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COGNITION

Chapter 12: Problem Solving

Cognitive Psychology *(Reed)*

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Defining a problem

There is a problem when a goal is not immediately able to be achieved (e.g., Reitman, 1965; Newell & Simon, 1972).

Problem-solving is the identification and selection of solutions to the problem.

Problem Solving

Directed and Undirected Thinking

- Directed: Goal-oriented and rational
  - Requires a clear well-defined goal
- Undirected: Meanders (day dreams, dreaming, drifting thoughts, etc.)
  - Plays a role in creativity and poorly-defined problems

Well-Defined and Ill-Defined Problems

Separating Problems by the skills needed to solve them (Greeno)

Greeno (1978) proposes that problems be arranged into three types: (+ combinations)

- **ARRANGEMENT (**anagrams, Luchin's water jug**):**
  - requires the problem solver to organize the objects into a way that satisfies some criterion
- **INDUCING STRUCTURE (**reasoning by analogy**)**
  - Requires the problem-solver to rearrange objects in some way that forms a new relation among them
- **TRANSFORMATIVE (**tower of Hanoi; missionaries**)**
  - **Initial State, Goal State, Sequence of Operations** (these define the **problem space**)

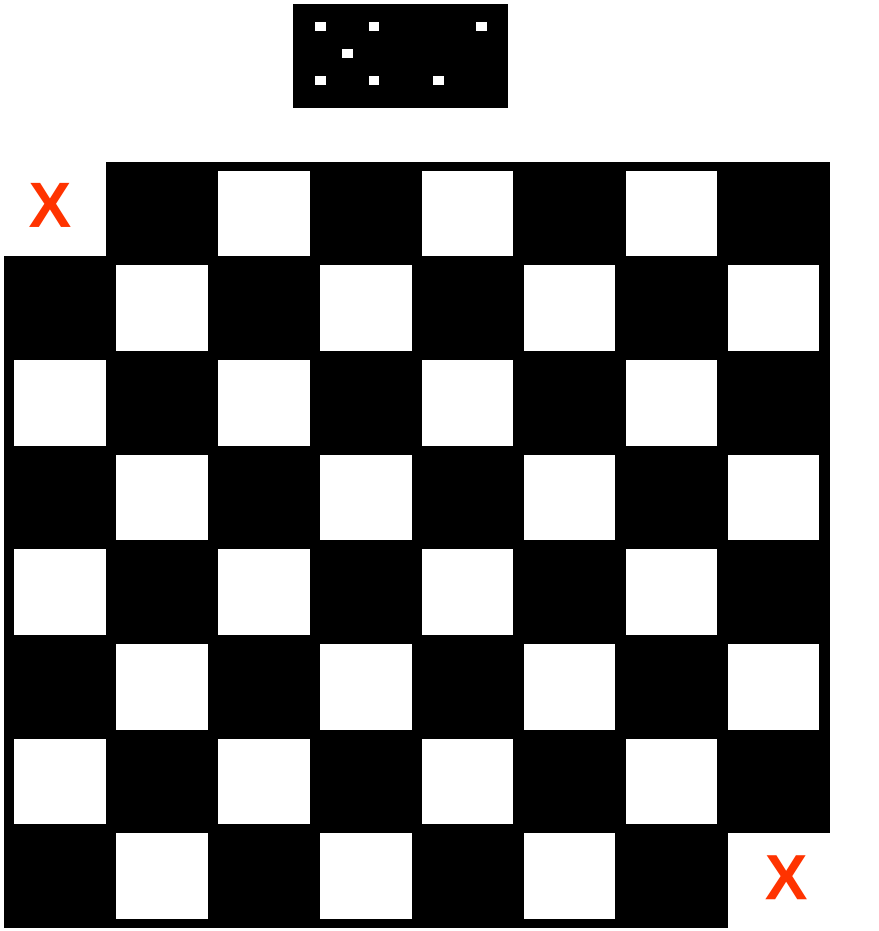
Arrangement Problems

- A Problem that requires rearranging its parts to satisfy a specified criterion
  - e.g., ANAGRAM:
    - rearranging a string of letters to form a word or words (KEROJ → ?; RWAET → ?)
  - Solving an arrangement problem often involves a lot of trial and error during which partial solutions are formed and evaluated.
- Dependent on: Fluency (generating possibilities), Retrieval of Solution Patterns, Knowledge of principles that constrain the search (e.g., _JR_ unlikely)

Inducing Structure

Chessboard / Domino Problem (Mutilated Chessboard Problem)

We have a chessboard with the two opposing corners removed, so that there are only 62 squares remaining. Now we take 31 dominoes shaped such that each domino covers exactly two squares. The question is: is it possible to arrange the 31 dominoes so that they cover all 62 squares on the chessboard?



