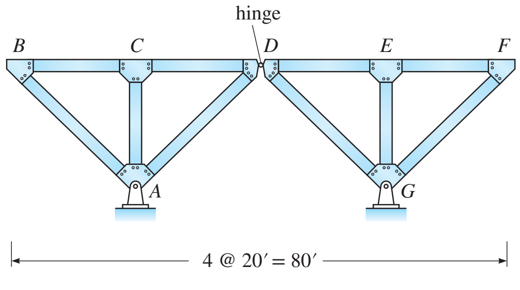
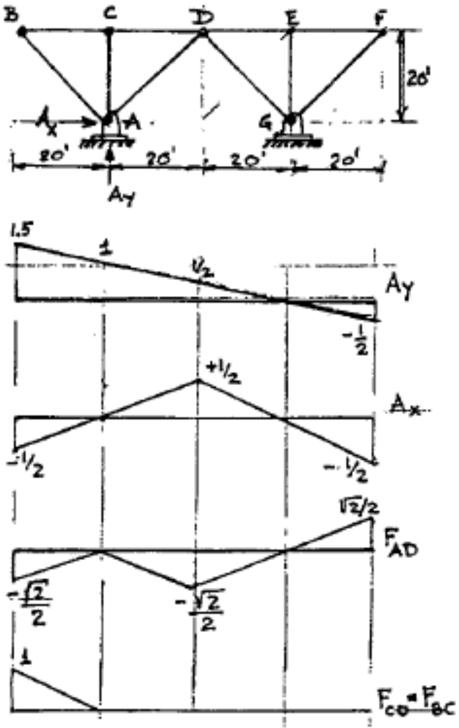
P12.36. Draw the influence lines for the vertical and horizontal reactions, A_X and A_Y, at support A and the bar forces in members AD, CD, and BC. If the truss is loaded by a uniform dead load of 4 kips/ft over the entire length of the top chord, determine the magnitude of the bar forces in members AD and CD.



P12.36

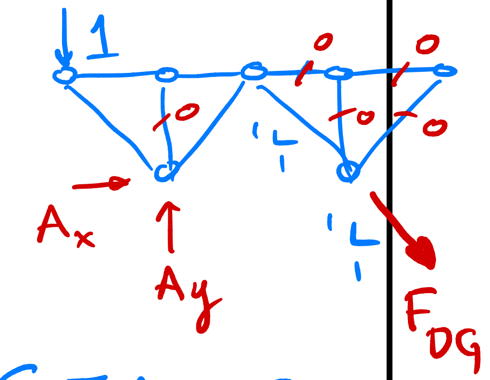


$$F_{BC} = F_{CD} = wA = 4^{\text{k/ft}}\left(\frac{1}{2}\right)(1)20 \boxed{= 40 \text{ kips}}$$

$$F^{*}_{AD} = \frac{1}{2}(40)\left(\frac{-\sqrt{2}}{2}\right)4 \text{ kips/ft} \qquad \boxed{= -40\sqrt{2} \text{ kips}}$$

Note* Positive and negative areas on ends cancel out.

KEY IS
ZFM



$\circlearrowleft + \; \Sigma M_A = 0$

$1(20) - \frac{1}{\sqrt{2}} F_{DG}(40) = 0$

$F_{DG} = \frac{\sqrt{2}}{2}$

$\xrightarrow{+} \Sigma F_x = 0$

$A_x + \frac{1}{\sqrt{2}} \cancel{F_{DG}}^{\sqrt{2}/2} = 0$

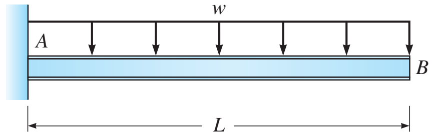
$\boxed{A_x = -1/2}$

$\uparrow \Sigma F_y = 0$

$-1 + A_y - \frac{1}{\sqrt{2}} \cancel{F_{DG}}^{\sqrt{2}/2} = 0$

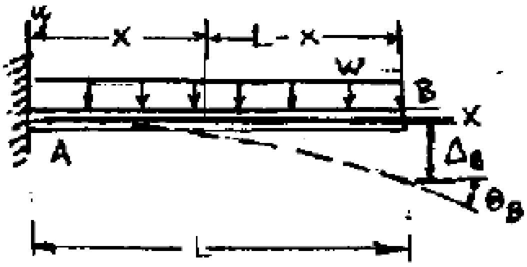
$\boxed{A_y = 1.5}$

P7.1. Derive the equations for slope and deflection for the beam in Figure P7.1. Compare the deflection at *B* with the deflection at midspan.



