# SJSU MATHEMATICS DEPARTMENT
# 2010 PROBLEM SOLVING COMPETITION

## PART B

### Information for Participants

The problems for PART B of the Competition appear below. You need not have submitted any solutions for Part A to participate in Part B. Even if you do only one or two problems, we encourage you to submit your solutions. It is important that you justify the key steps in your solutions to obtain full credit.

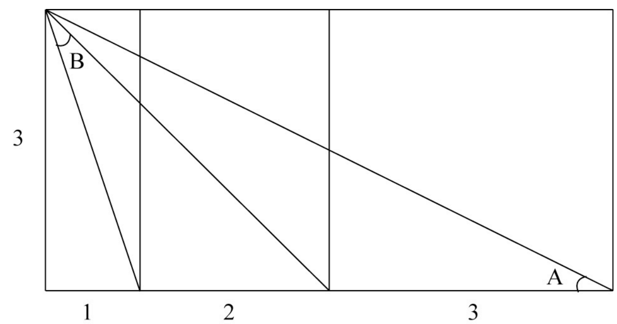
You can submit hard copies of your solutions at the Mathematics Department office (MH 308), or you can e-mail your solution to jackson@math.sjsu.edu. Include your name and ID number, your major and year in school, and your contact information (e-mail address and/or phone) with your submission. Solutions for Part B are due Thursday, October 28th at 4 pm, and late submissions will NOT be considered.

Final results of the Competition will be posted at the Department office, and also online at www.math.sjsu.edu by November 23, 2010. Winners will be contacted directly regarding their prizes.

If you have further questions about the Competition, please call the Department at (408) 924-5100. We again hope you enjoy working on the problems!

## Problem B-1

Rectangles of dimensions $3\times 1$, $3\times 2$, and $3\times 3$ are arranged horizontally as shown to form a $3\times 6$ rectangle. Show that $\angle A = \angle B$.




## Problem B-2

Let $y(x)$ be a solution of the differential equation $y' + 2xy = x^2$. Determine $\lim_{x\to\infty} \frac{y(x)}{x}$.

## Problem B-3

Let $f:[0,1]\to[0,1]$ be an increasing, twice-differentiable function such that $f(0)=0$, $f(1)=1$ and $f'(0)=f'(1)=0$. Show that $|f''(x)|\geq 4$ for some $x\in(0,1)$.

## Problem B-4

Let $P_0$ be the unit square. Assuming $P_n$, $n\geq 0$, has been defined, construct $P_{n+1}$ by subdividing each side of $P_n$ into three equal length segments, and "cutting-off" the corners at each vertex of $P_n$; i.e., the vertices of $P_{n+1}$ are the two subdivision points in the interior of each side of $P_n$, so that $P_n$ has $4\cdot 2^n$ sides. We indicate below how to go from $P_0$ to $P_1$. Determine $\lim_{n\to\infty} Area(P_n)$.