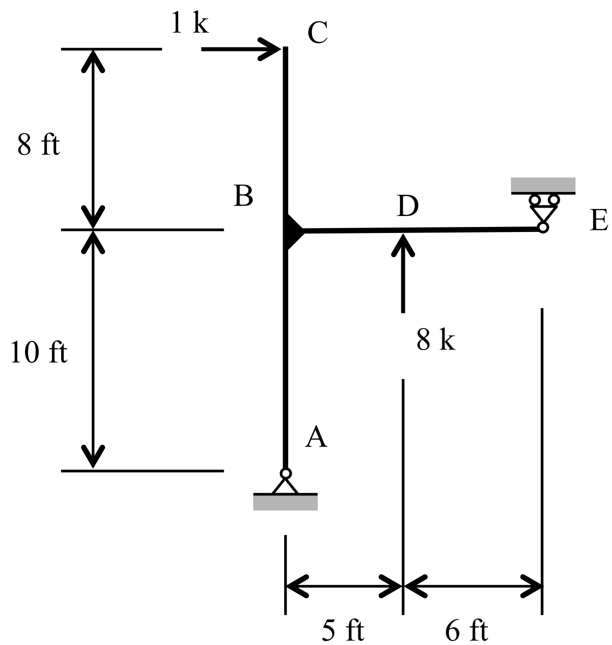
CE 160 Notes: Virtual Work Frame Example



The statically determinate frame from our internal force diagram example is made up of columns that are W8x48 (I = 184 in^4) members and a beam that is a W10x22 (I = 118 in^4). The modulus of elasticity of structural steel is 29,000 ksi, which yields the following bending stiffnesses:

- EI_{ABC} = 5,336,000 k-in^2
- EI_{BDE} = 3,422,000 k-in^2

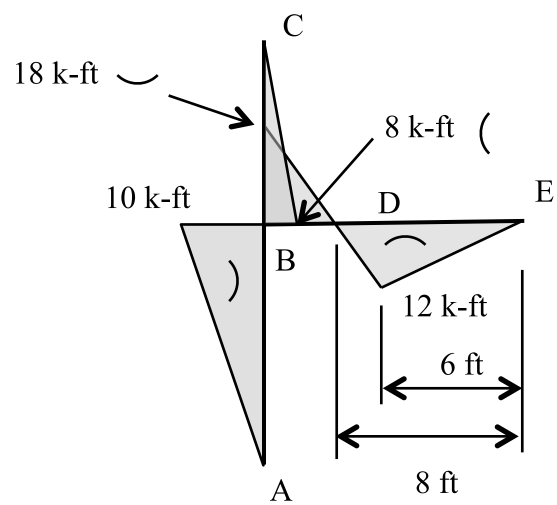
Find the horizontal displacement of point C using the method of virtual work.

Real Problem

We found the Shear, Moment, and Axial force diagrams for this frame in a previous example earlier in the semester.

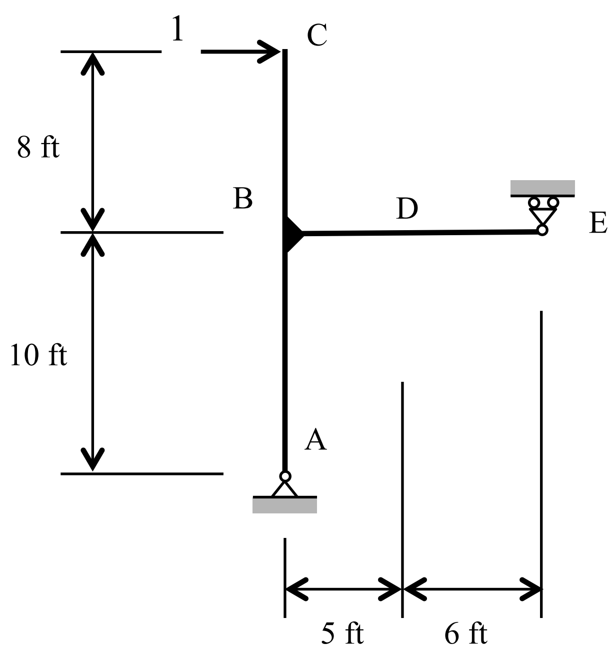
Moment Diagram of Real or P-System (M_P diagram)

(You should review your notes and verify this is the correct diagram)