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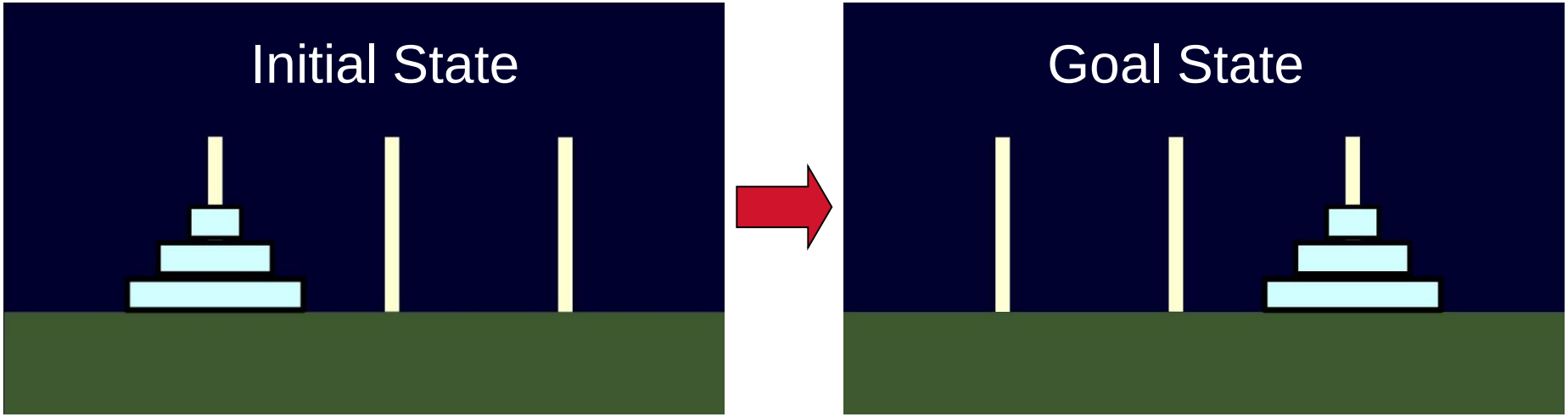
Tower Of Hanoi

(Transformative / well-defined problem)

Move from start state to end state by moving one disk at a time, and never placing a smaller disk on a larger disk.



Well-Defined Problem

Goal State (clear)

Initial State (clear)

Subgoals (problem can be broken down)

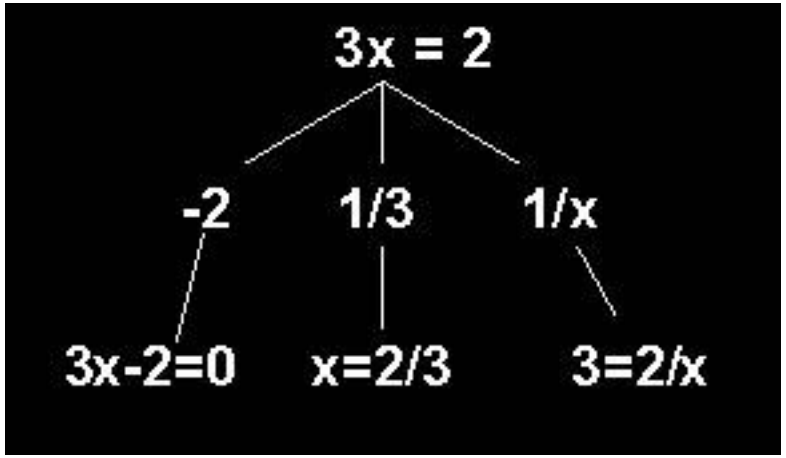
Problem Space (all possible legal moves [operators])

Problem Space (Newell & Simon)

all possible legal moves

Each legal move from the initial state to some intermediate state is specifically defined by an OPERATOR.

