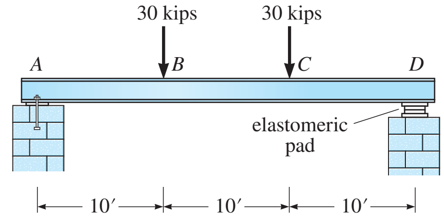
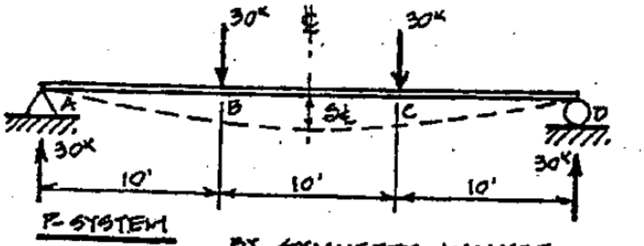
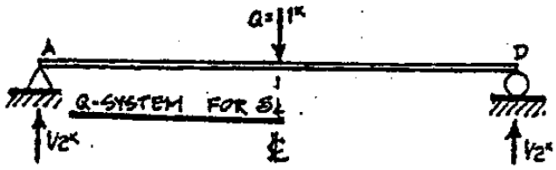
P8.19. Compute the deflection at midspan and the slope at A in Figure P8.19. EI is constant. Express the slope in degrees and the deflection in inches. Assume a pin support at A and a roller at D. $E = 29{,}000$ kips/in.^{2}, $I = 2000$ in.^{4}.



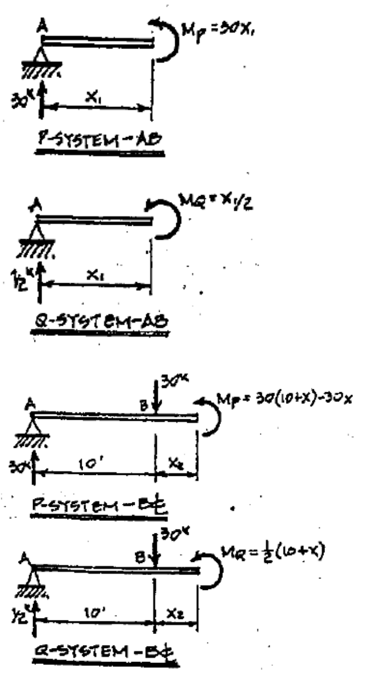
P8.19



BY SYMMETRY ANALYZE HALF OF THE STRUCTURE.



Compute δ £ (Midspan) Diagram	Segment	Orgin	Range	M_P	M_Q
	AB (= CD)	A	$0 \le x_1 < 10$	$30x_1$	$x_1/2$
	$\delta_{\text{center line}}$	B	$0 \le x_2 \le 5$	300	$5 + x_1/2$



$$W_Q = U_Q$$

$$1^k \cdot \delta_{\text{center line}} = \Sigma \int M_Q M_P \frac{dx}{EI}$$

$$\delta_{\text{center line}} = 2\int_{x=0}^{x=10} \frac{x}{2}(30x)\frac{dx}{EI} + 2\int_{x=0}^{x=5}\left(5+\frac{x}{2}\right)300\frac{dx}{EI}$$

$$\delta_{\text{center line}} = \frac{30}{EI}\left[\frac{x^2}{3}\right]_0^{10} + \frac{600}{EI}\left[5x+\frac{x^2}{4}\right]_0^{5}$$

$$\delta_{\text{center line}} = \frac{10{,}000}{EI} + \frac{18750}{EI}$$

$$\delta_{\text{center line}} = \frac{28{,}750(1728)}{29{,}000(2000)}$$

$$\boxed{\delta £ = 0.86 \text{ in} \downarrow}$$

P8.19. Continued

Compute θ_A:

