P5.2. Write the equations for shear and moment between points D and E. Select the origin at D.

\[ \Sigma M_A = -6(8) + 13(3 \cdot 10) - 18E_x = 0 \]
\[ E_x = 19 \text{ kips} \uparrow \]
\[ \Sigma F_x = A_x - 3 \cdot 10 + E_x = 0 \]
\[ A_x = 11 \text{ kips} \uparrow \]
\[ \Sigma F_y = E_y - 8 = 0 \]
\[ E_y = 8 \text{ kips} \rightarrow \]

\[ \Sigma F_y = V_y - 3(10 - x) + 19 \]
\[ V_y = 3x + 11 \text{ kips} \]
\[ \Sigma M_A = M_x + \left[ \frac{x}{2} \right] \cdot 3x - 19(x) = 0 \]
\[ M_x = 19x - \frac{3}{2} x^2 \text{ kip-ft} \]
P5.12. Consider the beam shown in Figure P5.12.
(a) Write the equations for shear and moment using an origin at end A.
(b) Using the equations, evaluate the moment at section 1.
(c) Locate the point of zero shear between B and C.
(d) Evaluate the maximum moment between points B and C.
(e) Write the equations for shear and moment using an origin at C.
(f) Evaluate the moment at section 1.
(g) Locate the section of maximum moment and evaluate $M_{\text{max}}$.
(h) Write the equations for shear and moment between B and C using an origin at B.
(i) Evaluate the moment at section 1.

\[ \Sigma M_y = 0; \quad 8 \times 4 - 48 \times 8 + 14 R_c = 0 \]
\[ R_c = 22 \text{kips} \]
\[ \Sigma F_x = 0; \quad -8 - 48 + R_y + 22 = 0 \]
\[ R_y = 34 \text{kips} \]

(a) A–B Origin at “A”: \(0 \leq x \leq 4\)
\[ \Sigma F_y = 0; \quad -8 - V = 0 \]
\[ V = -8 \]
\[ \Sigma M_x = 0; \quad M + 8x = 0 \]
\[ M = -8x \]
\[ \Sigma F_y = 0; \quad -8 + 34 - 3(x - 4) - V = 0 \]
\[ V = 3x + 38 \quad \text{(EQ. 1)} \]
\[ \Sigma M_z = 0; \quad 8x - 34(x - 4) + 3(x - 4)(x - 4) + M = 0 \]
\[ M = -1.5x^2 + 38x - 160 \quad \text{(EQ. 2)} \]

(b) Moment at (1) set \(x = 9\)
\[ M_1 = -1.5(9)^2 + 38 \times 9 - 160 \]
\[ = -121.5 + 340 - 160 \]
\[ = 60.5 \text{ kip-ft} \]

(c) Locate Point \(V = 0\) \(B - C\)
Use EQ. 1: 
\[ 0 = -3x + 38 \]
\[ x = 12.67 \]

(d) \(M_{\text{max}}\): Set \(x = 12.67\) in (EQ. 2)
\[ M_{\text{max}} = -1.5(12.67)^2 + 38 \times 12.67 - 160 \]
\[ = -240.79 + 481.46 - 160 \]
\[ = 80.67 \text{ kip-ft} \]
P5.12. Continued

(e) \( B-C \)
\[ \Sigma V = 0; \quad V - 3x + 22 = 0 \]
\[ V = -22 + 3x \]
\[ \Sigma M_z = 0; \quad M + 3x \cdot \frac{x}{2} - 22x = 0 \]
\[ M = 22x - \frac{3}{2}x^2 \]

\( A-B \)
\[ \Sigma F_y = 0; \quad -84 - V = 0 \]
\[ V = -84 \]
\[ \Sigma M_z = 0; \quad -8(20 - x) - M = 0 \]
\[ M = 8x - 160 \]

(f) Moment at section (1) \( R_x + x = 11^2 \)
\[ M = 22(11) - \frac{3}{2}(11)^2 = 60.5 \text{ kip} \cdot \text{ft} \]

(g) \( M_{\text{max}} \)
Set \( V = 0; \quad -22 + 3x = 0 \)
\[ V = \frac{22}{3} = 7.33^2 \]
\[ M_{\text{max}} = 22 \times 7.33 - \frac{3}{2}(7.33)^2 \]
\[ = 161.26 - 80.59 \]
\[ M_{\text{max}} = 80.67 \text{ kip} \cdot \text{ft} \]