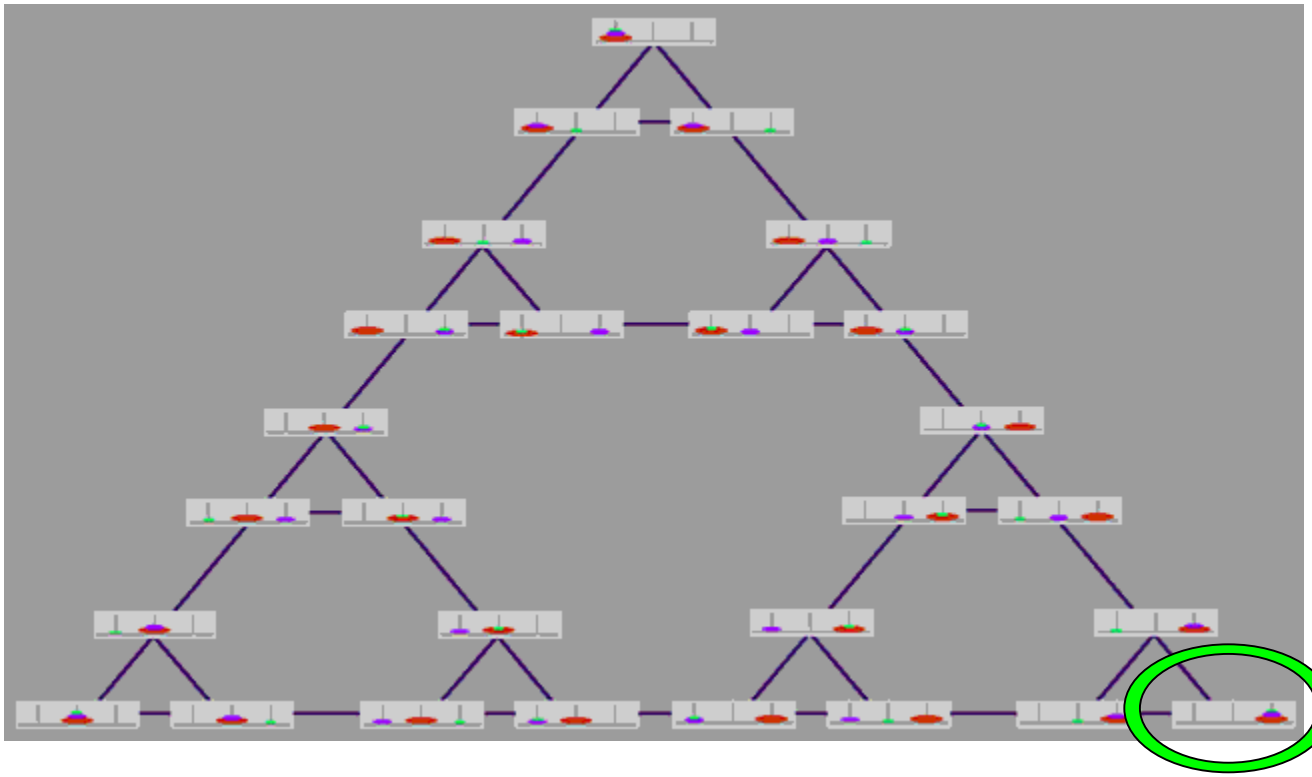
Solve for *x*:



Tower Of Hanoi (the problem space)