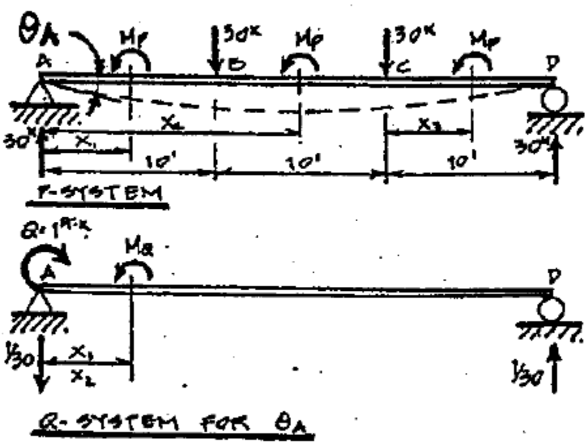
Orgin	Range	M_P	M_Q
A	$0 \le x_1 \le 10$	$30x$	$1 - x/30$
A	$10 \le x_2 \le 20$	300	$1 - x/3$
C	$0 \le x_3 \le 10$	$30x$	$x/30$

$$\theta_A = \int_0^{10} (30x)\left(1-\frac{x}{30}\right)\frac{dx}{EI} + \int_{10}^{20} (300)\left(1-\frac{x}{30}\right)\frac{dx}{EI} + \int_0^{10} (30x)\left(\frac{x}{30}\right)\frac{dx}{EI}$$

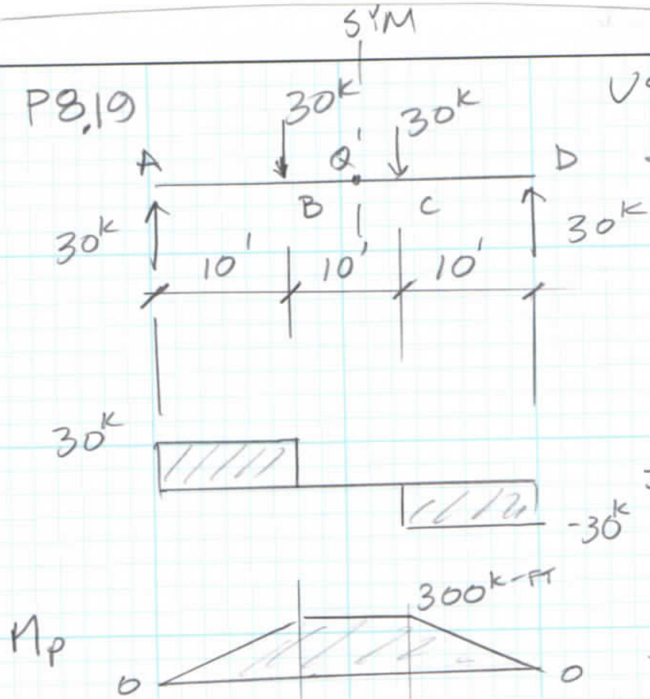
Integrating Gives:

$$\theta_A = \frac{3000}{EI} = \frac{3000(144)}{29000(2000)} = 0.00745 \text{ Radians}$$

