HW#9; Phys 263. Due Mon 5/8

1. A) Construct the gate/operator that will rotate any state on the Bloch sphere by 90 degrees around the x-axis. (This is known as the $\sqrt{\mathbf{X}}$ or $\sqrt{NOT}$ gate!)
  B) Show that it works by applying it to the +h/2 eigenstate of Sx, Sy, and Sz.

2) A) For a generic mixed state (corresponding to some vector **n** in the Bloch Ball), construct the 2x2 density matrix ρ out of the 3 components of **n**.
   B) Find out how Tr(ρ^2) is related to the magnitude of **n**.

3) A) For a pure 2-qubit state, A|00>+B|01>+C|10>+D|11>, generate the 4x4 density matrix, as well as the reduced 2x2 density matrix corresponding to the first qubit.

  B) Compare your 2x2 density matrix (for qubit#1) to your answer from question #2, and determine how the components of the vector **n** in the Bloch Ball are related to A,B,C,D.

  C) Show that $\mathbf{n}^2$ + (concurrence)^2 = 1, where the concurrence is 2|AD-BC|. (Hint: Don't forget that the original 2-qubit state is normalized!)

  D) When the concurrence is maximum, one has a "maximally entangled state". What can you say about measurement outcomes on one of the qubits in a maximally entangled state? (Think about your answer to part C).

4. In this problem you'll determine how to calculate measurement probabilities on a mixed-state qubit, corresponding to an arbitrary vector in the Bloch Ball, **n**.

  A) Show that if you build a mixed state out of a probabilistic-mix of two pure states $\rho = A\rho_1 + B\rho_2$, then the Bloch Ball vector of the mixed state will be a weighted sum of the Bloch-SPHERE vectors of the two pure states, **n1** and **n2**. (Hint: just write everything in terms of Identity and Pauli matrices.)

  B) Although there are many possible ways to sum up two (unit) Bloch Sphere vectors to make a (non-unit) Bloch-Ball vector, there is usually only one way to do this if your original Bloch Sphere vectors are Orthogonal. (Orthogonal in Hilbert space, which means **opposite** on the Bloch Sphere!) Find how to decompose a generic **n** in this manner; solve for the probability weights A and B in terms of the magnitude of **n,** and figure out what directions **n1** and **n2** should be pointing. Don't forget that your probabilities should add to 1!

  C) With this decomposition in hand, if the original mixed state represents a spin-1/2 system, find the probabilities of measurement outcomes for a spin-component aligned at an angle θ with the Bloch-Ball vector **n**. (Hint: we've already worked out the probabilities for pure states; check the notes!) Check that your answer matches known results in the limits |**n**|=1 and |**n**|=0.