Well-Defined Problem

- Goal State, Initial State, Subgoals, Problem Space

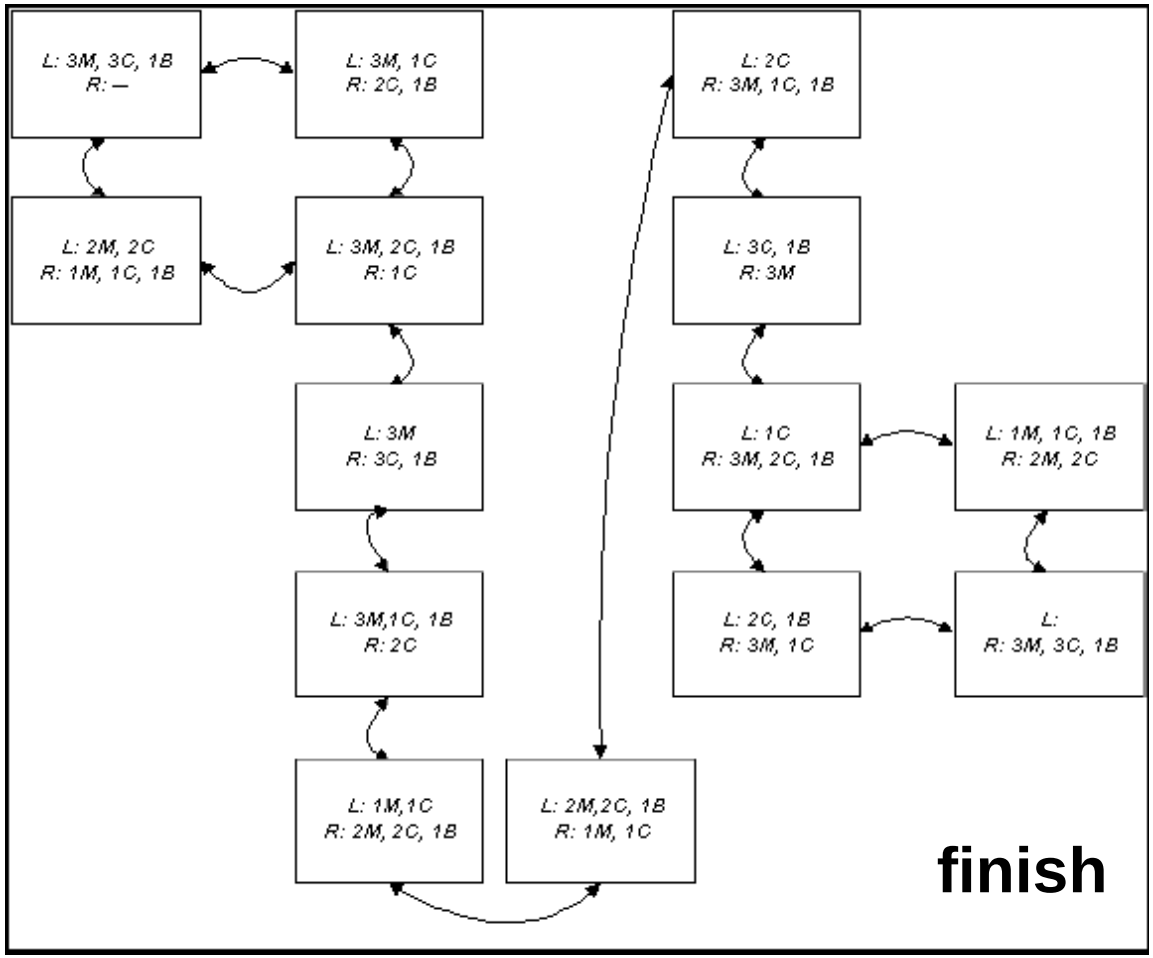


- Goal State

all possible legal moves (operator actions)

Missionaries and Cannibals (transformative)

- Your task is to take 3 missionaries and 3 cannibals across the river.
- The boat holds 2.
- The cannibals must never outnumber the missionaries (or they will eat them).



Ill-Defined Problems

- How do you advance in your chosen career?
  - What is the goal state?
  - What is your current state (initial state)
  - What are the intermediate goals (subgoal decomposition)
  - What are all possible operations [OPERATORS] that could be employed (i.e., what is the problem space?)

Where we are at

Well-defined problems

- Your car doesn't start in the morning and you want to try and find out what's wrong with it
- You want to beat an opponent at chess
- You want to find a street in an unfamiliar city

Ill-defined problems

- You want to be happy
- You want to be successful
- Draw a picture
- Write an essay

Productive and Reproductive Problem Solving

The GESTALT approach to problem solving differentiates between:

Productive Thinking

- insight and creativity

Reproductive Thinking

- following a sequence known to produce a workable answer

Gestalt approach to problem-solving

Adapted their problem-solving approach from perception. Perception inherently involves **restructuring**.

- Two views of Necker cube seen by restructuring image to see as “right” or “left”

Person often has to **restructure** a problem in order to gain **insight** into its solution