$$\boxed{\theta_A = 0.43 \text{ Degrees}}$$

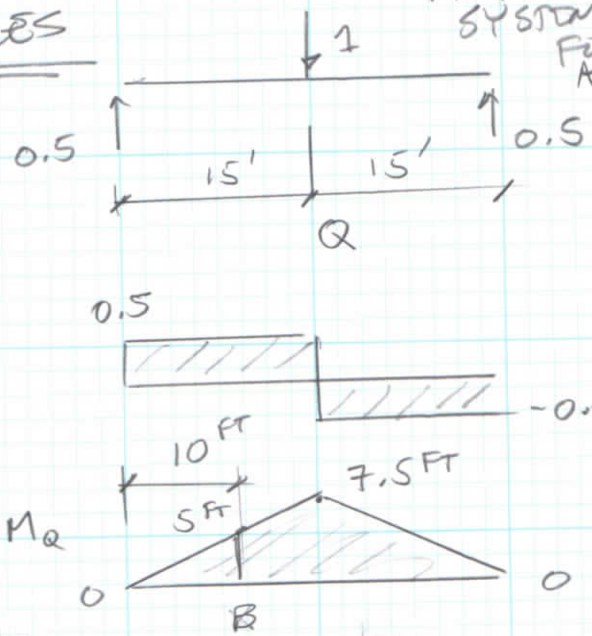


P8.19



USING TABLES

VIRTUAL SYSTEM FOR δ AT MIDSPAN



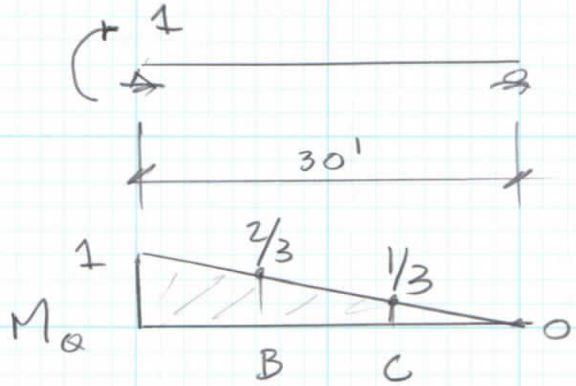
$$1 \cdot \delta = \left[\frac{1}{EI} \int^{L_{AB}} M_P M_Q \, dx + \frac{1}{EI} \int^{L_{BQ}} M_P M_Q \, dx \right] * 2$$ ← DUE TO SYMMETRY

$$\delta = \frac{1}{EI}\left[\left(\frac{1}{3}\right) \underbrace{(5^{FT})}_{M_1} \underbrace{(300^{k\text{-}FT})}_{M_3} \underbrace{(10^{FT})}_{L} + \frac{1}{EI} \left(\frac{1}{2}\right) (\underbrace{5^{FT}}_{M_1} + \underbrace{7.5^{FT}}_{M_2}) \underbrace{(300^{k\text{-}FT})}_{M_3} \underbrace{(5^{FT})}_{L} \right] * 2$$

$$\delta = \frac{1}{EI}\left[5000^{k\text{-}FT^3} + 9375^{k\text{-}FT^3} \right] * 2$$

$$\delta = \frac{1}{EI}\left[10000^{k\text{-}FT^3} + 18750^{k\text{-}FT^3} \right]$$ ← SEE SOLN FOR REST.

VIRTUAL SYSTEM FOR θ_A



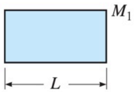
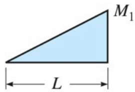
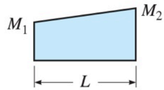
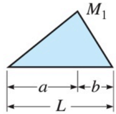
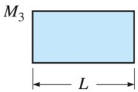
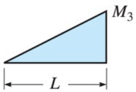
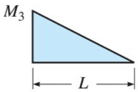
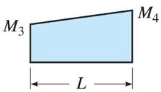
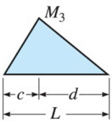
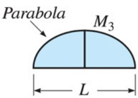
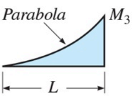
$$1\cdot\theta_A = \frac{1}{EI}\int_0^{L_{AB}} M_P M_Q\,dx + \frac{1}{EI}\int_0^{L_{BC}} M_P M_Q\,dx + \frac{1}{EI}\int_0^{L_{CD}} M_P M_Q\,dx$$

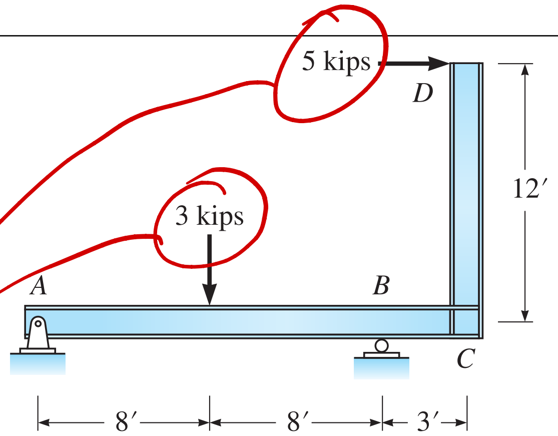
$$\theta_A = \frac{1}{EI}\Big[\frac{1}{6}\Big[(2)\overset{M_1}{(2/3)} + \overset{M_2}{1}\Big]\overset{M_3}{300^{k\text{-}ft}}\overset{L}{(10^{ft})} + \frac{1}{2}\Big(\overset{M_2}{\tfrac{2}{3}} + \overset{M_1}{\tfrac{1}{3}}\Big)\overset{M_3}{(300^{k\text{-}ft})}\overset{L}{(10^{ft})}$$

