# Answer to Selected Chapter-end Problems of Chapter 9

## ME 130-01 Applied Engineering Analysis

Offered by Tai-Ran Hsu, the Instructor, 4/23/2019

9-1) (a) $\frac{\partial f}{\partial x} = 15x^2 + 20xy + 8y^2$

$$\frac{\partial^2 f}{\partial x^2} = 30x + 20y$$

$$\frac{\partial f}{\partial y} = 10x^2 + 16xy + 21y^2$$

$$\frac{\partial^2 f}{\partial y^2} = 16x + 42y$$

(b) $\frac{\partial x}{\partial t} = -\sin 10t + \frac{1}{20}\cos 10t + \frac{1}{2}\sin 10t$

$$\frac{\partial^2 x}{\partial t^2} = -10\cos 10t - \frac{1}{2}\sin 10t + 5\sin 10t$$

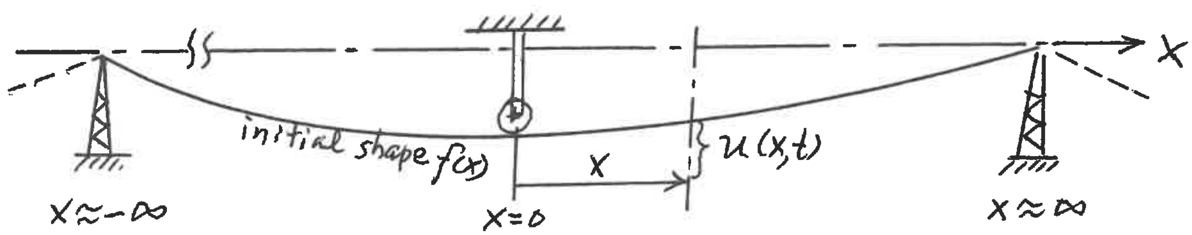
(c) $\frac{\partial u}{\partial x} = \sum_{n=1}^{\infty} \frac{n\pi}{L}\left(b_n \cos\frac{n\pi a}{L}t \cos\frac{n\pi}{L}x\right)$

$$\frac{\partial^2 u}{\partial x^2} = -\sum_{n=1}^{\infty} \frac{n^2\pi^2}{L^2}\left(b_n \cos\frac{n\pi a}{L}t \sin\frac{n\pi}{L}x\right)$$

$$\frac{\partial u}{\partial t} = \sum_{n=1}^{\infty} \frac{n\pi a}{L}\left(b_n \sin\frac{n\pi a}{L}\sin\frac{n\pi}{L}x\right)$$

$$\frac{\partial^2 u}{\partial t^2} = -\sum_{n=1}^{\infty}\left(\frac{n\pi a}{L}\right)^2\left(b_n \cos\frac{n\pi a}{L}t \sin\frac{n\pi}{L}x\right)$$

9-2)




A Long power transmission cable structure

PDE: $a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}$ $\quad -\infty < x < +\infty$

with $u(x,t)\big|_{t=0} = u(x,0) = f(x)$

$$\frac{\partial u(x,t)}{\partial t}\bigg|_{t=0} = u'(x,0) = g(x)$$

Use Fourier transform method to solve the PDE:

$$\mathcal{F}_x[u(x,t)] = u^*(\omega, t) = \int_{-\infty}^{\infty} u(x,t)\, e^{-i\omega x}\, dx$$

The PDE is converted to ODE:

$$a^2\omega^2 u^*(\omega,t) = \frac{d^2u^*(\omega,t)}{dt^2}$$

with $u^*(\omega,0) = F(\omega)$

$$\left.\frac{du^*(\omega,t)}{dt}\right|_{t=0} = G(\omega)$$

Solve for $u^*(\omega,t) = C_1 e^{ia\omega t} + C_2 e^{-ia\omega t}$

or $u^*(\omega,t) = F(\omega)\cosh a\omega t - \frac{G(\omega)}{ia\omega}\sinh a\omega t$

By inverse Fourier transform

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[F(\omega)\cosh at\omega - \frac{G(\omega)}{ia\omega}\sinh at\omega\right]d\omega$$

9-3) (a) Separation of variable technique can be used for the solution of the PDE.

(b) Same as in (a)

(c) same as in (a)

(d) Separation of variable technique cannot be used to solve the PDE,

9-4) PDE: $\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}$

with $T(0,t)=0$ and $T(L,t)=0$, $\alpha = \frac{k}{\rho c} = 1.159\ cm^2/s$

Solution: $T(x,t) = \sum_{n=1}^{\infty} b_n e^{-\frac{\alpha n^2\pi^2}{L^2}t}\sin\frac{n\pi}{L}x$

where $b_n = \frac{2}{L}\int_0^L (100-40x)\sin\frac{n\pi}{L}x\,dx$

9-5) $T(x,t) = \sum_{n=1}^{\infty} b_n \exp\left[-\alpha\left(n-\frac{1}{2}\right)^2\left(\frac{\pi}{L}\right)^2 t\right]\sin\frac{n\pi}{L}x$

