Answer to Selected Chapter-end Problems of Chapter 8

ME 130-01 Applied Engineering Analysis

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8-1)

(a) $u(x) = c_1 \cos 3x + c_2 \sin 3x$
where c_1 & c_2 are arbitrary constants

(b) $u(x) = c_1 \cos\left(\frac{x}{2}\right) + c_2 \sin\left(\frac{x}{2}\right)$

(c) $u(x) = \frac{5}{4} e^{2x} + \frac{3}{4} e^{-2x}$

(d) $u(x) = 6 e^{-2x} - 4 e^{-3x}$

8-2) $V(1.055) = -8.66$ m/s towards the origin of the path

8-3) (a) $y_1(t) = \frac{1}{2} \cos 5t$

(b) $y_2(t) = 2 \sin 5t$

8-4) (a) with $m = m_1 = 2$ kg and $y'(0) = 10$
$y_1(t) = 2 \sin 5t$

(b) with $m_1 = m_2 = 10$ kg and $y'(0) = 0$
$y_2(t) = 4.472 \sin 2.236\, t$

8-5) (a) with $C = 17.7$ kg/s:
$y(t) = e^{-1.735t}(c_1 \cos 2.61t + c_2 \sin 2.61t)$

(b) $y(t) = -2.66\, e^{-1.56t} + 0.66\, e^{-6.288t}$

8-6) $y(t) = 1.337 e^{-8t}(e^{3.742t} - e^{-3.742t})$

8-7) $y(t) = e^{-3t}(\cos 3.315t + 0.905 \sin 3.315t)$

8-8) (a) Case 1: $C = 8700$ N-m/s:
$y(t) = e^{-16.11t}(-0.3186 e^{0.5216t} + 0.2986 e^{-0.5216t})$
$y_{max} = -6.02$ m

Answer to Chapter 8 Problems – Cont'd

8-8) cont'd

(b) The amplitude of the vehicle after runs over the small road block in Figure 8.34(b) is given in Eq.(d) in part (a)

(c) Amplitude of vibration at time $t = 1$ sec. is:

$$y(1) = e^{-16.11}[-0.3186\, e^{0.5216} + 0.2986\, e^{-0.5216}]$$

$$\doteq 0$$

For the case of critical damping with $C = 8694.8$ N-m/s:

$$y(t) = 0.02\, e^{-16.1015t}(1 + 16.1015)$$

$$y(1) = e^{-16.1015}(0.02 + 0.322) \longrightarrow 0$$

8-9) computed vehicle velocity: $V = 10$ Km/hr $= 277.78$ cm/s

(a) Time to pass the speed bump is:

$$\Delta t = W/V = 18/277.78 = 0.065\ s$$

where W = projected width of the bump

(b) $C = \sqrt{4mK} = 7070$ N-m/s

(c) $y(t) = -e^{-14.14t}(0.848t + 6\times10^{-2})$

(d) With $y(t_f) = -1$ mm

$$y(t_f) = 0.5575\ s$$

8-10) (a) $u(x) = 1.667e^{x} + 0.1693e^{-5x} - 0.2x^2 - 0.32x - 1.336$

(b) $u(x) = 0.0833e^{-2x} + 0.0574\, e^{-5x}$

$+ 0.0258 \cos 2x + 0.0603 \sin 2x$

8-11) $y(t) = 745.25 - 745.25 e^{0.2t} - 49.05t$

$y_{max.} = y(5.557) = 228 \text{ m}$

8-12) $m \dfrac{d^2 y(t)}{dt^2} + c \dfrac{dy(t)}{dt} = -mg$

with $y(0) = 0 \quad y'(0) = 0$

$y(t) = -4073.32 + 4073.32 e^{-0.0491t} + 200t$

$V_\infty = v(t)\big|_{t\to\infty} = 200 \text{ m/s}$

$t_f = 93.79 \text{ s}$

8-13) $y(t) = 3.125\times10^{-3} \sin 4t - 0.0125 t \cos 4t$

$y(t_f) = 0.05 = 3.125\times10^{-3} \sin 4t_f - 0.0125 t_f \cos 4t_f$

$t_f = 4 \text{ s}$

8-14)

(a) $y(t) = 2.108 \sin(0.0335t) \sin(7t)$

$f_{beat} = \dfrac{1}{t_{beat}} = \dfrac{e}{2\pi} = \dfrac{0.0335}{2\times3.14} = 0.00533 \text{ Hz}$

(b) $y_{max} = 2.108 \sin(0.0335 t_m)$

t_m from $\sin(7t_m) = 1.0 \quad t_m = 6.2243$

$y_{max} = 1.58 \text{ cm}$

8-15)

(a) DE: $\dfrac{d^2 y(t)}{dt^2} + 16 y(t) = 0.16 \cos 4t$

$y(0) = 0.1$ and $y'(0) = 0$

(b) $y(t) = 0.1 \cos 4t + 0.02 t \sin 4t$

(c) $t_f = 4.735 \text{ s}$

8-16)

(a) DE: $\frac{d^2y(t)}{dt^2} + 16y(t) = \cos 4t$

$y(0) = -0.005m$ and $y'(0) = 0$

$y(t) = -0.005 \cos 4t + \frac{t}{8} \sin 4t$

(b) The elastic support will deform with increasing amplitudes and will break when the amplitude exceed 0.05 m.

(c) $y(t_f) = 0.05 = -0.005 \cos 4t_f + \frac{t_f}{8} \sin 4t_f$

Solve for $t_f = 1.686s$