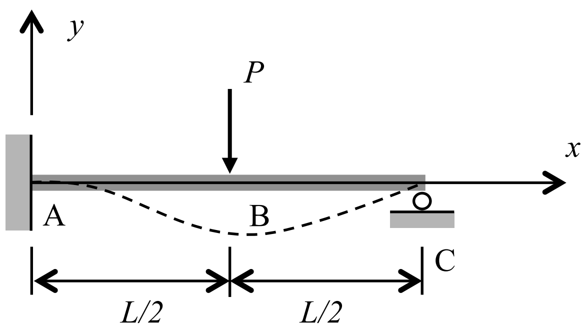
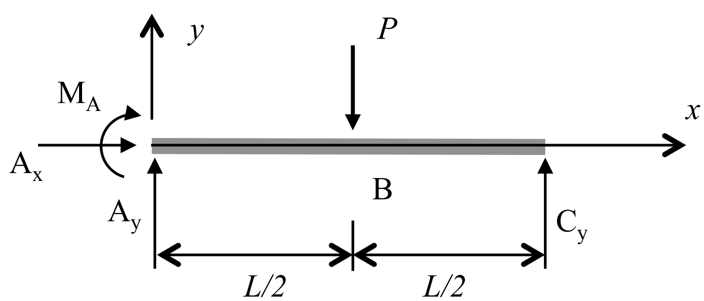
CE 160 Notes: Indeterminate Beam Example

For the indeterminate beam loaded with the point load, P, find the support reactions at A and B. EI is constant.



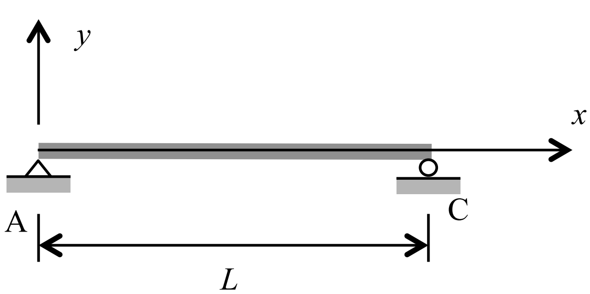
F.B.D neglecting axial effects



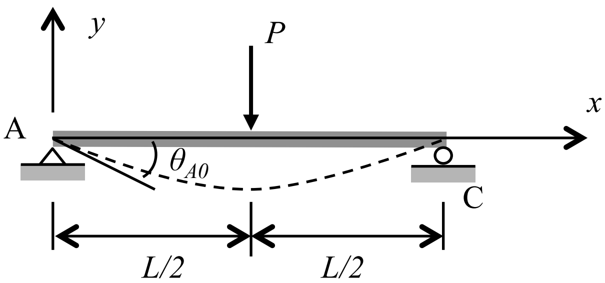
4 unknowns, 3 equations of equilibrium

Indeterminate to the First Degree

Define M_A as the redundant, so the released or primary structure is:



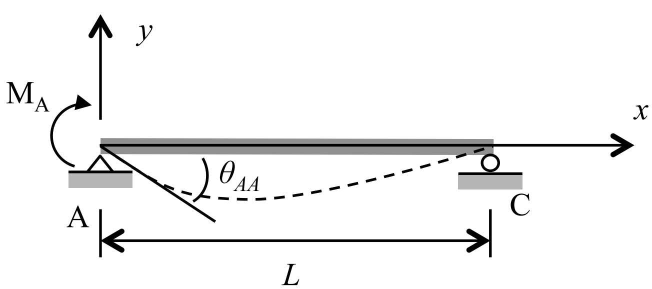
Released Structure Problem – Need to find the rotation at point A, θ_{A0}