$$+ \frac{1}{3}\overset{M_1}{(1/3)}\overset{M_3}{(300^{k\text{-}ft})}\overset{L}{(10^{ft})}\Big]$$

$$\theta_A = \frac{1}{EI}\left(1166.67^{k\text{-}ft^2} + 1500^{k\text{-}ft^2} + 333.33^{k\text{-}ft^2}\right)$$

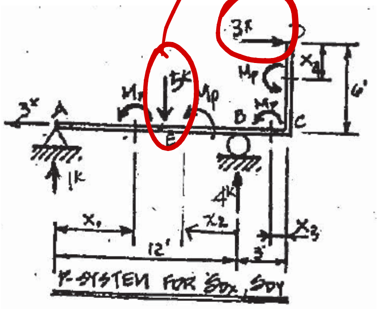
$$\theta_A = \frac{1}{EI}\left(3000^{k\text{-}ft^2}\right)$$ ← SEE SOLN FOR REST.

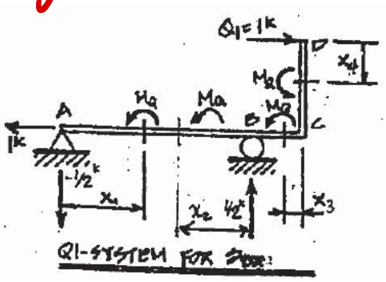
M_P \ M_Q				
	M_1M_3L	$\frac{1}{2}M_1M_3L$	δ_{BQ} $\boxed{\frac{1}{2}(M_1 + M_2)M_3L}$ θ_{ABC}	$\frac{1}{2}M_1M_3L$
	$\frac{1}{2}M_1M_3L$	δ_{AB} $\boxed{\frac{1}{3}M_1M_3L}$ θ_{ACD}	$\frac{1}{6}(M_1 + 2M_2)M_3L$	$\frac{1}{6}M_1M_3(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3L$	θ_{A} AB $\boxed{\frac{1}{6}(2M_1 + M_2)M_3L}$	$\frac{1}{6}M_1M_3(L + b)$
	$\frac{1}{2}M_1(M_3 + M_4)L$	$\frac{1}{6}M_1(M_3 + 2M_4)L$	$\frac{1}{6}M_1(2M_3 + M_4)L$ $+ \frac{1}{6}M_2(M_3 + 2M_4)L$	$\frac{1}{6}M_1M_3(L + b)$ $+ \frac{1}{6}M_1M_4(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3(L + c)$	$\frac{1}{6}M_1M_3(L + d)$ $+ \frac{1}{6}M_2M_3(L + c)$	for $c \leq a$: $\left(\frac{1}{3} - \frac{(a-c)^2}{6ad}\right)M_1M_3L$
	$\frac{2}{3}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{3}(M_1 + M_2)M_3L$	$\frac{1}{3}M_1M_3\left(L + \frac{ab}{L}\right)$
	$\frac{1}{3}M_1M_3L$	$\frac{1}{4}M_1M_3L$	$\frac{1}{12}(M_1 + 3M_2)M_3L$	$\frac{1}{12}M_1M_3\left(3a + \frac{a^2}{L}\right)$

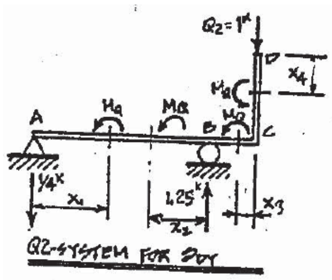
P8.28. Compute the horizontal and vertical components of deflection at point D in Figure P8.28. EI is constant, $I = 120$ in.^{4}, $E = 29{,}000$ kips/in.^{2}.	

P8.28

ERROR in soln !!