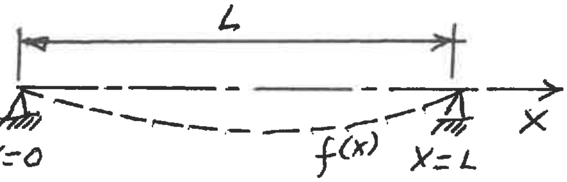
9-7) PDE: $\dfrac{\partial^2 T(x,y)}{\partial x^2} + \dfrac{\partial^2 T(x,y)}{\partial y^2} = 0$ $\quad 0 \le x \le 30\,cm$, $\; 0 \le y \le 20\,cm$

BC: $T(0,y)=0, \; T(30,y)=0$

$T(x,0)=0, \; T(x,20)=100°$

$$T(x,y) = \sum_{n=1}^{\infty} \frac{200}{n\pi \sinh \frac{2n\pi}{3}} \left[ (-1)^n - 1 \right] \sin \frac{n\pi}{30} x \sinh \frac{n\pi}{30} y$$

9-8) PDE: $\dfrac{\partial^2 u(x,t)}{\partial t^2} = a^2 \dfrac{\partial^2 u(x,t)}{\partial x^2}$




$u(x,t)\big|_{t=0} = f(x) = x(L-x)$

$\dfrac{\partial u(x,t)}{\partial t}\Big|_{t=0} = \dot{u}(x,0) = 0$

$u(x,t)\big|_{x=0} = u(0,t) = 0$ and $u(L,t) = 0$

Solution: $u(x,t) = \displaystyle\sum_{n=1}^{\infty} \frac{2}{L} \left[ \int_0^L [x(L-x)] \sin \frac{n\pi x}{L} dx \right] \cos \frac{n\pi a}{L} t \sin \frac{n\pi x}{L}$

9-9) Solution:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{L} \left[ \int_0^L \left( \sin \frac{\pi x}{L} \sin \frac{n\pi x}{L} \right) dx \right] \cos \frac{n\pi a}{L} t \sin \frac{n\pi x}{L}$$

9-10) Beam vibration analysis.

PDE: $a^2 \dfrac{\partial^4 y(x,t)}{\partial x^4} = -\dfrac{\partial^2 y(x,t)}{\partial t^2}$ $\quad 0 \le \; \le L \;\; t>0$

$a^2 = EI/m$

Given conditions $y(x,0) = f(x)$

$y'(x,0) = g(x)$

Solution: $y(x,t) = \displaystyle\sum_{n=1}^{\infty} \left( A_n \cos \frac{n^2\pi^2}{L^2} a t - B_n \sin \frac{n^2\pi^2}{L^2} a t \right) \sin \frac{n\pi}{L} x$

where $A_n = \dfrac{2}{L} \displaystyle\int_0^L f(x) \sin \frac{n\pi}{L} x \, dx$

$B_n = \dfrac{2L}{n^2\pi^2 a} \displaystyle\int_0^L g(x) \sin \frac{n\pi}{L} x \, dx$

9-11) $$Y(x,t)=\sum_{n=2}^{\infty} A_n \cos\frac{n^2\pi^2}{L^2}at \sin\frac{n\pi}{L}x + \sum_{n=1}^{\infty} B_n \sin\frac{n^2\pi^2}{L^2}at \sin\frac{n\pi}{L}x$$

where $$A_n = \frac{\sin(n-1)\pi/L}{2(n-1)\pi} - \frac{\sin(n+1)\pi/L}{2(n+1)\pi}$$

$$B_n = \frac{2L}{n^2\pi^2 a}\int_0^L g(x)\sin\frac{n\pi}{L}x\,dx$$

9-12)

$$Y(x,t)=\sum_{n=1}^{\infty}\Big[\cosh\beta_n x-\cos\beta_n x - \frac{(\cosh\beta_n L-\cos\beta_n L)^2}{(\sinh\beta_n L)^2-(\sin\beta L)^2}(\sinh\beta_n x-\sin\beta_n x)x \times (A_n\cos a^2\beta_n^4 t + B_n \sin a^2\beta_n^4 t)\Big]$$

where $\beta$ are solution of characteristic equation:

$$1-\cosh\beta L\cos\beta L=0$$

$A_n$ obtained from:

$$f(x)=\sum_{n=1}^{\infty} A_n\Big[\cosh\beta_n x-\cos\beta_n x - \frac{(\cosh\beta_n L-\cos\beta_n L)^2}{(\sinh\beta_n L)^2-(\sin\beta_n L)^2}(\sinh\beta_n x-\sin\beta_n x)\Big]$$

$B_n$ obtaine from

$$g(x)=\sum_{n=1}^{\infty} B_n\Big[\cosh\beta_n x-\cos\beta_n x - \frac{(\cosh\beta_n L-\cos\beta_n L)^2}{(\sinh\beta_n L)^2-(\sin\beta_n L)^2}(\sinh\beta_n x-\sin\beta_n x)\Big]$$