(h) \( \Sigma F_y = 0; \quad -8 + 34 - 3x - V = 0 \)
\[ V = 26 - 3x \]
\[ \Sigma M_z = 0; \quad -8(4 + K) + 34x - \frac{3x \cdot x}{2} - M = 0 \]
\[ M = -32 + 26x - \frac{3x^2}{2} \]

(i) Moment at section (1)
Let \( x = 5' \)
\[ M = -32 + 26(5) - \left( \frac{3}{2} \right) 5^2 \]
\[ M = 60.5 \text{ kip} \cdot \text{ft} \]
P5.17. For each beam, draw the shear and moment curves label the maximum values of shear and moment, locate points of inflection, and sketch the deflected shape.
P5.40. (a) Draw the shear and moment curves for the frame in Figure P5.40. Sketch the deflected shape. (b) Write the equations for shear and moment in column AB. Take the origin at A. (c) Write the shear and moment equations for girder BC. Take the origin at joint B.

Segment ABC: Origin CD, Range 0 ≤ x₂ ≤ 2.4

\[ \Sigma F_y = 0; \]
\[ 24.8 - 2.4x_2 - V_y = 0 \]
\[ V_y = 24.8 - 2.4x_2 \]

\[ \Sigma M_o = 0; \]
\[ 24.8x_2 + 4'(15') - 2.4x_2 \left( \frac{x_2}{2} \right) - M_e = 0 \]
\[ M_e = 60 + 24.8x_2 - 1.2x_2^2 \]

Segment AB, Origin @ A, Range 0 ≤ x₁ ≤ 15'

\[ \Sigma M_o = 0; \quad +4x_1 - M_i = 0 \]
\[ M_i = 4k_1 \]

Segment CE
P5.47. For the frame in Figure P5.47, draw the shear and moment curves for all members. Next sketch the deflected shape of the frame. Show all forces acting on a free-body diagram of joint C.

**M Diagrams**

**Deflected Share**

**Joint C**
The beam shown is pin supported at point A; roller supported at points B, D, and F (note that roller supports resist movement both up and down); and has internal hinges at points C and E. Neglecting the weight of the beam, for the point loads applied at points C and G as shown:

1. Show that the beam is statically determinate and find the support reactions at points A, B, D, and F.

**Show that beam is statically determinate**

To evaluate determinacy, we must cut beam at supports and all places where we know an internal force (i.e. \( M = 0 \) at hinges at C and E.

**F.B.D.s**

\[
\begin{align*}
\text{X} &= \text{total number of unknowns} = 9 \\
\text{n} &= \text{number of F.B.D.} \\
3\text{n} &= \text{total number of equations of equilibrium} = 3(3) = 9 \\
\text{beam is statically determinate}
\end{align*}
\]
Find support reactions

F.B.D. of piece EFG

![F.B.D. diagram of piece EFG]

Apply equations of equilibrium to find unknown forces

Moment Equilibrium

\[ \sum M_E = 0 \]

Counterclockwise moments about point E positive

\[-(3 \text{k})(27 \text{f}t) + F_y (15 \text{f}t) = 0\]

\[ F_y = 5.4 \text{k} \]  (F_y is positive so F_y acts upward as assumed)

Force Equilibrium

\[ + \sum F_y = 0 \]

Upward forces positive

\[ V_E - 3 \text{k} + F_y = 0 \]
\[ V_E - 3 \text{k} + 5.4 \text{k} = 0 \]

\[ V_E = -2.4 \text{k} \]  (V_E is negative so V_E acts downward)

Force Equilibrium

\[ + \sum F_x = 0 \]

forces positive to the right
\[-F_E = 0\]

\[F_E = 0\] (no axial force in beam segment)

F.B.D. of piece CDE with known forces shown

Apply equations of equilibrium to find unknown forces

Moment Equilibrium

\[\sum M_C = 0\]

Counterclockwise moments about point C positive

\[(2.4 \, k)(27 \, ft) + D_y (15 \, ft) = 0\]

\[D_y = -4.32 \, k\] (\(D_y\) is negative so \(D_y\) acts downward)

Force Equilibrium

\[\sum F_y = 0\]

Upward forces positive

\[V_C + 2.4 \, k + D_y = 0\]

\[V_C + 2.4 \, k + (-4.32 \, k) = 0\]

\[V_C = 1.92 \, k\] (\(V_C\) is positive so \(V_C\) acts upward as assumed)