Ignore axial deformations, $E = 29000$ ksi $I = 120$ in.^{4}

Segment	Orgin	Range	M_{Q1}	M_{Q2}	M_p
AE	A	$0 \le x_1 \le 6'$	$-0.5x_1$	$-0.25x$	$1x_1$
EB	B	$0 \le x_2 \le 6'$	$0.5x_2 - 6$	$1.25x_2 - 3$	$4x_2 - 18$
CB	C	$0 \le x_3 \le 3'$	-6	$-1x_3$	-18
DC	D	$0 \le x_4 \le 6'$	$-1x_4$	0	$-3x_4$

$\boxed{\delta_{DX}}$ $\quad \Sigma Q \cdot \delta_p = \Sigma \int M_{Q_1} \cdot M_P \frac{dx}{EI}$

$$1^k \cdot \delta_{DX} = \frac{1728}{EI}\int_0^{6'}(-0.5x^2)\,dx + \int_0^{6'}(0.5x-6)(4x-18)\,dx + \int_0^{3'}(-6)(-18)\,dx + \int_0^{6}(-x)(-3x)\,dx$$

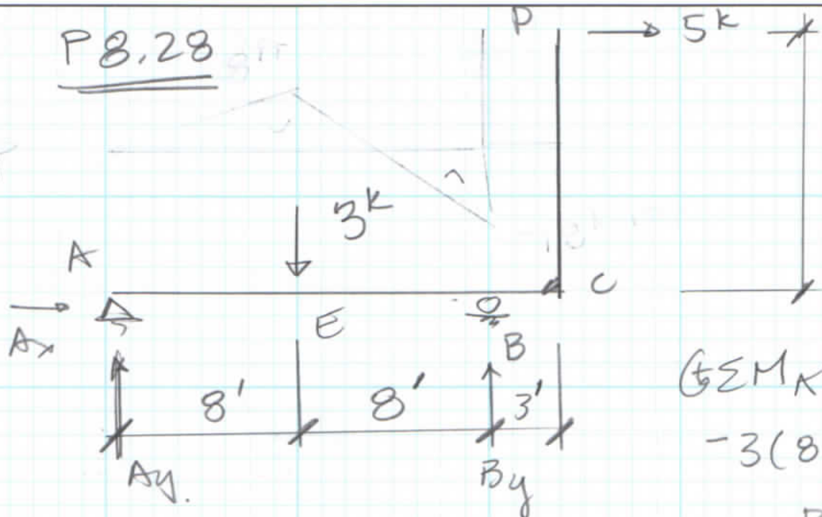
$$= \frac{1728}{EI}\left|-0.167x^3\right|^{6'} + (0.67x^3 - 16.5x^2 + 108x)\Big|^6 + 108x\Big|^3 + x^3\Big|^6 = \frac{702(1728)}{(120)(29000)} = \boxed{0.349'' \rightarrow}$$

$\boxed{\delta_{DX}}$

$$1^k \cdot \delta_{Dy} = \frac{1728}{EI}\int_0^{6'}(-0.25x^2)\,dx + \int_0^{6'}(1.25x-3)(4x-18)\,dx + \int_0^{3}18x\,dx$$

$$= -0.0833x^3\Big|^6 + (1.67x^3 - 17.25x^2 + 54x)\Big|^6 + 9x^2\Big|^3 \frac{1728}{EI} = \frac{126.7(1728)}{(120)(29000)} = 0.063'' \downarrow$$

