HW#10; Due Monday 5/15 (the day I hand out the take-home final!)

Note: There are two pages to this assignment -- but I think it should be less work than last week.

1) No-Signaling: An example

A) Last week you found the partial trace of the generic entangled state A|00>+B|01>+C|10>+D|11>, for qubit #1. Copy it out, or work it out again. Don't forget A,B,C,D are complex.

B) Now, suppose Bob measures his qubit (qubit #2) in the |0>,|1> basis. He either gets |0> or |1>. For each case, work out the collapse and renormalized state that results from his measurement result. (Hint: Two of the terms will collapse to zero in each case, but two will survive. Don't forget to renormalize!)

C) Use your results from part B) to find the partial trace of Alice's qubit (qubit #1), after each of Bob's possible two measurements. They will be different!

D) Now, combine your answers from part C) with the probabilities that correspond to each of Bob's outcomes, and build the mixed state that corresponds to Alice not knowing which measurement outcome Bob actually got. If you did everything right, it should be the same as your answer to part A), which proves that Bob can't change Alice's probabilities. (This follows from the no-signaling theorem.)

E) Now, if Bob calls up Alice and tells her about his measurement result, that **will** change her expected probabilities. Explain how this is consistent with your answer from D!

2) The CHSH Inequality

Background: A singlet state is generated; one particle is sent to Alice and one is sent to Bob. Both Alice and Bob measure their spin-1/2 particle along a chosen axis. If the result is aligned with the axis ($+\hbar/2$), that result is labeled as a "1". (For Alice, a=1; for Bob, b=1.). If the result is anti-aligned with the chosen axis ($-\hbar/2$), that result is labeled as a "-1". (For Alice, a=-1; for Bob, b=-1.)

A) Show that (for a singlet state) the expectation value <ab> will always be $-\cos(\theta)$, where θ is the angle between Alice and Bob's chosen measurement axes. (Think about how a singlet state collapses under measurement. Also, don't forget how to get expectation values from probabilities and outcomes!)

B) Alice can choose to measure along the z-axis (value a1), or along the x-axis (value a2). Bob also has a choice. Value "b1" is when he measures along the α-axis (some angle α from the z-axis, towards the x-axis, in the x-z plane). Value "b2" is when he measures along the β-axis (some angle β from the z-axis, towards the **negative** x-axis, in the x-z plane). Find the values of α and β for which the value "S" in the CHSH inequality is maximized.

C) If $\alpha = \beta$, find the range of values of α for which the classical CHSH inequality is violated.

3. The Bell Basis

Background: This is a complete basis in the 4D Hilbert space of 2-qubit states, but unlike the usual basis |00>,|01>,|10>,|11>, each of the 4 Bell states is already maximally entangled! (Of course, you can build non-entangled states out of superpositions of Bell states, since you can build **any** state out of a complete basis.)

A) One way to generate the 4 Bell states is to start with a singlet state, and do one of 4 things: 1) Do nothing, 2) Rotate qubit #1 180 degrees around X, 3) Rotate qubit #1 180 degrees around Y, 4) Rotate qubit #1 180 degrees around Z. Find the 4 states using this technique.

B) A simpler way is to start with separable qubits and put them through a CNOT gate. Show that you can also generate the same states this way, so long as Qubit #1 is on the x-axis (either +x or -x on the Bloch sphere). And Qubit #2 is on the z-axis.

C) Express the generic state A|00>+B|01>+C|10>+D|11>, in the Bell Basis! (Find the coefficients of each Bell state using Fourier's trick. Don't forget A,B,C,D are complex.)