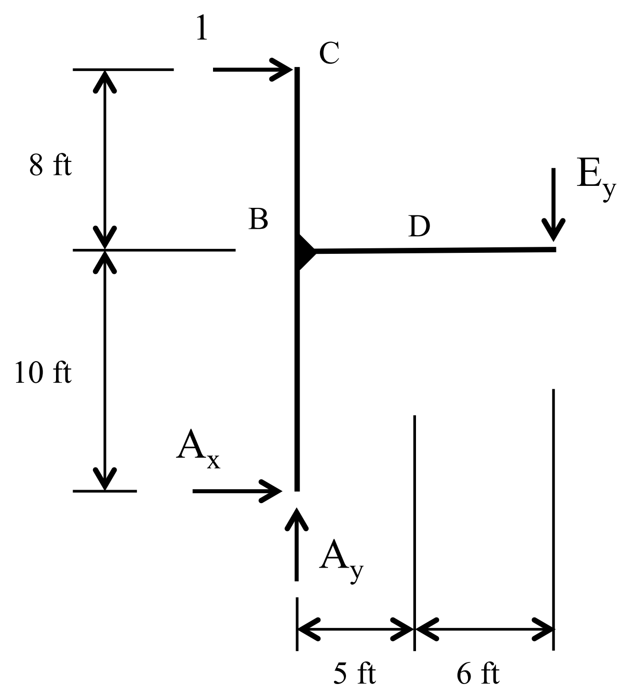


Virtual Problem

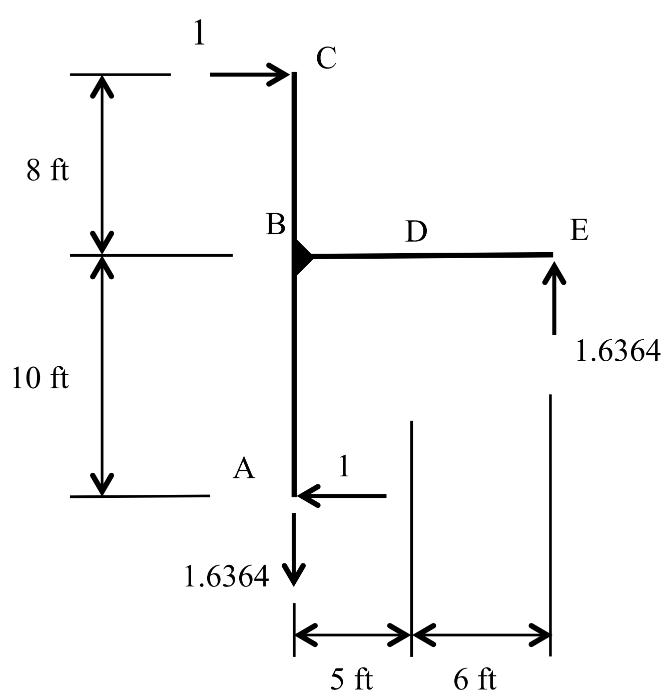
Virtual System to measure δ_C



Free Body Diagram of Virtual System (or Q-system)

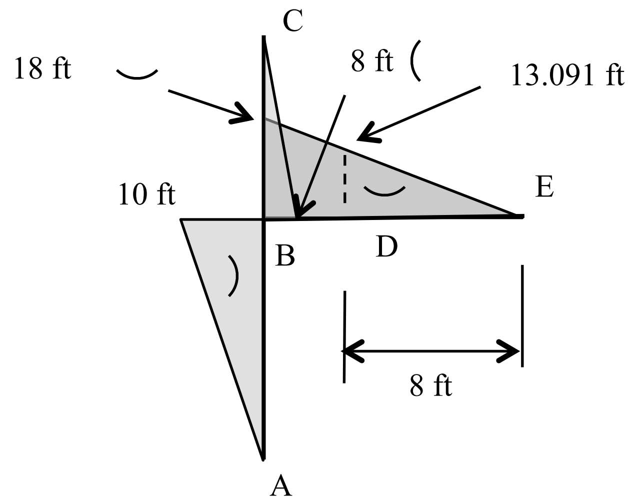


Equilibrium ($\sum M_A = 0;\ \sum F_y = 0;\ \sum F_x = 0$) yields the following support reactions:



Virtual Moment Diagram (M_Q diagram)

(You should be able to verify this is the correct diagram)