Insight

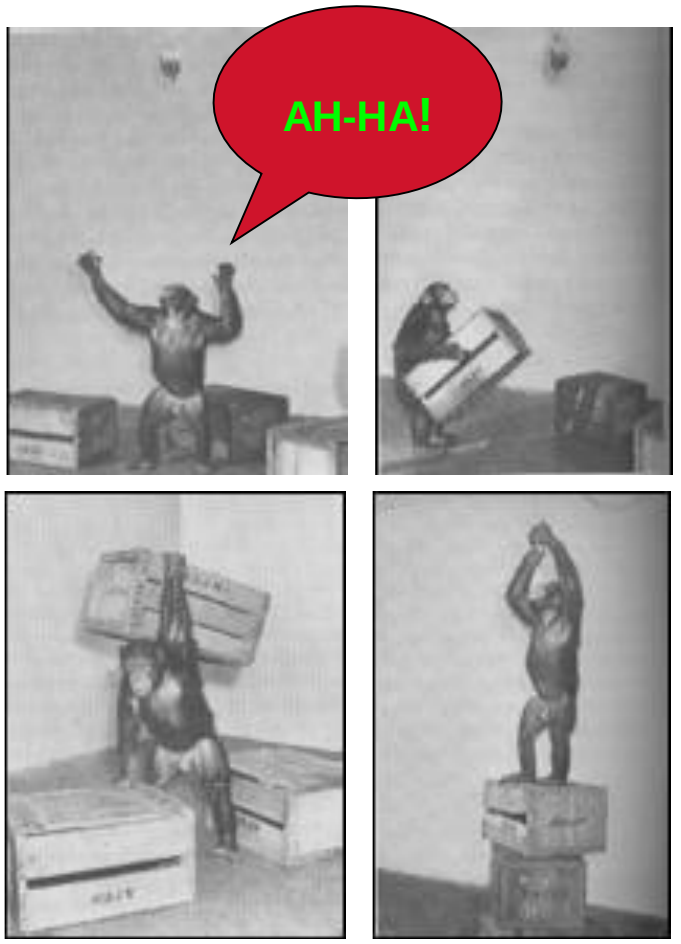
problem solving in monkeys: Kohler (1927)

Monkeys showed “insight” during problem-solving.

- Demonstrated productive “new way of structuring elements showing insight and creativity”
- Deep useful understanding of the nature of the problem
- versus trial-and-error (a.k.a., reproductive // tried and true // rule-based) problem solving (e.g., of the cat-in-the-box of Thorndike, 1898).

Solved problems by using tools.

Sultan stacks boxes to reach bananas.



Gestalt approach to problem-solving

THE CANDLE PROBLEM (Dunker, 1945)

- Fix lit candle to wall, with candle, box of nails & box of matches.

THE TWO-STRING PROBLEM (Maier, 1931).

Subjects in room with 2 strings hanging from ceiling and set of other objects (nut, bowl, pliers, sandwich, …).

- Task is to attach the strings. However, cannot reach them at same time.

WATER-JUG PROBLEM

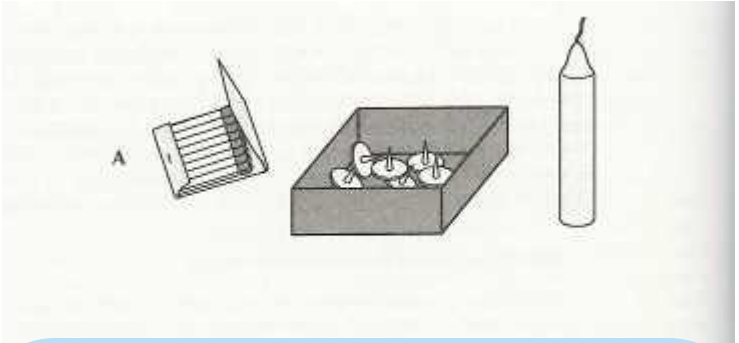
- The subject is given a set of jugs of various stated capacities, and is asked to measure out a desired quantity of water

NINE-DOT PROBLEM (Scheerer, 1931)

- Connect all nine dots by drawing four continuous lines?

Duncker's (1945) Candle Problem (Functional Fixedness)

- Participants were provided a candle, a box of nails, and several other objects, and asked to attach the candle to the wall so that it did not drip onto the table below.
- Participants tried to nail the candle directly to the wall or to glue it to the wall by melting it. Very few thought of using the inside of the nail box as a candle-holder and nailing this to the wall.
- The participants were "fixated" on the box's normal function of holding nails and could not reconceptualize it in a manner that allowed them to solve the problem.