Force Equilibrium

\[\sum F_x = 0\]

forces positive to the right
\[ -F_c = 0 \]

\[ F_c = 0 \] (no axial force in beam segment)

F.B.D. of piece ABC with known forces shown

Apply equations of equilibrium to find unknown forces

Moment Equilibrium

\[ \sum M_A = 0 \]

\[ + \quad \text{Counterclockwise moments about point A positive} \]

\[-(2 \text{ k})(27 \text{ ft}) - (1.92 \text{ k})(27 \text{ ft}) + B_y (12 \text{ ft}) = 0 \]

\[ B_y = 8.82 \text{ k} \] (B_y is positive so B_y acts upward as assumed)

Force Equilibrium

\[ + \uparrow \sum F_y = 0 \]

Upward forces positive

\[ A_y + B_y - 2 \text{ k} - 1.92 \text{ k} = 0 \]

\[ A_y + 8.82 \text{ k} - 2 \text{ k} - 1.92 \text{ k} = 0 \]

\[ A_y = -4.9 \text{ k} \] (A_y is negative so A_y acts downward)
Force Equilibrium

\[ + \rightarrow \sum F_x = 0 \]

forces positive to the right

\[ A_x = 0 \]

\[ A_x = 0 \] (no axial force in beam segment)

F.B.D. of entire beam showing support reactions

![F.B.D. diagram of entire beam showing support reactions](image)
2. Construct the internal shear and bending moment diagrams for the beam. Label all local maximum and minimum values and their locations. Use the “usual” Civil Engineering sign convention for each of your diagrams (i.e. the sign convention we used in Lab #2). Show your results on the axes provided on the next page.

Find internal shear and moment from A to B
Make a cut at an arbitrary point between points A and B

F.B.D of beam to left of cut, show unknown internal forces V and M in their positive sense

![Diagram of beam with forces and moments]

Moment Equilibrium of segment

\[ \sum M_a = 0 \]

Clockwise moments about point a are positive

\[ (4.9)(x) + M = 0 \]

\[ M = -4.9x \text{ (units in k and ft)} \]

Force Equilibrium

\[ +\uparrow \sum F_y = 0 \]

Upward forces positive

\[ -4.9 - V = 0 \]

\[ V = -4.9 \text{ (units in k)} \]
In general, \( F = 0 \) for all of the beam segments

Plot \( V \) and \( M \) between points \( A \) and \( B \)

At point \( A \)
Evaluate at \( x = 0 \)
\[
V_A = -4.9 \ k
\]
\[
M_A = -(4.9)(0) = 0 \ kf
\]

At point \( B \) just the left of point \( B \) (\( x = 11.99999\ldots \) ft)
Evaluate at \( x = 12 \) ft
\[
V_B = -4.9 \ k
\]
\[
M_B = -(4.9)(12) = -58.8 \ kf
\]

Note that \( M \) is linear in \( x \) between \( A \) and \( B \)

Shear between \( B \) and \( C \), \( C \) and \( D \), \( D \) and \( F \), \( F \) and \( G \) will be constant since there is no distributed load in these regions.

Find internal shear and moment from \( B \) to \( C \)

Make a cut at an arbitrary point between points \( B \) and \( C \)

F.B.D of beam to left of cut, show unknown internal forces \( V \) and \( M \) in their positive sense
Moment Equilibrium of segment

\[
\sum M_a = 0
\]

\[\text{Counterclockwise moments about point a are positive}\]

\[ (4.9)(x) - (8.82)(x - 12) + M = 0 \]

\[ M = 3.92x - 105.84 \text{ (units in k and ft)} \]

Force Equilibrium

\[ + \sum F_y = 0 \]

Upward forces positive

\[-4.9 + 8.82 - V = 0 \]

\[ V = 3.92 \text{ (units in k)} \]

Plot V and M between points B and C

At point B⁺ just the right of point B (x = 12.000…001 ft)
Evaluate at x = 12
\[ V_{B+} = 3.92 \text{ k} \]
\[ M_{B+} = (3.92)(12) - 105.84 = -58.8 \text{ k ft} \]

At point C⁻ just the left of point C (x = 26.999999… ft)
Evaluate at x = 27 ft
\[ V_{C-} = 3.92 \text{ k} \]
\[ M_{C-} = (3.92)(27) - 105.84 = 0 \text{ k ft} \]
Note that M is linear in $x$ between B and C, the bending moment should be zero at the internal hinge at C, and there is continuity in the moment diagram across point B.
Can also use the area under shear diagram to find moment at C

\[ M_C - M_B = \text{Area of V diagram between points B and C} \]

Area of shear diagram between B and C = (3.92 k) (15 ft) = 58.8 k-ft

\[ M_C - M_B = 58.8 \text{ k-ft} \]
\[ M_C = M_B + 58.8 \text{ k-ft} \]
\[ M_C = -58.8 + 58.8 = 0 \text{ k-ft} \]

Shear between C and D, D and F, and F and G will be constant since there is no distributed load in these regions. Make cuts in these regions and fill in the rest of the shear diagram.