P8.28



$\circlearrowleft \Sigma M_A = 0$

$-3(8) + B_y(16) - 5(12) = 0$

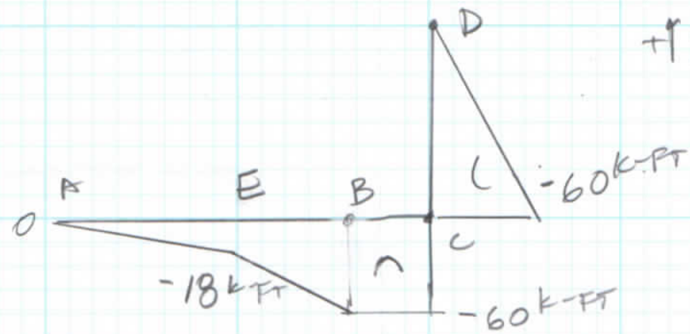
$B_y = \underline{5.25^k}$

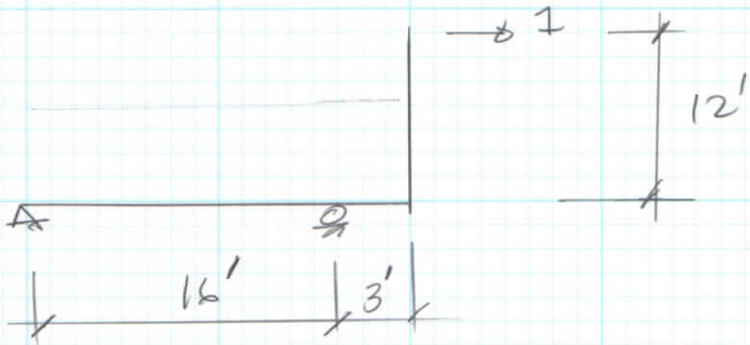
$+\uparrow \Sigma F_y = 0 \quad A_y + B_y - 3 = 0$

$A_y = 3 - 5.25$

$A_y = \underline{-2.25^k}$

$A_x = \underline{-5^k}$

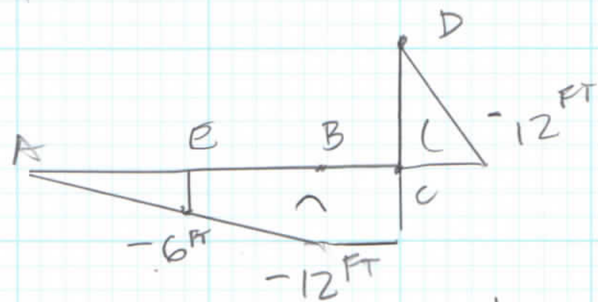




$\Sigma M_A = 0$

$-1(12') + B_y(16') = 0$

$B_y = 0.75$



$$1 \cdot \delta_H = \frac{1}{EI}\left[\int_0^{L_{AE}} M_P M_Q\,dx + \int_0^{L_{EB}} M_P M_Q\,dx + \int_0^{L_{BC}} M_P M_Q\,dx + \int_0^{L_{CD}} M_P M_Q\,dx\right]$$

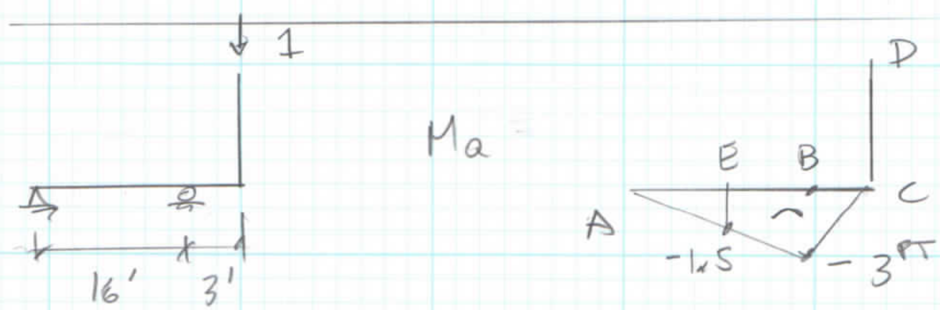
$$\delta_H = \frac{1}{EI}\Big[\left(\tfrac{1}{3}\right)\overset{M_1}{(-6)}\overset{M_3}{(-18)}\overset{L}{(8)} + \tfrac{1}{6}\overset{M_1}{(-18)}\left(2\overset{M_3}{(-6)}+\overset{M_4}{(-12)}\right)\overset{L}{(8)}$$

$$+\tfrac{1}{6}\overset{M_2}{(-60)}\left(\overset{M_3}{-6}+2\overset{M_4}{(-12)}\right)(8) + \overset{M_1}{(-60)}\overset{M_3}{(-12)}\overset{L}{(3)}$$

$$+\left(\tfrac{1}{3}\right)\overset{M_1}{(-60)}\overset{M_3}{(-12)}\overset{L}{(12)}\Big]^{L}$$

$$\delta_H = \frac{1}{EI}\left(288 + 576 + 2400 + 2160 + 2880\right)$$

$$\delta_H = \frac{8304^{k\text{-}FT^3}}{EI} = \frac{(8304)(12^3)}{(120)(29000)} = \underline{\underline{4.123^{IN}}} \rightarrow$$