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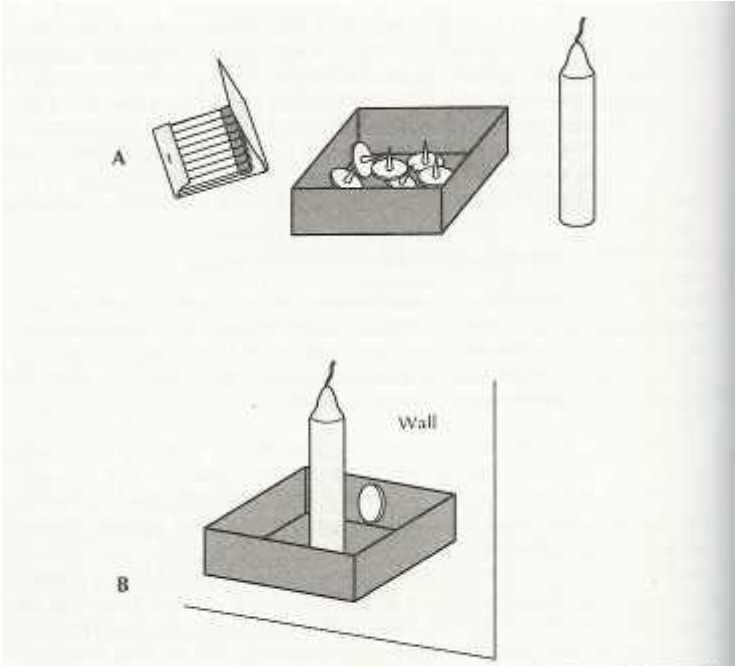
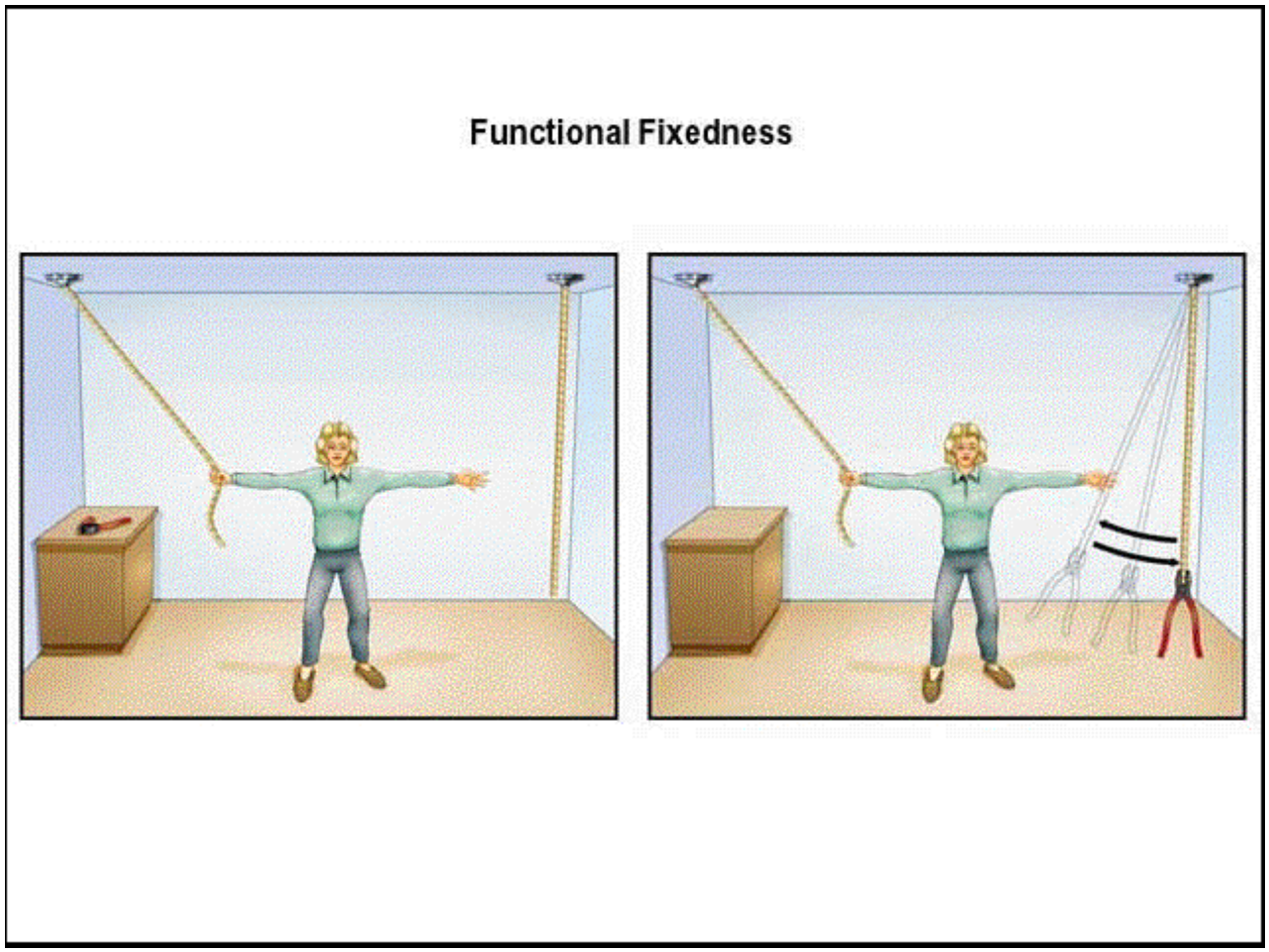


