com <br> - Students need to complete homework Section 8.1 by Monday, 09/02/2019. <br> - When finishing this homework assignment, students will be able to see the homework solutions. |

## Appendix 3

## A List of Examples for in-class Activities

Example 1: Multiply complex numbers
a) $6 i(5-2 i)$
b) $(3+4 i)(-8 i)$
c) $(\sqrt{14}+i \sqrt{10})(\sqrt{14}-i \sqrt{10})$
d) $(2-5 i)(6+3 i)$

Example 2: Divide complex numbers
a) $\frac{3}{4-5 i}$
b) $\frac{6+2 i}{4 i}$
c) $\frac{4+7 i}{1-2 i}$
d) $\frac{(5-3 i)+(5+i)}{(1+6 i)-(2-4 i)}$

Example 3: Solve the quadratic equations
a) $x^{2}+4=0$
b) $3 x^{2}-2 x+5=0$
c) $\frac{3}{2} t^{2}-6 t+9=0$
d) $1.4 t^{2}-2 t=10$

Example 4: Find the absolute values of the complex numbers
a) $-8 i$
b) $5 i$
c) $-6+i$
d) $9-4 i$
e) $-4-7 i$
f) $8+14 i$

Example 5: The application of complex numbers in a real-world problem
ELECTRICITY. Circuit components such as resistors (R), inductors (L), and capacitors (C) all oppose the flow of current. This opposition is called resistance for resistors and reactance for inductors and capacitors. A circuit's total opposition to current flow is impedance. All of these quantities are measured in ohms ( $\Omega$ ). The impedance for a series circuit is the sum of the impedances for the individual components.

The following table shows the relationship between a component's resistance or reactance and its contribution to impedance:

| Components | Resister (R) | Inductor (L) | Capacitor (C) |
| :---: | :---: | :---: | :---: |
| Symbol |  |  | - |
|  |  | $-\gamma \gamma \sim$ | c |
|  |  | $\mathrm{b} \Omega$ | $\mathrm{c} \Omega$ |
| Impedance | bi | $-c i$ |  |

Note: While a component's resistance or reactance is a real number, its impedance is a complex number. Find the impedance of the circuit shown below:


[^0]
## Appendix 4

## Exit-ticket Activities

Please finish these problems and remember to submit them to me before you leave.
This activity will count 8 participation points.
Problem 1: Multiply and divide complex numbers
a) Multiply $(7-5 \mathrm{i})(4+6 \mathrm{i})$
b) Divide $(12+10 \mathrm{i})$ by $(2-3 \mathrm{i})$

Problem 2: Solve the quadratic equation $x^{2}-8=-36$

Problem 3: Find the absolute value of the complex number $-4+3 \mathrm{i}$

# Advanced Post-Class Practice 

(Page 13-18)

## Advanced Post-Class Practice

## Topic: Operations with Complex Numbers (Imaginary Numbers)

Date assigned: 08/30/2019
Date due: 09/02/2019
Time estimate to complete this assignment: 120 minutes
Materials needed: an account on MyOpenMath.com, any devices connected to the Internet, online videos involving the topic of complex numbers, post-class worksheets (see Appendix 5), pencils, and papers.

The advanced post-class practice is given for students to complete after the class meeting.
Post-class Activities and Deliverables: You will apply your knowledge of complex numbers to solve homework assignments Section 8.1 (see Appendix 5). After completing this lesson, you will be able to:

Give a definition and examples of complex numbers.
Give examples of complex conjugates.
Plot a point on a complex plane.
Add complex numbers.
Subtract complex numbers.
Multiply complex numbers.
Divide complex numbers.
Solve a quadratic equation involving complex numbers.
Find the absolute values of complex numbers.
Use complex numbers to solve real-world problems.
Connections to Future Lesson Plan(s): In this lesson plan, you have learned about the operations with complex numbers in a rectangular form. In the future lesson plan (probably on Wednesday 11/13/2019), you will continue learning more topics of complex numbers in a polar form (which belong to the trigonometric part).

Question? If you have any questions, please come to my office hours (Monday and Wednesday 12:00 pm - 1:00 pm at ST-F213) or contact me via e-mail: hvo14@calstatela.edu.

Note: If you do not have access to the Internet at your area, please let me know in advance and we will make alternative arrangements.

The Flipped AFTER CLASS Work Plan [120 minutes]

| Advanced Learning Objectives | Activity and Rationale | Instructions to Students |
| :---: | :---: | :---: |
| Multiply complex numbers. <br> Divide complex numbers. <br> Solve a quadratic equation involving complex numbers. <br> Find the absolute values of complex numbers. <br> Use complex numbers to solve real-world problems. | Instructor: <br> On www.MyOpenMath.com: <br> - Assign students some most important practice problems involving the complex numbers in a rectangular form. <br> - Prepare and upload online homework for Section 8.1. <br> - Provide students homework solutions after they complete submitting their homework . <br> Students: <br> - Step 1: Review Section 8.1 from the course website www.MyOpenMath.com <br> - Step 2: Write one-page summary sheet for Section 8.1 Complex Numbers in rectangular form. <br> - Step 3: Practice problems involving the complex numbers. <br> - Step 4: Complete homework Section 8.1 online on the course website www.MyOpenMath.com | - Homework Section 8.1 (see Appendix 5) must be done online on the course website: www.MyOpenMath.com. <br> - Homework Due: on Monday, 09/02/2019 by 12:00pm. <br> - Students need to carefully follow all instructions steps by steps. <br> - Homework solutions: when students finish submitting their homework assignments, they will be able to see homework solutions under each homework question. <br> Purpose: <br> * It is a great time for students to learn, review, and practice homework assignments that involve the complex numbers. <br> * Thanks to the homework solutions, students have an opportunity to check on their understanding or any misconceptions about the complex numbers. |

## Reflection Assignment:

## Please answer three following reflection questions:

1. How are the absolute value of a complex number and the geometry - Pythagorean Theorem related?
2. How do you perceive a flipped-classroom contributing to your ability to learn this math content and improve your critical thinking (Math 1040 Pre-Calculus - Section 8.1 Complex Numbers)?
3. What concepts are the most complicated for you to understand in the topic of complex numbers?