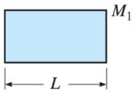
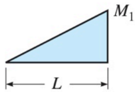
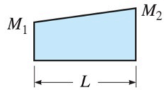
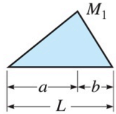
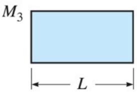
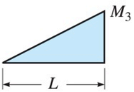
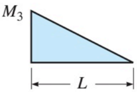
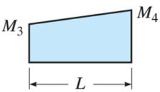
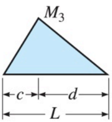
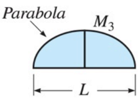
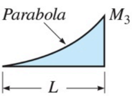
$$1\cdot\delta_V = \frac{1}{EI}\left[\int^{L_{AE}} M_P M_Q\,dx + \int^{L_{EB}} M_P M_Q\,dx + \int^{L_{BC}} M_P M_Q\,dx\right]$$

$$\delta_V = \frac{1}{EI}\Big[\tfrac{1}{3}\overset{M_1}{(-18)}\overset{M_3}{(-1.5)}\overset{L}{(8)} + \tfrac{1}{6}\overset{M_1}{(-18)}\left(2\overset{M_3}{(-1.5)}+\overset{M_4}{(-3)}\right)\overset{L}{(8)}$$

$$+\tfrac{1}{6}\overset{M_2}{(-60)}\left(\overset{M_3}{-1.5}+2\overset{M_4}{(-3)}\right)\overset{L}{(8)} + \tfrac{1}{2}\underset{M_1}{(-60)}\underset{M_3}{(-3)}\underset{L}{(3)}\Big]$$

$$\delta_V = \frac{1}{EI}\left[72 + 144 + 600 + 270\right]$$

$$\delta_V = \frac{1086^{k\text{-}FT^3}}{EI} = \frac{(1086)(12^3)}{(120)(29000)} = \underline{\underline{0.539^{IN}}} \downarrow$$

M_P \ M_Q				
	$\boxed{M_1M_3L}$ δ_H BC	$\boxed{\frac{1}{2}M_1M_3L}$ δ_V BC	$\frac{1}{2}(M_1+M_2)M_3L$	$\frac{1}{2}M_1M_3L$
	$\frac{1}{2}M_1M_3L$	δ_H AE CD $\boxed{\frac{1}{3}M_1M_3L}$ δ_V AE	$\frac{1}{6}(M_1+2M_2)M_3L$	$\frac{1}{6}M_1M_3(L+a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3L$	$\frac{1}{6}(2M_1+M_2)M_3L$	$\frac{1}{6}M_1M_3(L+b)$
	$\frac{1}{2}M_1(M_3+M_4)L$	$\frac{1}{6}M_1(M_3+2M_4)L$	δ_H EB $\boxed{\frac{1}{6}M_1(2M_3+M_4)L + \frac{1}{6}M_2(M_3+2M_4)L}$ δ_V EB	$\frac{1}{6}M_1M_3(L+b) + \frac{1}{6}M_1M_4(L+a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3(L+c)$	$\frac{1}{6}M_1M_3(L+d) + \frac{1}{6}M_2M_3(L+c)$	for $c \leq a$: $\left(\frac{1}{3}-\frac{(a-c)^2}{6ad}\right)M_1M_3L$
	$\frac{2}{3}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{3}(M_1+M_2)M_3L$	$\frac{1}{3}M_1M_3\left(L+\frac{ab}{L}\right)$
	$\frac{1}{3}M_1M_3L$	$\frac{1}{4}M_1M_3L$	$\frac{1}{12}(M_1+3M_2)M_3L$	$\frac{1}{12}M_1M_3\left(3a+\frac{a^2}{L}\right)$