Duncker's (1945) Candle Problem The subjects are asked to attach a candle to the wall and are given a box of tacks, candles, and matches, as shown in panel A. The solution is shown in panel B.

The Two-String Problem



Functional Fixedness

Birch and Rabinowitz (1951) adapted the two-cord problem from *Maier* (1930, 1931), where subjects would be given 2 cords hanging from the ceiling, and 2 heavy objects in the room.

They are told they must connect the cords, but they are just far enough apart that one cannot reach the other easily.

The solution was to tie one of the heavy objects to a cord and be a weight, and swing the cord as a pendulum, catch the rope as it swings while holding on to the other rope, and then tie them together.

The participants are split into 3 groups:

- Group R, which completes a pretask of completing an electrical circuit by using a relay
- Group S, which completes the circuit with a switch
- and Group C which is the control group given no pretest experience.

Group R participants were more likely to use the switch as the weight, and Group S were more likely to use the relay. Both groups did so because they were previous experience led them to use the objects a certain way, and functional fixedness did not allow them to see the objects as being used for another purpose.

the water-jug experiment (Luchin 1942, 1959)

The subject is given a set of jugs of various stated capacities, and is asked to measure out a desired quantity of water.

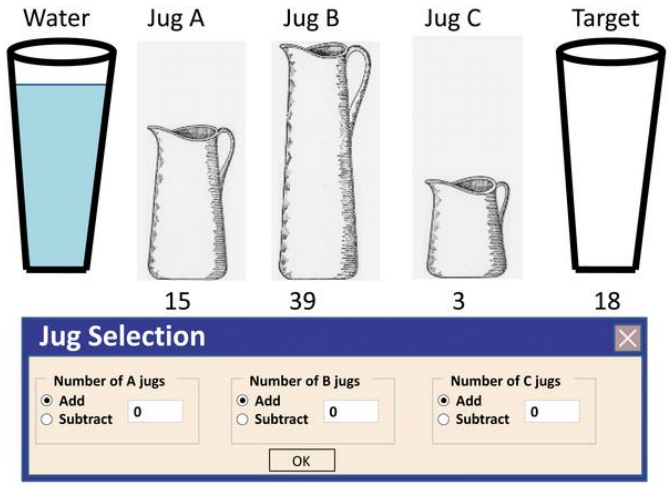
Problem	Capacity Jug A	Capacity Jug B	Capacity Jug C	Desired quantity
1	21	127	3	100
2	14	163	25	99
3	18	43	10	5
4	9	42	6	21
5	20	59	4	31
6	23	49	3	20
7	15	39	3	18
8	28	76	3	25
9	18	48	4	22
10	14	36	8	6

the water-jug experiment (Luchin 1942, 1959)

All problems except 8 can be solved by B - 2C - A.
For problems 1 through 5 this solution is simplest.
For problem 7 and 9 the simpler solution is A + C.
Problem 8 cannot be solved by B - 2C - A, but can be solved by A - C.
Problems 6 and 10 can be solved more simply as A - C.

<u>Subjects who worked through all problems in order:</u>
83% used B- 2C - A on problems 6 and 7.
64% failed to solve problem 8.
79% used B - 2C - A on problems 9 and 10.

<u>Subjects who saw only last 5 problems.</u>
Fewer than 1% used B - 2C - A.
Only 5% failed to solve problem 8.
Problem can be overcome by warning subjects.



After problem 5, Lurchins told some subjects “Don't be blind”, which caused more than 50% to find the simpler solution on the remaining problems.

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Gestalt approach to problem-solving

THE CANDLE PROBLEM (Dunker, 1945)

→ **Functional Fixedness: limited by thinking about the normal functional uses of an object**

THE TWO-STRING PROBLEM (Maier, 1931).

→ **Functional Fixedness: limited by thinking about the normal functional uses of an object**

WATER-JUG PROBLEM

→ **Fixation** occurs when solver is fixated on wrong approach to problem. It often is result of past experience.