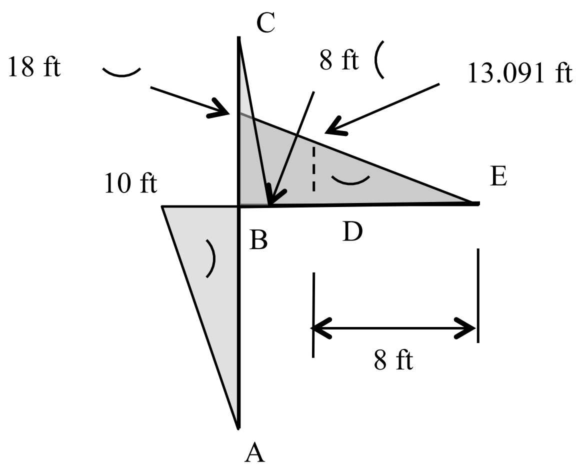


Use Table 4 in the text to evaluate the virtual work product integrals that come from the principle of virtual work:

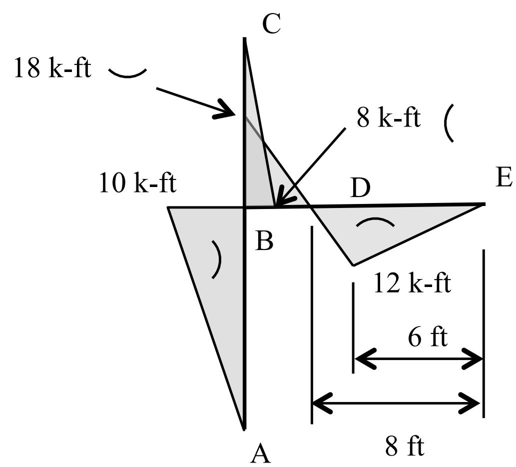
$$1 \cdot \delta_C = \frac{1}{EI}\int_0^L M_Q M_P \, dx$$

Integrate over three segments: AB, BC, and BDE

M_Q Diagram



M_P Diagram



Segment AB

From Virtual Work Integral Table (see page 8):

$$\frac{1}{3}M_1 M_3 L = \left(\frac{1}{3}\right)(10 \text{ ft})(10 \text{ k-ft})(10 \text{ ft})$$

$= 333.333$ k-ft^3

$= 576{,}000$ k-in^3

Segment BC

From Virtual Work Integral Table (see page 8):

$$\frac{1}{3}M_1 M_3 L = \left(\frac{1}{3}\right)(8 \text{ ft})(8 \text{ k-ft})(8 \text{ ft})$$

$=170.6667$ k-ft^3

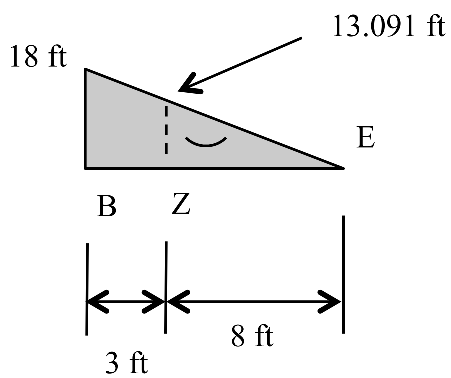
$= 294{,}912$ k-in^3

Segment BDE -- Split into two segments at inflection point of M_P diagram (point Z)

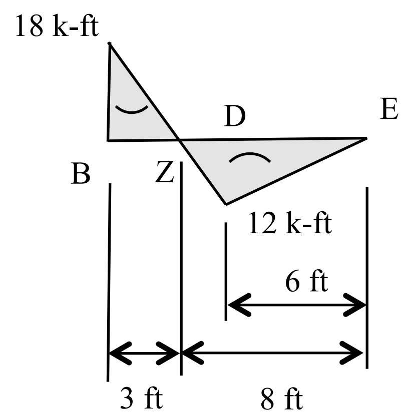
$$\frac{18\,ft}{x} = \left(\frac{12\,ft}{5 - x}\right)$$

$$x = 3 \text{ ft}$$

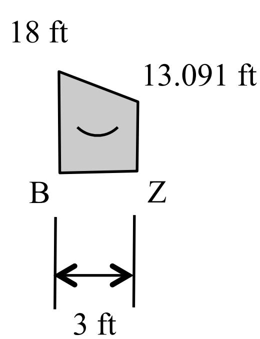
M_Q Diagram

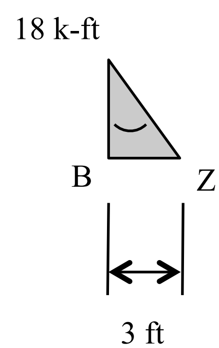


M_P Diagram



Segment BZ



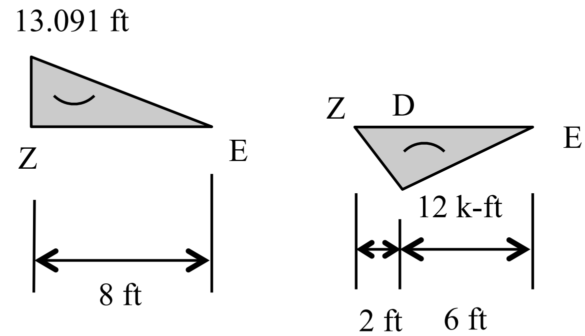


From Virtual Work Integral Table (see page 8):