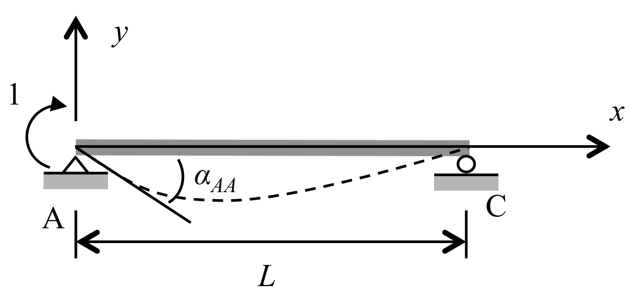


Use the solutions tabulated in the text Figure 11.3

$$\boxed{\theta_{A0} = -\frac{PL^2}{16EI}}$$

Redundant Problem – Need to find the flexibility coefficient, α_{AA}

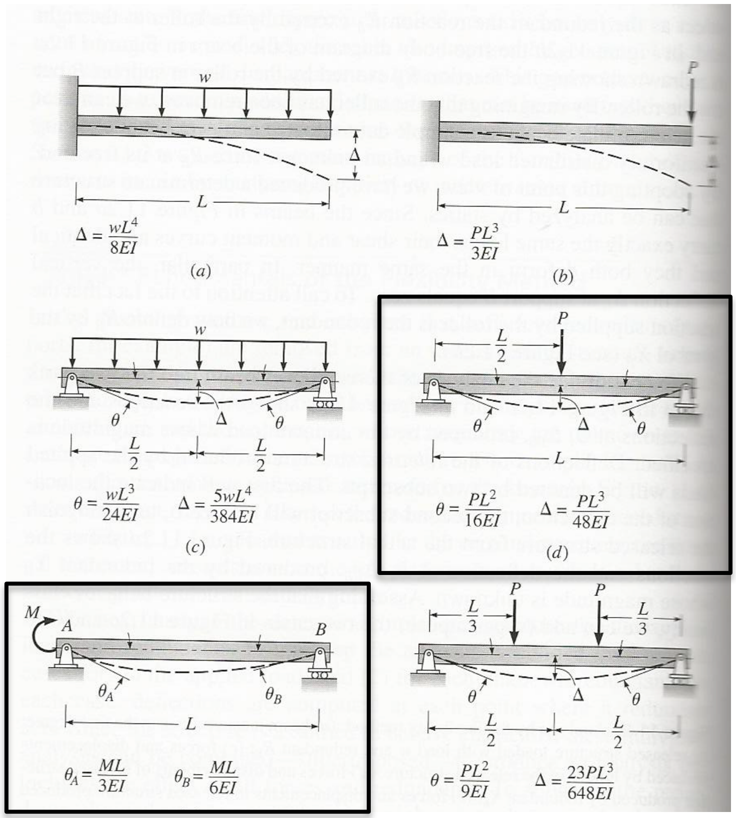




$$\theta_{AA} = M_A \alpha_{AA}$$

Use the solutions tabulated in the text Figure 11.3

$$\boxed{\alpha_{AA} = -\frac{L}{3EI}}$$



Compatibility Equation at Point A

$$\theta_{A0} + M_A \alpha_{AA} = \theta_A = 0$$

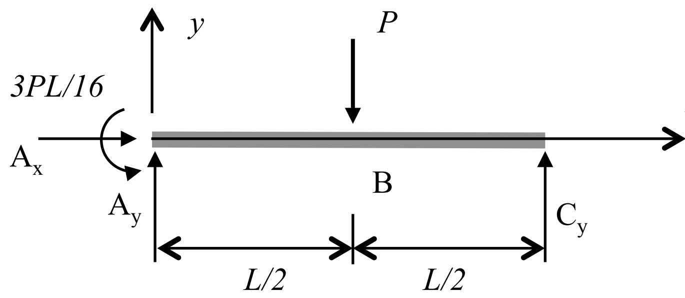
Substituting released problem deflections and flexibility coefficient:

$$-\frac{PL^2}{16EI} + M_A\left(-\frac{L}{3EI}\right) += 0$$

$$M_A = \frac{PL^2}{16EI}\left(-\frac{3EI}{L}\right)$$

Solving the system of compatibility equations yields:

$$\boxed{M_A = -\frac{3PL}{16}}$$



We can find the other reactions using equilibrium;

$$+\rightarrow \sum F_x = 0$$

$$\boldsymbol{A_x = 0}$$

$$\sum M_A = 0$$

$+\circlearrowleft$ Counterclockwise moments about point A are positive

$$\frac{3PL}{16} - P\left(\frac{L}{2}\right) + C_y(L) = 0$$

$\boldsymbol{C_y = \frac{5}{16}P}$ (C_y is positive so it acts upward as assumed)

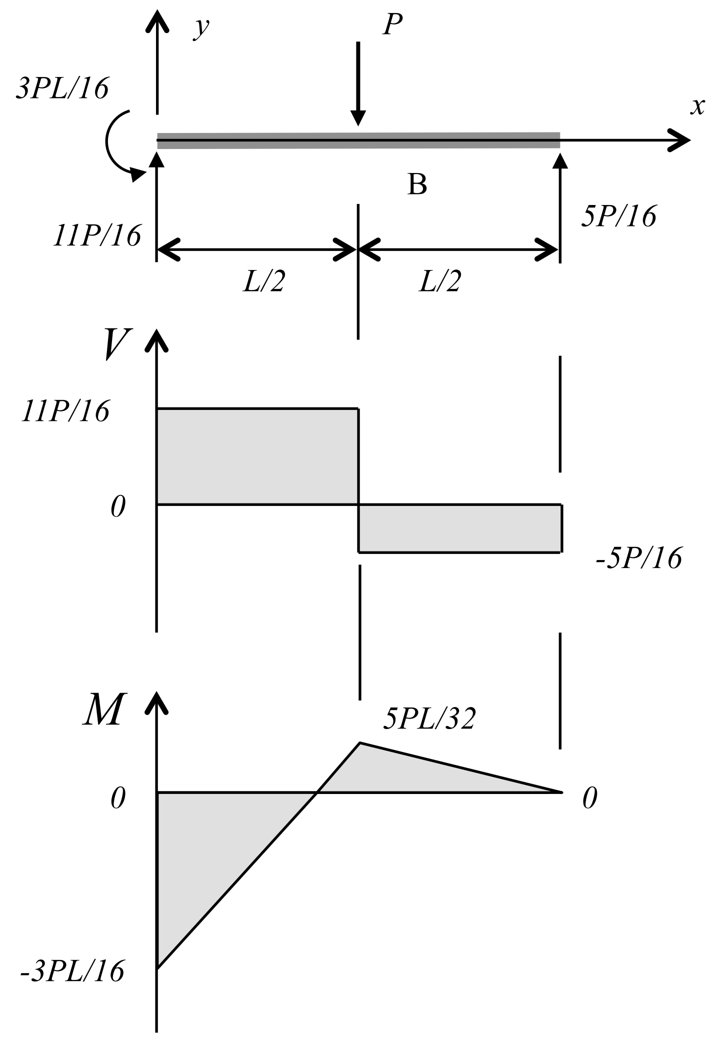
$$+\uparrow \sum F_y = 0$$

$$A_y + C_y - P = 0$$

$$A_y + \frac{5}{16}P - P = 0$$

$\boldsymbol{A_y = \frac{11}{16}P}$ (A_y is positive so it acts upward as assumed)

Shear and bending moment diagrams for the indeterminate beam.



We can find the maximum positive moment at point B using the area under the shear diagram

$$M_B - M_A = \left(\frac{11P}{16}\right)\left(\frac{L}{2}\right)$$

$$M_B = -\frac{3PL}{16} + \frac{11PL}{32} = \boldsymbol{\frac{5PL}{32}}$$