→ **Fixation** refers to the blocking of solution paths to a problem that is caused by past experiences related to the problem

→ **NEGATIVE SET (set effects)** - bias or tendency to solve a problem a particular way

NINE-DOT PROBLEM (Scheerer, 1931)

→ fixation, negative set

Restructuring Using Analogies (Duncker, 1945)

- One special kind of restructuring (Inducing Structure), is analogical problem solving.
- To find a solution to one problem - the so called target problem, an analogous solution to another problem - the source problem, is presented.

> *As a doctor you have to treat a patient with a malignant, inoperable tumor, buried deep inside the body. There exists a special kind of ray, which is perfectly harmless at a low intensity, but at the sufficient high intensity is able to destroy the tumor - as well as the healthy tissue on his way to it. What can be done to avoid the latter?*

When this question was asked to participants in an experiment, most of them couldn't come up with the appropriate answer to the problem. Then they were told a story that went something like this:

Reasoning by Analogy

This is the “source” problem
In this case it is an ISOMORPH of the “target” problem

A General wanted to capture his enemy's fortress. He gathered a large army to launch a full-scale direct attack, but then learned, that all the roads leading directly towards the fortress were blocked by mines. These roadblocks were designed in such a way, that it was possible for small groups of the fortress-owner's men to pass them safely, but every large group of men would initially set them off. The General divided his troops into several smaller groups and made each of them march down a different road, timed in such a way, that the entire army would reunite exactly when reaching the fortress and could hit with full strength.

Note that superficial similarities can cause people to attempt to use similar solutions but may not detect isomorphs when presented with models from other domains.

Newell and Simon's
General Problem Solver (GPS)

- Information processing view (information passed through a system
- Restricting to computer programing meant surfacing assumptions and steps
- The hope was that it might assist with recursively providing a better model of human problem-solving (informed by verbal report to guide programming)
- Constraints include both task and operator characteristics. Operator characteristics include: STM/LTM Capacity, storage, and retrieval time.

Problem-Solving Strategy:
Means-End analysis (heuristic)

- Identify difference between current & goal state.
- Create sub-goal & select operator that achieves it.
- If operator cannot be applied then use means-end-analysis recursively (i.e. repetitively) to remove blocking conditions.

- Newell and Simon's GPS forms a reasonable model of how people go about solving transformational problems

Novice vs. Expert Problem Solving

- Backwards Chaining (working backwards)… working back from the end state until reaching the origin state. This is common for novice problem-solvers in the domain (also used by experts).
- Forwards Chaining (working forwards). Experts are much more likely to start from the origin point and move forward whenever the problem strikes them as readily solvable.

10 suggestions for improving problem solving

1. Increase domain knowledge
2. Automate some components
3. Follow a systematic plan
4. Draw inferences
5. Develop subgoals
6. Work backwards (if the goal state is well-defined)
7. Search for contradictions
8. Search for relations among problems (analogies)
9. Find a difference problem representation (diagram)
   - Think of the chess board / dominoes vs. slices of bread
10. Practice! (learn general strategies to approach problems with)

Key Terms (partial list)

- Directed / undirected thinking
- Well / ill –defined problems
- Initial state
- Goal state
- Subgoal
- Operator
- Problem space
- (Re)Productive thinking
- Isomorphic problems
- Algorithm
- Heuristic
- Metacognition
- Creativity
- Incubation
- Verification
- Functional fixedness
- Negative set
- fixation

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Assignment #10

(Biases in Reasoning and Decision Making):

GOAL: To have you demonstrate your grasp three different cognitive biases in decision-making. One of these must be the "*Anchoring and Adjustment*" heuristic; another must relate to the work of Kahneman. The remaining one is any cognitive biases in decision-making not already included in your write-up that was discussed in class or which is mentioned in the text (group think, framing, etc.).

REQUIREMENT: Write a report discussing the heuristics that have influenced three separate decisions that you have personally made. Provide a brief description of each of three scenarios in which you had to make a decision (or were involved in the decision making) and the cognitive biases that may have been involved (possibly in retrospect). Each of the three sections of this assignment will include a clear definition of the heuristic(s), the description of the situation, and statements that indicate why each heuristic applies to the situation as you have described it. It is possible (but not required) that multiple heuristics may have influenced each decision; you may note this, but you must describe three separate decision-making episodes. One of the episodes must use the "anchoring and adjustment" heuristic, another as discussed by Kahneman. The other heuristic is up to you.

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