DUE: This reflection assignment will be due on Monday, $09 / 02 / 2019$ by 12:00 pm on www.MyOpenMath.com (under Math 1040/ Students and Instructor Reflection Forum).

## Appendix 5

## Online Homework (Assignment 8.1: Complex Numbers) for Post-class activities

## Learning Objectives: All Basic and Advanced Learning Objectives

## 1. (1 point)

Express $\sqrt{-36}$ as a complex number in standard form: $a+b i$
Answer: $0+6 \mathrm{i}$
2. (1 point)

Express $1+\sqrt{-9}$ as a complex number in standard form: $\mathrm{a}+\mathrm{bi}$
Answer: $1+3 \mathrm{i}$
3. (1 point)

Perform the indicated operations \& simplify, give your answer in standard form.
Add: $\quad(10-5 i)+(3+22 i)$
Answer: $13+17 \mathrm{i}$
Subtract: $\quad(10-5 i)-(3+22 i)$
Answer: 7-27i
4. (1 point)

Express $\sqrt{-121}-\sqrt{-64}$ as a complex number in standard form:

Answer: 3i
5. (1 point)

Simplify the following expression $7(3+18 i)$
Answer: $21+126 \mathrm{i}$

## 6. (1 point)

Evaluate the expression $(2+4 i)(-1+4 i)$ and write the result in the form $a+b i$.

$$
\text { Answer: }-18+4 \mathrm{i}
$$

7. (1 point)

Evaluate the expression $(2+i)(-2-i)$ and write the result in the form $\mathrm{a}+\mathrm{bi}$.
Hint: Watch the following video "Multiplying Complex Numbers"
https://www.myopenmath.com/assessment/watchvid.php?url=http\%3A\%2F\%2Fwww.youtube.com\%2Fwatch
\%3Fv\%3DFmr3o2zkwLM

$$
\text { Answer: }-3-4 \mathrm{i}
$$

8. (1 point)

What is the complex conjugate of the number $8+9 \mathrm{i}$ ?
Answer: 8-9i
9. (1 point)

The complex conjugate of $-2-18 \mathrm{i}$ is?

$$
\text { Answer: }-2+18 \mathrm{i}
$$

10. (1 point)

Evaluate the expression $\frac{4-3 i}{2 i}$ and write the result in standard form $\mathrm{a}+\mathrm{bi}$.
Hint: Watch the following video "Dividing Complex Numbers"
https://www.myopenmath.com/assessment/watchvid.php?url=http\%3A\%2F\%2Fwww.youtube.com $\% 2$ Fwatch \%3Fv\%3DXBJjbJAwM1c

$$
\text { Answer: }-\frac{3}{2}-2 i
$$

11. (1 point)

Simplify the following Expression $\frac{7+5 i}{3+2 i}$
Enter the answer as reduced fraction, when necessary.

$$
\text { Answer: } \frac{31}{13}+\frac{1}{13} i
$$

12. (1 point)

Simplify the following expression $\frac{1+7 i}{7-6 i}$
Enter the answer as reduced fraction, when necessary.

$$
\text { Answer: }-\frac{7}{17}+\frac{11}{17} i
$$

13. (1 point)

Let $f(x)=x^{2}+3 x+3$.
Evaluate the function for the following, write your answers in standard form a +bi
a) $f(4+i)$

Answer: $30+11 \mathrm{i}$
b) $f(-i)$

Answer: $2-3 \mathrm{i}$
14. (1 point)

Find the absolute value of the complex number $-3+10 \mathrm{i}$
Answer: $\sqrt{109}$
15. (1 point)

Solve the equation: $-5(n-3)^{2}=10$
Answer: $3 \pm i \sqrt{2}$

A bonus question: The application of complex numbers in a real-world problem

- This question is considered as an Extra credit question.
- It counts 2 points.

ELECTRICITY. Circuit components such as resistors (R), inductors (L), and capacitors (C) all oppose the flow of current. This opposition is called resistance for resistors and reactance for inductors and capacitors. A circuit's total opposition to current flow is impedance. All of these quantities are measured in ohms ( $\Omega$ ). The impedance for a series circuit is the sum of the impedances for the individual components.

Hints: The following table shows the relationship between a component's resistance or reactance and its contribution to impedance:

| Components | Resister (R) | Inductor (L) | Capacitor (C) |
| :---: | :---: | :---: | :---: |
| Symbol | W | -88n- | $11$ |
|  | $\mathrm{a} \Omega$ | $\mathrm{b} \Omega$ | $\mathrm{c} \Omega$ |
| Impedance | a | $b i$ | $-c i$ |

Find the impedance of the circuit shown below:



[^0]:    Answer for example 5:
    Impedance of circuit $=8+6 i+(-2 i)=8+4 i(\Omega)$