Find internal shear from C to D

F.B.D of beam to left of cut between C and D, showing unknown internal forces V and M in their positive sense

<table>
<thead>
<tr>
<th>Force Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( + \sum F_y = 0 )</td>
</tr>
</tbody>
</table>

Upward forces positive

\[ -4.9 + 8.82 - 2 - V = 0 \]

\[ V = 1.92 \text{ (units in k)} \]
Find internal shear from D to F
F.B.D of beam to left of cut between D and F, showing unknown internal forces V and M in their positive sense

\[ + \sum F_y = 0 \]

Upward forces positive

\[-4.9 + 8.82 - 2 - 4.32 - V = 0 \]

\[ V = -2.4 \text{ (units in k)} \]

Find internal shear from F to G
F.B.D of beam to left of cut between F and G, showing unknown internal forces V and M in their positive sense

\[ + \sum F_y = 0 \]

Upward forces positive

\[-4.9 + 8.82 - 2 - 4.32 + 5.4 - V = 0 \]
\[ V = 3 \text{ (units in k)} \]

Note that we get the same result looking at the F.B.D. to the right of the cut

\[ \begin{align*}
V &= 3 \\
F &= 3 \text{ (units in k)}
\end{align*} \]

Force Equilibrium

\[ +\sum F_y = 0 \]

Upward forces positive

\[ V - 3 = 0 \]

\[ V = 3 \text{ (units in k, checks with previous result)} \]

Now use areas under shear diagram to complete the moment diagram

**Moment at point D**

Area of shear diagram between C and D = (1.92 k) (15 ft) = 28.8 k-ft

\[ M_D - M_C = 28.8 \text{ k-ft} \]
\[ M_D = M_C + 28.8 \text{ k-ft} \]
\[ M_D = 0 + 28.8 \text{ k-ft} \]

\[ M_D = 28.8 \text{ k-ft} \]

M is linear in between C and D

**Moment at point E**

Area of shear diagram between D and E = (-2.4 k) (12 ft) = -28.8 k-ft

\[ M_E - M_D = -28.8 \text{ k-ft} \]
\[ M_E = M_D - 28.8 \text{ k-ft} \]
\[ M_E = 28.8 - 28.8 \text{ k-ft} \]

\[ M_E = 0 \text{ k-ft} \]

M is linear in between D and E and the bending moment should be zero at the internal hinge at E
Moment at point F

Area of shear diagram between E and F = (-2.4 k) (15 ft) = -36 k-ft

\[ M_F - M_E = -36 \text{ k-ft} \]
\[ M_F = M_E - 36 \text{ k-ft} \]
\[ M_F = 0 - 36 \text{ k-ft} \]

\[ M_F = -36 \text{ k-ft} \]

M is linear in between E and F

Moment at point G

Area of shear diagram between F and G = (3 k) (12 ft) = 36 k-ft

\[ M_G - M_F = 36 \text{ k-ft} \]
\[ M_G = M_F + 36 \text{ k-ft} \]
\[ M_G = -36 + 36 \text{ k-ft} \]

\[ M_G = 0 \text{ k-ft} \]

M is linear in between F and G and the bending moment should be zero at the free end at point G.
Plot $V$ and $M$ diagrams under the F.B.D of the beam

Note that a shear discontinuity will be expected at $B$, $C$, $D$, and $F$ due to the point loads.
Note the differential and integral relationships between $V$, $M$, and $w$:

\[\frac{dV}{dx} = w \quad [\text{slope of tangent to } V \text{ diagram at a point} = \text{distributed load intensity at that point}]\]

\[\frac{dM}{dx} = V \quad [\text{slope of tangent to } M \text{ diagram at a point} = \text{value of } V \text{ at that point}]\]

$V_B - V_A = \text{Area of distributed load between points } A \text{ and } B$

$M_B - M_A = \text{Area of } V \text{ diagram between points } A \text{ and } B$