$$\frac{1}{6}(M_1 + 2M_2)M_3L = \left(\frac{1}{6}\right)[13.091 \text{ ft} + 2(18 \text{ ft})](18 \text{ k-ft})(3 \text{ ft})$$

$$= 441.818 \text{ k-ft}^3$$
$$= \mathbf{763{,}461.8167 \text{ k-in}^3}$$

Segment ZDE



From Virtual Work Integral Table (see page 8):
Note that result is negative due to dissimilar curvatures

$$-\frac{1}{6}M_1M_3(L+c) = -\left(\frac{1}{6}\right)(13.091\text{ ft})(12\text{ k-ft})[8\text{ ft} + 6\text{ ft}]$$

= -366.5454 k-ft^3
= -633,390.54 k-in^3

Total for segment BDE

763,461.8167 k-in^3 - 633,390.54 k-in^3 = 130,071.28

Divide by EI of each segment

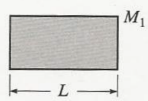
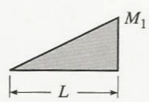
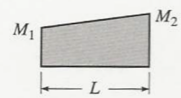
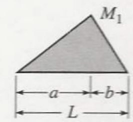
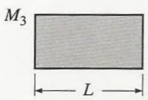
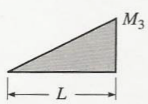
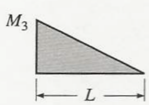
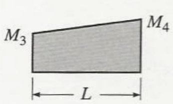
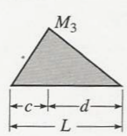
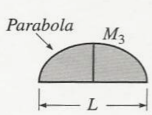
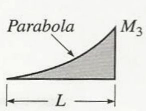
$$\delta_C = \frac{576{,}000\text{ k-in}^3}{5{,}336{,}000\text{ k-in}^2} + \frac{294{,}912\text{ k-in}^3}{5{,}336{,}000\text{ k-in}^2} + \frac{130{,}071.28\text{ k-in}^3}{3{,}422{,}000\text{ k-in}^2}$$

$$\delta_C = 0.1079\text{ in} + 0.05527\text{ in} + 0.03801\text{ in}$$

$$\boldsymbol{\delta_C = 0.201\text{ in}}$$
Result is positive, so deflection is in the direction of the unit load – **to the right**

Product Integral Evaluation using Table 4 in text:

Table 4: Values of Product Integrals $\int_{x=0}^{x=L} M_Q M_P \, dx$

M_Q / M_P				
	M_1M_3L	$\frac{1}{2}M_1M_3L$	$\frac{1}{2}(M_1 + M_2)M_3L$	$\frac{1}{2}M_1M_3L$
	$\frac{1}{2}M_1M_3L$	$\boxed{\frac{1}{3}M_1M_3L}$	$\boxed{\frac{1}{6}(M_1 + 2M_2)M_3L}$	$\frac{1}{6}M_1M_3(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3L$	$\frac{1}{6}(2M_1 + M_2)M_3L$	$\frac{1}{6}M_1M_3(L + b)$
	$\frac{1}{2}M_1(M_3 + M_4)L$	$\frac{1}{6}M_1(M_3 + 2M_4)L$	$\frac{1}{6}M_1(2M_3 + M_4)L$ $+\frac{1}{6}M_2(M_3 + 2M_4)L$	$\frac{1}{6}M_1M_3(L + b)$ $+\frac{1}{6}M_1M_4(L + a)$
	$\frac{1}{2}M_1M_3L$	$\boxed{\frac{1}{6}M_1M_3(L + c)}$	$\frac{1}{6}M_1M_3(L + d)$ $+\frac{1}{6}M_2M_3(L + c)$	for $c \le a$: $\left(\frac{1}{3} - \frac{(a - c)^2}{6ad}\right)M_1M_3L$
	$\frac{2}{3}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{3}(M_1 + M_2)M_3L$	$\frac{1}{3}M_1M_3\left(L + \frac{ab}{L}\right)$
	$\frac{1}{3}M_1M_3L$	$\frac{1}{4}M_1M_3L$	$\frac{1}{12}(M_1 + 3M_2)M_3L$	$\frac{1}{12}M_1M_3\left(3a + \frac{a^2}{L}\right)$