P7.1

Analysis by Double Integration



$$\frac{w(L-x)^2}{2} = M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{w}{2EI}(L^2 - 2xL + x^2)$$

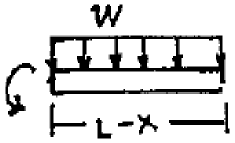
$$2EI\frac{d^2y}{dx^2} = -lL^2 + 2wxL - wx^2$$

$$2EI\frac{dy}{dx} = -wL^2x + 2wL\frac{x^2}{8} - \frac{wx^3}{3} + \cancel{C_1'}^{0} = 0 \qquad (1)$$

$$At \;\; x = 0, \; \frac{dy}{dx} = 0 \therefore C_1 = 0$$

$$2EIy = -\frac{wL^2x^2}{2} + \frac{wLx^3}{3} - \frac{wx^4}{12} + \cancel{C_2'}^{0} = 0 \qquad (2)$$

$$\text{At} \;\; x = 0, \; y = 0 \therefore C_2 = 0$$



$\underline{\text{Compute } \Delta_B;}$ Set $x = L$ in Eq(2)

$$\Delta_B = y = \frac{1}{2EI}\left(-\frac{wL^4}{2} + \frac{wL^4}{3} - \frac{wL^4}{12}\right)$$

$$\boxed{\Delta_B = -\frac{wL^4}{BEI}\downarrow}$$

$\underline{\text{Compute } \theta_B;}$ Set $x = L$ in Eq1

$$\theta_B = \frac{1}{2EI}\left(-wL^3 + wL^3 - \frac{wL^3}{3}\right)$$

$$\theta_B = \frac{wL^3}{2EI}\left(-1 + 1 - \frac{1}{3}\right) = -\frac{wL^3}{6EI}$$

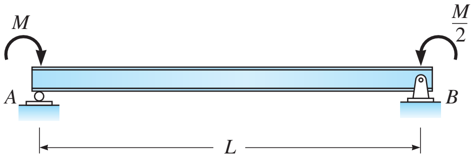
$$\boxed{\theta_B = -\frac{wL^3}{6EI}}$$

$\Delta \; at \; \frac{L}{2};$ Set $x = \frac{L}{2}$ in Eq2

$$\boxed{\Delta = -\frac{17wL^4}{384EI}\downarrow = -\frac{wL^4}{22.59}}$$

$$\text{Compare}\frac{\Delta \; at \; \frac{L}{2}}{\Delta \; at \; B} = \frac{\frac{1}{22.59}}{\frac{1}{8}} = 0.35$$

P7.3. Derive the equations for slope and deflection for the beam in Figure P7.3. Compute the maximum deflection. *Hint*: Maximum deflection occurs at point of zero slope.



P7.3

Compute $R_A : \overset{\curvearrowright +}{\Sigma M_B} = 0$

$$0 = M - \frac{M}{2} - R_A L$$

$$R_A = \frac{M}{2L}$$

Ecaluate $Mx : \Sigma M$

$$Mx = M - R_{Ax} = \boxed{M - \frac{M}{2L}x}$$

$$\frac{d^2y}{dx^2} = \frac{Mx}{EI} - \left(M - \frac{Mx}{2L}\right)\frac{1}{EI} \qquad (1)$$

$$EI\frac{dy}{dx} = Mx - \frac{Mx^2}{2L2} + C_1 \qquad (2)$$

$$EIy = \frac{Mx^2}{2} - \frac{Mx^3}{12L} + C1x + C_2 \qquad (3)$$

Substi $y = 0$ @ $x = 0$ in Eq (3)

$$0 = 0 - 0 + 0 + C_2$$

$$\therefore C_2 = 0$$

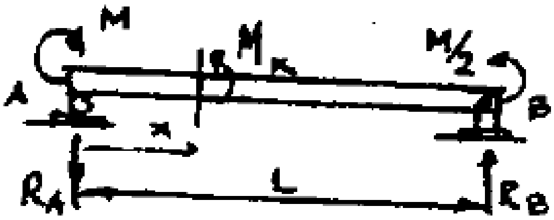
Substi $y = 0$ @ $x - L$ in Eq (3)

$$0 = \frac{ML^2}{2} - \frac{ML^2}{12} + C_1 L$$

$$C_1 = -\frac{5}{12}ML$$

$$EI\frac{dy}{dx} = Mx - \frac{Mx^2}{4L} - \frac{5}{12}ML \qquad (2a)$$

$$EIy = \frac{Mx^2}{2} - \frac{Mx^3}{12L} - \frac{5}{12}MLx \qquad (3a)$$



Compute $\Delta_{\max}$: Set $\frac{dy}{dx} = 0$ in Eq 2a to locate position $\Delta_{\max}$

$$0 = Mx - \frac{Mx^2}{4L} - \frac{5}{12}ML$$

$$x^2 - 4Lx = -\frac{5L}{12}(4L^2)$$

$$(x - 2L)^2 = \frac{7}{3}L^2$$

$$\boxed{x = 0.4725L}$$

Substi $x = 0.4725L$ into (Eq. 3a)

$$\Delta_{\max \cdot y} = \frac{M}{EI}\left[\frac{(0.4725L)^2}{2} - \frac{(0.4725L)^3}{12L} - \frac{5L(0.4725L)}{12}\right]$$

$$= \frac{-0.094ML^2}{EI}$$