Virtual Work Truss Example
Support Settlement
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Consider the idealized truss structure with a pin support at A and a roller support at C.

Find the horizontal displacement of point E due to the pin support settling 9.0 cm and moving 2.0 cm to the right using the Principle of Virtual Work.

For all truss members use:

\[ A = 25 \text{ cm}^2 \]
\[ E = 210 \text{ GPa} = 210 \text{ kN/mm}^2 \]
Virtual System to Measure $\delta_{Eh}$

1. Remove all loads from the structure;
2. Apply a unit, dimensionless virtual load \textbf{in-line} with the real displacement, $\delta_{Eh}$, that we want to find;
3. Perform a truss analysis to find all truss member support reactions, $R_Q$
Find Support Reactions

\[ \sum M_C = 0 \]

\[ A_y = -0.667 \]
Find Support Reactions

\[ A_x = -1.0 \]

\[ \sum F_x = 0 \]
Virtual System Support Reactions

- A to B: 1.0
- B to D: 0.667
- D to E: 0.667
- E to C: 0.667
- A to C: 0.667

Dimensions:
- AB = 3 m
- BC = 3 m
- AC = 4 m
Use the Principle of Virtual Work to Find $\delta_{Eh}$

$$1 \cdot \delta_{Eh} + \sum R_Q \delta_s = 0$$

From the virtual analysis

No internal work in this problem
Evaluate the Virtual Work Expression

\[ 1 \cdot \delta_{Eh} + \sum R_Q \delta_s = 0 \]

\[ 1 \cdot \delta_{Eh} + A_y \delta_{sv} - A_x \delta_{sh} = 0 \]

\[ \delta_{sv} \text{ and } A_y \text{ are in the same direction} \]

\[ \delta_{sh} \text{ and } A_x \text{ are in opposite directions} \]

Internal work is zero for this problem

\[ \delta_{Eh} + (0.667)(9.0 \text{ cm}) - (1.0)(2.0 \text{ cm}) = 0 \]

\[ \delta_{Eh} + 6.0 \text{ cm} - 2.0 \text{ cm} = 0 \]

\[ \delta_{Eh} = -4.0 \text{ cm} \]

\[ \delta_{Eh} = 4.0 \text{ cm to the left} \]

\[ A_y \text{ and } A_x \text{ are the support reactions in the virtual system} \]

\[ \delta_{sv} \text{ and } \delta_{sh} \text{ are the real support movements} \]

Negative result, so deflection is in the opposite direction as the virtual unit load
Results for $\delta_{Eh}$

$\delta_{Eh} = 4.0 \text{ cm to the left}$