

SJSU MATHEMATICS DEPARTMENT 2010 PROBLEM SOLVING COMPETITION

PART A

Information for Participants

The problems for PART A of the Competition appear below. It is important that you justify the key steps in your solutions to obtain full credit.

You can submit hard copies of your solutions at the Mathematics Department office (MH 308), or you can e-mail your solution to jackson@math.sjsu.edu. Include your name and id number, your major and year in school, and your contact information (e-mail address and/or phone) with your submission. Solutions for Part A are due Thursday, October 14th at 4 pm, and late submissions will NOT be considered.

The problems for Part B will be available on Monday, October 18th at 9 am. Final results of the Competition will be posted at the Department office, and also online at www.math.sjsu.edu by November 23, 2010. Winners will be contacted directly regarding their prizes.

If you have further questions about the Competition, please call the Department at (408) 924-5100. We hope you enjoy working on the problems!

Problem A-1

You have two children, one of which is a girl born on a Sunday. What is the probability that both of your children are girls?

Problem A-2

Evaluate $\int_0^1 \left\{ \frac{1}{\sqrt{x}} \right\} dx$, where $\{ \alpha \} = \alpha - \lfloor \alpha \rfloor$ denotes the fractional part of $\alpha \in \mathfrak{R}$.

Problem A-3

(a) Show that a regular tetrahedron can be placed in \mathfrak{R}^3 so that each vertex occurs at an integer lattice point.

(b) Show that an equilateral triangle cannot be placed in \mathfrak{R}^2 so that each vertex occurs at an integer lattice point.

Problem A-4

Let $f(n)$ denote the number of integers in the set $\{1, 2^1, 2^2, \dots, 2^n\}$ with leading decimal digit 1. Find $\lim_{n \rightarrow \infty} \frac{f(n)}{n}$.